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(54) **ADAPTIVE ADJUSTMENT OF TIME AND FREQUENCY DOMAIN EQUALIZERS IN COMMUNICATIONS SYSTEMS**  
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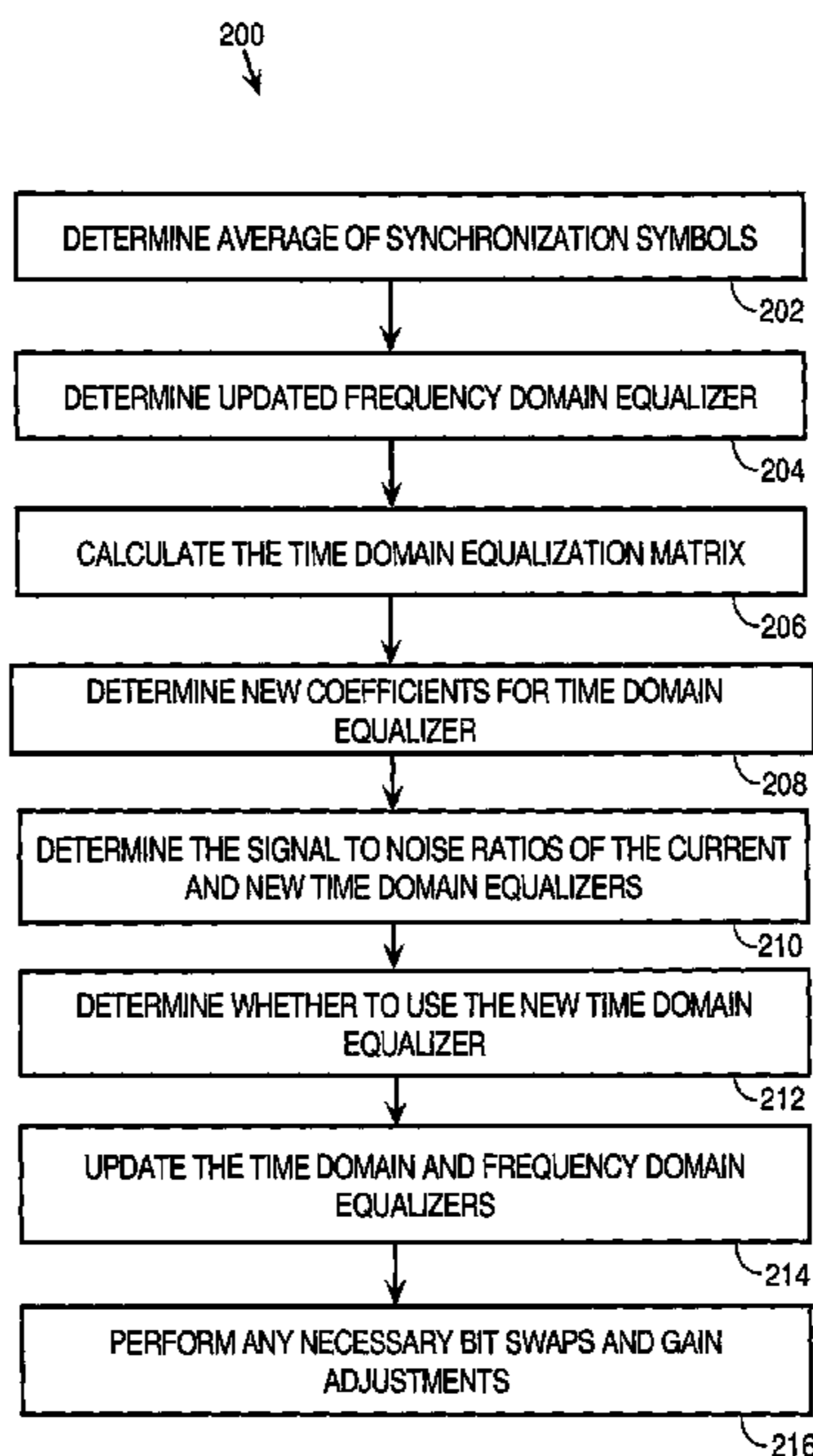
**Related U.S. Patent Documents**  
Reissue of:  
(64) Patent No.: **7,272,199**  
Issued: **Sep. 18, 2007**  
Appl. No.: **10/057,133**  
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(51) **Int. Cl.**  
**H04B 1/10** (2006.01)  
**H03H 7/30** (2006.01)  
**H04L 27/28** (2006.01)  
(52) **U.S. Cl.** ..... **375/350; 375/229; 375/260**  
(58) **Field of Classification Search** ..... **375/222, 375/229, 231-233, 260, 346, 348, 350; 370/210; 708/402-405**  
See application file for complete search history.

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(57) **ABSTRACT**  
An update mechanism and approach are provided for configuring a communications receiver. According to the approach, a time domain equalizer and a frequency domain equalizer in a communications receiver are dynamically updated based upon performance data that indicates the performance of a communications channel from which the communications receiver receives data. This approach accounts for changes in the communications channel attributable to changes in the transmission medium or changes in interference sources.

**44 Claims, 4 Drawing Sheets**



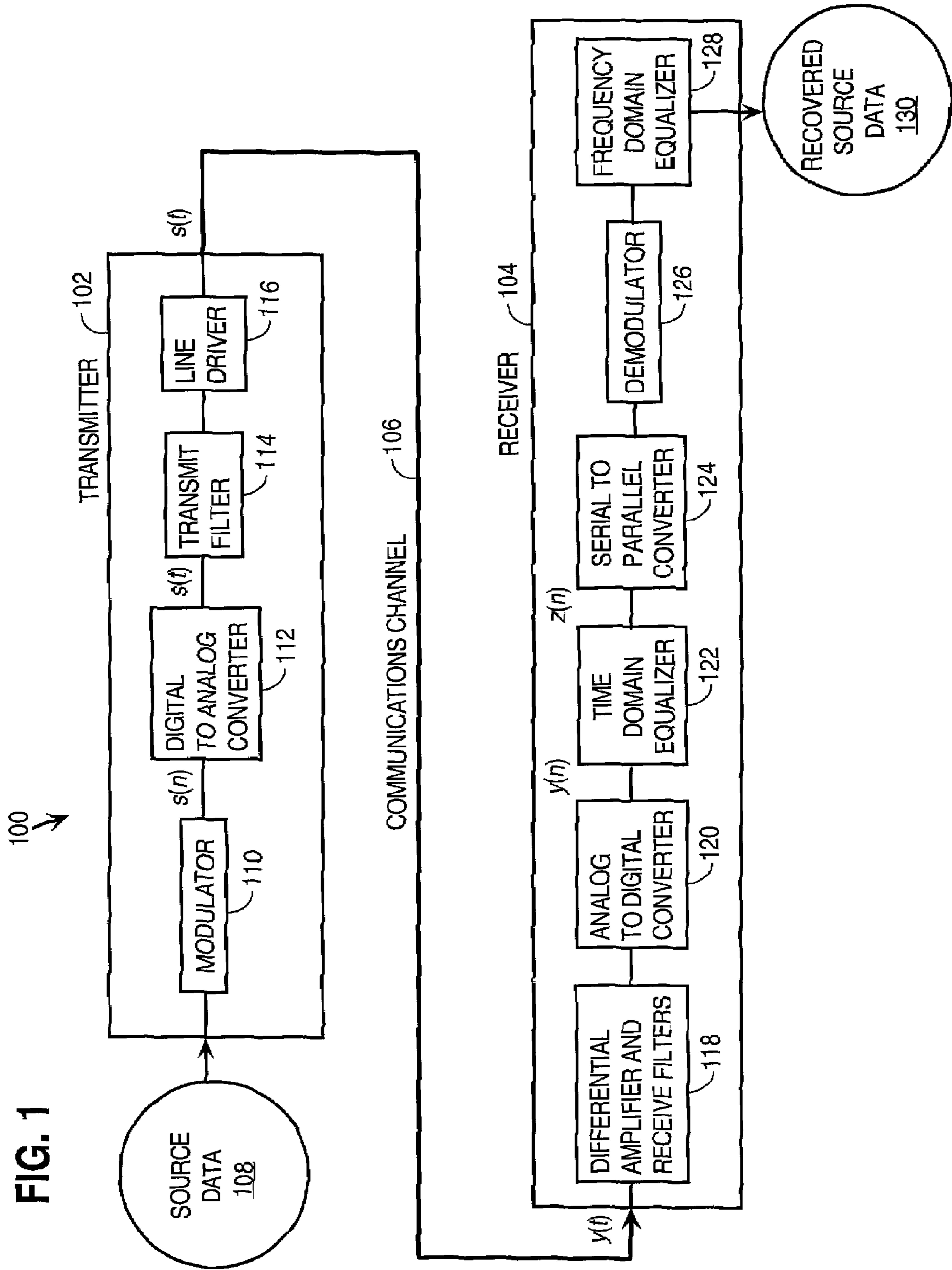


FIG. 2

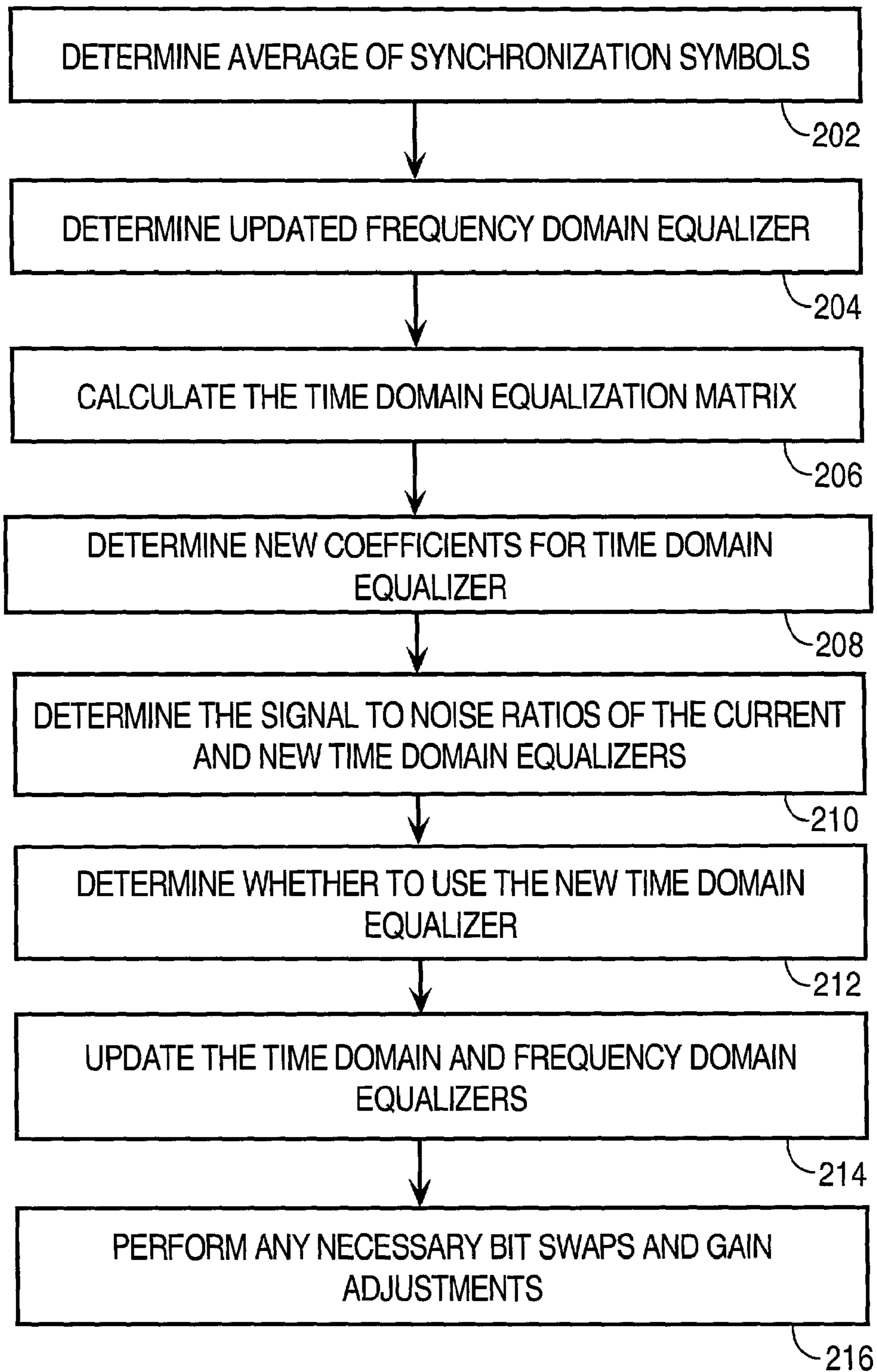
200  
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FIG. 3A

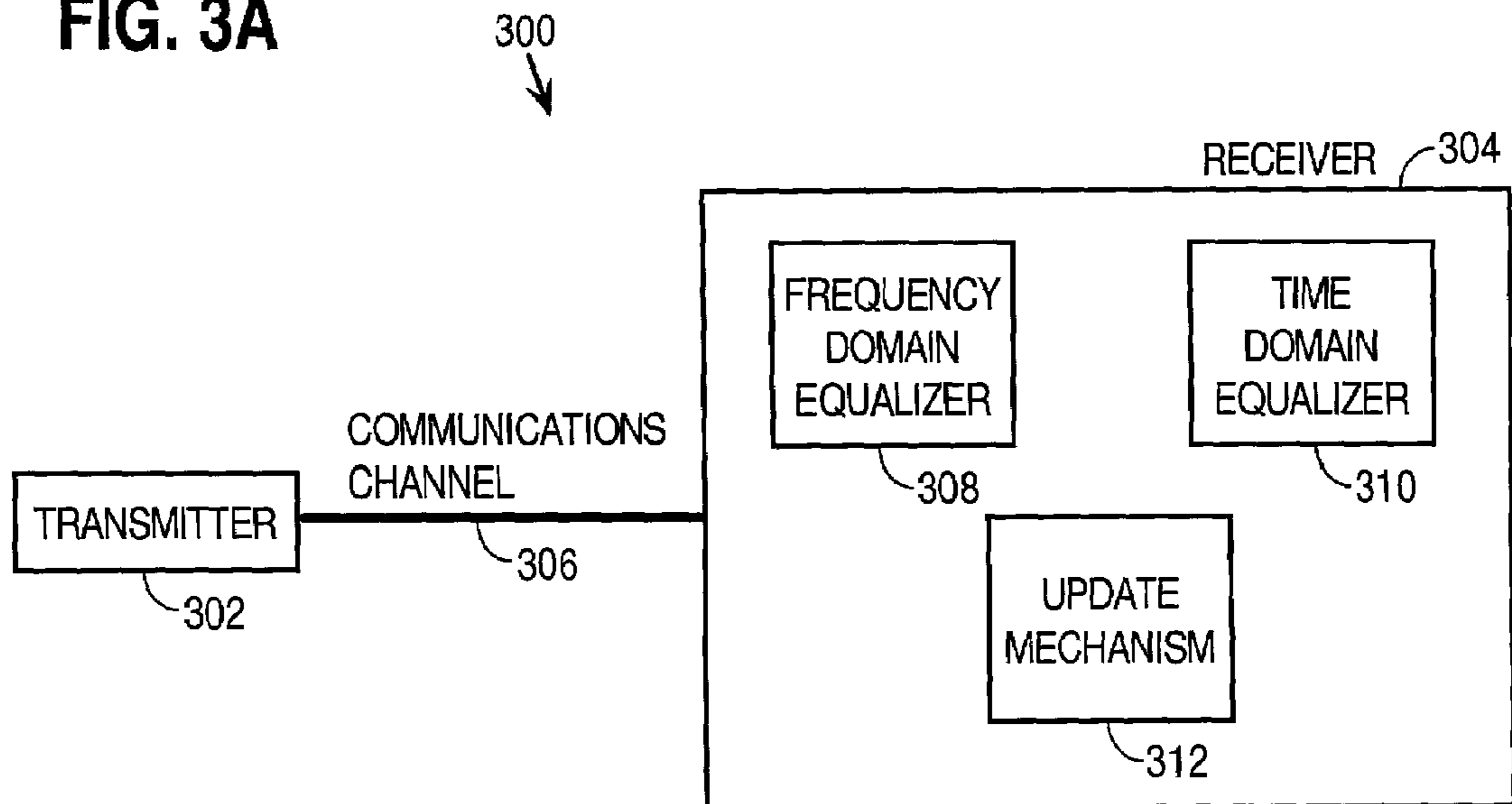


FIG. 3B

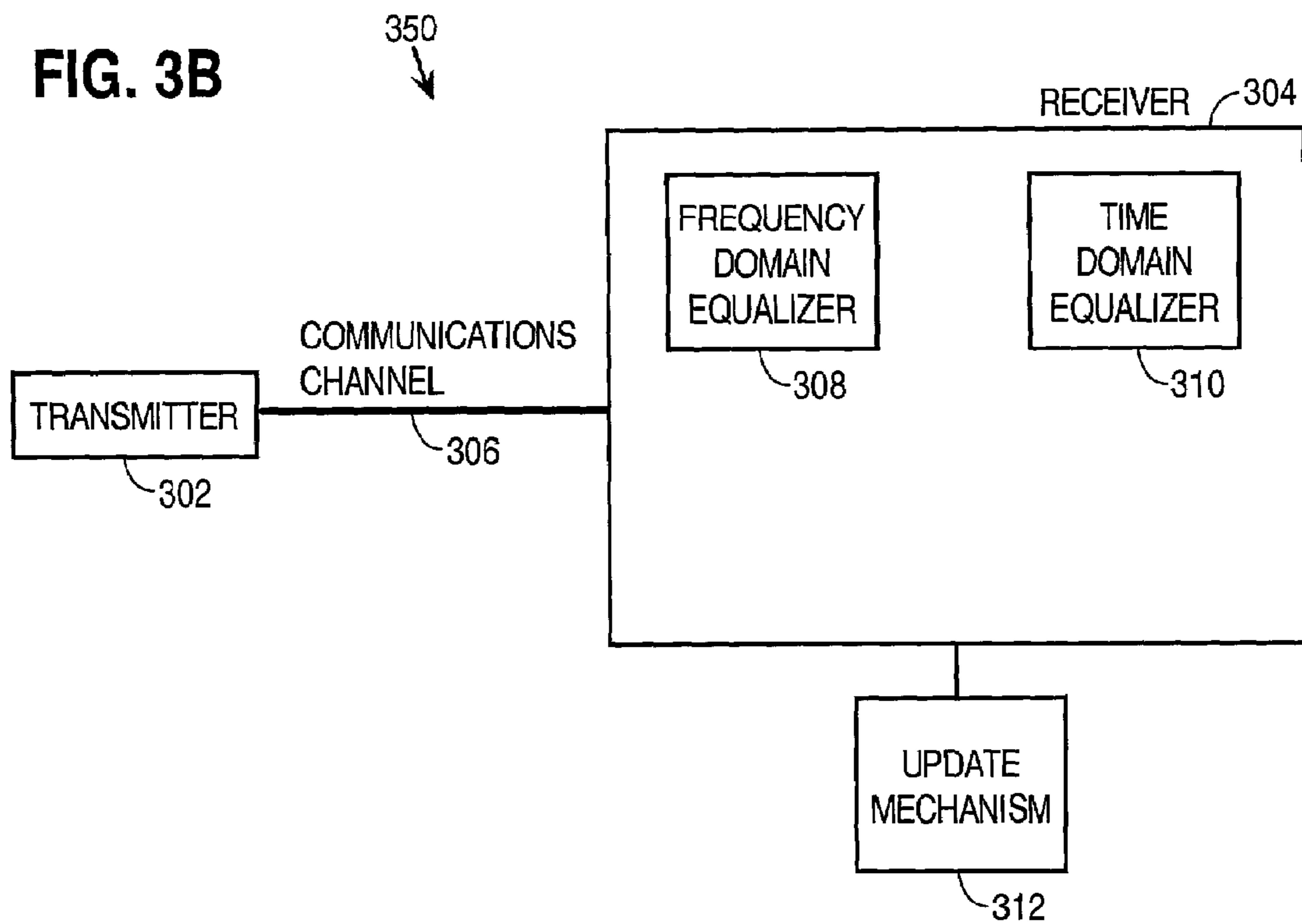
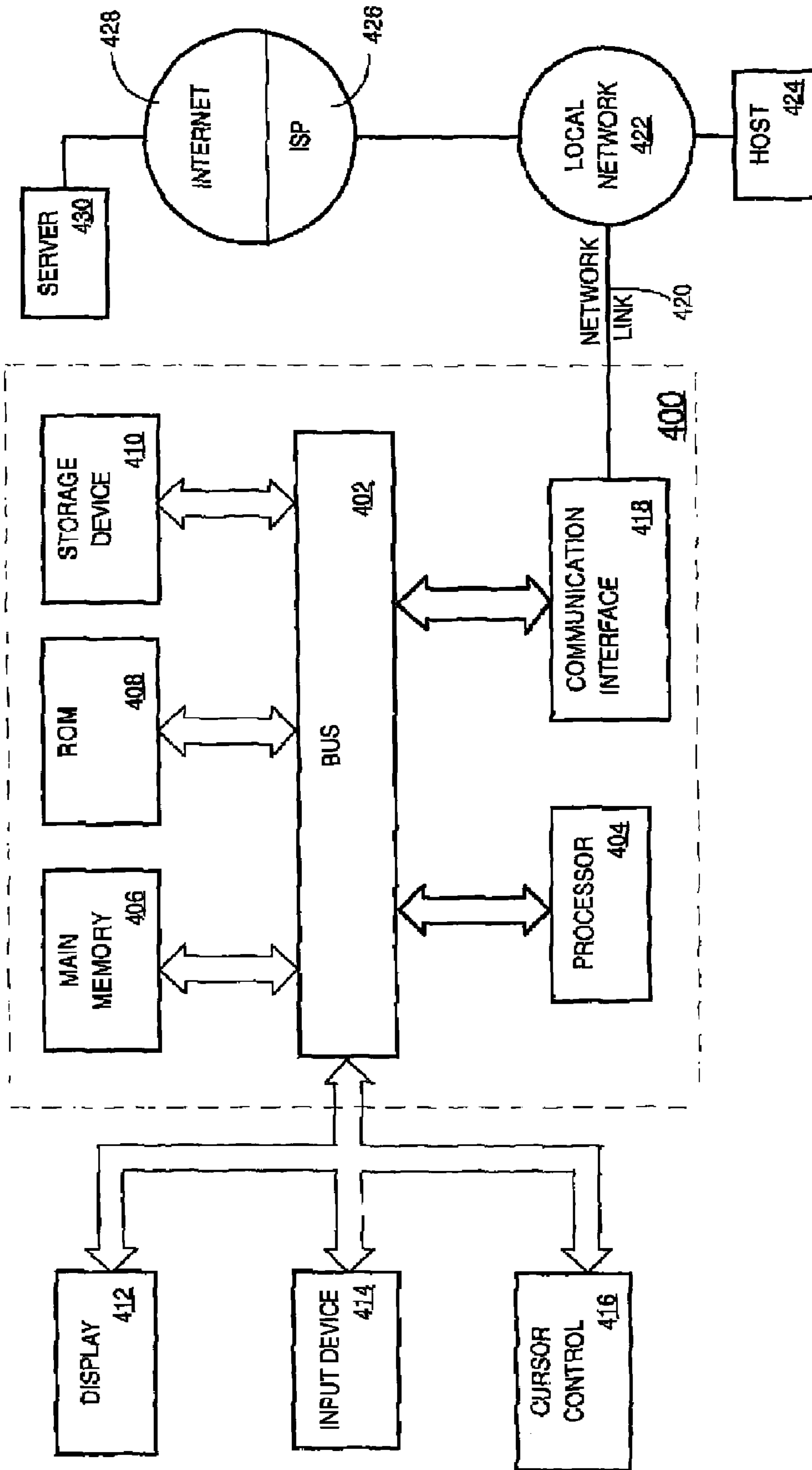


FIG. 4



**ADAPTIVE ADJUSTMENT OF TIME AND  
FREQUENCY DOMAIN EQUALIZERS IN  
COMMUNICATIONS SYSTEMS**

**Matter enclosed in heavy brackets [ ] appears in the original patent but forms no part of this reissue specification; matter printed in italics indicates the additions made by reissue.**

RELATED APPLICATIONS

This application is a Reissue Application of U.S. Pat. No. 7,272,199 granted on Sep. 18, 2007. U.S. Pat. No. 7,272,199 claims priority from U.S. Provisional Patent Application No. 60/264,369, entitled "Method for the Adaptive Adjustment of Time and Frequency Domain Equalizers in an ADSL/VDSL Receiver," filed Jan. 25, 2001, by A. Storm and D. Wu, the contents of which are incorporated herein by reference in its entirety for all purposes.

FIELD OF THE INVENTION

The present invention relates generally to digital communications systems, and more specifically, to an approach for adaptively adjusting time and frequency domain equalizers in digital communications systems.

BACKGROUND OF THE INVENTION

There is a continuing need for higher performance digital data communications systems. Perhaps nowhere is this need more evident than on the worldwide packet data communications network now commonly referred to as the "Internet." On the Internet, the "richness" of content is constantly increasing, requiring an ever-increasing amount of bandwidth to provide Internet content to users. As a result of this increased demand for bandwidth, significant efforts have been made to develop new types of high-speed digital data communications systems. For example, optical fiber based networks are being built in many large metropolitan areas and undersea to connect continents. As another example, new wireless protocols are being developed to provide Internet content to many different types of small, portable devices.

One of the significant drawbacks of deploying many of these new types of high-speed digital data communications systems is the high cost and amount of time required to develop and build out the new infrastructure required by the systems. Because of these high costs, many new high-speed digital data communications systems are initially deployed only in densely populated areas, where the cost of building out the new infrastructure can be quickly recovered. Less populated areas must often wait to receive the new communications systems and some rural areas never receive the new systems where it is not cost effective to build the infrastructure.

Significant efforts are being made to utilize conventional twisted pair telephone lines to provide high-speed digital data transmission. The use of twisted pair telephone lines provides two important benefits. First, a significant amount of twisted pair telephone line infrastructure already exists in many countries. Thus, using conventional twisted pair telephone lines avoids the cost of building expensive new infrastructure. Second, conventional twisted pair telephone lines extend into customers' homes and businesses, avoiding the so-called "last mile" problem. As a result of recent development efforts in this area, several new communications protocols, such as

Asymmetric Digital Subscriber Line (ADSL), G.Lite and Very high speed Digital Subscriber Line (VDSL), have been developed for providing high-speed digital transmission over conventional twisted pair telephone lines.

Despite the advantages to using conventional twisted pair telephone lines to provide high-speed digital communications, there are some problems with this approach. First, conventional twisted pair telephone lines cause signal attenuation per unit length that increases rapidly with frequency. A moderate length twisted pair line, for example around fifteen thousand feet, may cause only a few decibels (dB) of attenuation in the voice band, for which the line was originally designed, but many tens of dB of attenuation at higher transmission frequencies, for example around 1.1 MHz for ADSL. This results in a transfer function with a wide dynamic range, making channel equalization more difficult. The transfer function is further complicated by bridge taps and impedance mismatches between line sections that cause reflections and echoes at the receiver. Furthermore, filtering performed at the transmitter and receiver also increases the complexity of the transfer function.

The standards for ADSL and G.Lite specify Discrete Multitone (DMT) modulation. DMT is also under consideration for use in VDSL systems. DMT modulation generally involves transmitting digital data on a number of carriers simultaneously. Modulation and demodulation are performed using a Fast Fourier Transform (FFT). A cyclic prefix is introduced to ensure separation between successive DMT symbols and eliminate inter-symbol interference (ISI). In practice, the cyclic prefix is necessarily quite short, generally much shorter than the impulse response of the communications channel. This often results in significant ISI being present in the received data. Large amounts of ISI cause a large reduction in the available communications bandwidth. This is especially true for long twisted pair telephone lines likely to be encountered in ADSL and VDSL communications systems. The effect of this ISI is to reduce the Signal-to-Noise Ratio (SNR) in each bin of the FFT demodulator employed in a DMT system.

Standard equalizers used in digital communication systems, such as adaptive Least Means Squares (LMS) and Recursive Least Squares (RLS) equalizers, are generally inappropriate for DMT systems since they do not adequately shorten the communications channel impulse response and thereby fail to sufficiently eliminate ISI. Some attempts have been made to shorten the overall channel plus equalizer impulse response to be less than the cyclic prefix length. See for example, A Multicarrier Primer, by J. M. Cioffi; and Impulse Response Shortening for Discrete Multitone Transceivers, by P. Melsa, R. Younce and C. Rohrs, IEEE Transactions on Communications, Vol. 44, No. 12, Dec. 1996.

In addition to the equalization problem, twisted pair lines suffer from various forms of interference. Up to fifty twisted pairs are conventionally grouped together in binders. As a result, a signal on one pair can cause interference on other pairs in the same binder. This interference is called crosstalk and causes a reduction in the SNR at the receiver. Current approaches to mitigate crosstalk require access to the signal transmitted on the interfering line. This makes current approaches useful only in a central office environment, where the signals on all pairs in a binder are available. Thus, none of the existing crosstalk mitigation approaches are suitable when only the received signal is available. The foregoing problems are exacerbated in dynamic communications environments. For example, equalizer performance may be further degraded by changes on the transmission medium, e.g.,

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the transmission line, attributable to temperature changes or changes in the interference sources.

Based on the foregoing, there is a need for an approach for processing data received from a communications channel that does not suffer from the limitations of conventional approaches.

### SUMMARY OF THE INVENTION

An update mechanism and approach are provided for configuring a communications receiver. According to the approach, a time domain equalizer and a frequency domain equalizer in a communications receiver are dynamically updated based upon performance data that indicates the performance of a communications channel from which the communications receiver receives data. This approach accounts for changes in the communications channel attributable to changes in the transmission medium or changes in interference sources, and is particular useful in digital subscriber line (DSL) applications.

According to one aspect of the invention a communications receiver is provided. The communications receiver includes a time domain equalizer, a frequency domain equalizer and an update mechanism configured to update both the time domain equalizer and the frequency domain equalizer based upon performance data that indicates performance of a communications channel from which the communications receiver receives data.

According to another aspect of the invention, an update mechanism is provided and configured to update both a time domain equalizer and a frequency domain equalizer in a communications receiver based upon performance data that indicates performance of a communications channel from which the communications receiver receives data.

According to another aspect of the invention, a method is provided for configuring a communications receiver. The method includes updating both a time domain equalizer and a frequency domain equalizer contained in the communications receiver based upon performance data that indicates performance of a communications channel from which the communications receiver receives data.

### BRIEF DESCRIPTION OF THE DRAWINGS

Embodiments are illustrated by way of example, and not by way of limitation, in the figures of the accompanying drawings in which like reference numerals refer to similar elements and in which:

FIG. 1 is a block diagram that depicts a digital data communications arrangement to which embodiments of the invention may be applied;

FIG. 2 is a flow diagram that depicts an approach for configuring and updating a communications receiver according to an embodiment of the invention;

FIG. 3A is a block diagram that depicts a communications receiver arrangement on which embodiments of the invention may be implemented;

FIG. 3B is a block diagram that depicts another communications receiver arrangement on which embodiments of the invention may be implemented; and

FIG. 4 is a block diagram that depicts a computer system on which embodiments of the invention may be implemented.

### DETAILED DESCRIPTION OF THE INVENTION

In the following description, for the purposes of explanation, specific details are set forth in order to provide a thor-

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ough understanding of the invention. However, it will be apparent that the invention may be practiced without these specific details. In some instances, well-known structures and devices are depicted in block diagram form in order to avoid unnecessarily obscuring the invention.

Various aspects and features of the approach described herein for processing data received from a communications channel are described in more detail in the following sections: (1) Functional Overview; (2) Architecture Overview; (3) Mathematical Overview; (4) Dynamically Updating the Time Domain and Frequency Domain Equalizers; and (5) Implementation Mechanisms.

#### I. Functional Overview

An update mechanism and approach are provided for configuring a communications receiver. According to the approach, a time domain equalizer and a frequency domain equalizer in a communications receiver are dynamically updated based upon performance data that indicates the performance of a communications channel from which the communications receiver receives data. This approach accounts for changes in the communications channel attributable to changes in the transmission medium or changes in interference sources, and is particular useful in digital subscriber line (DSL) applications.

#### II. Architecture Overview

FIG. 1 is a block diagram of a communications system arrangement 100 to which embodiments of the invention may be applied. Arrangement 100 includes a transmitter 102 communicatively coupled to a receiver 104 via a communications channel 106. Communications channel 106 may be any type of medium or mechanism for providing data from transmitter 102 to receiver 104. For purposes of explanation only, various embodiments of the invention are described herein in the context of communications channel 106 as a landline, such as one or more conventional twisted pair telephone lines.

Transmitter 102 receives digital source data 108, e.g., a digital stream, that is modulated by a modulator 110 to generate a sampled data signal  $s(n)$ , where  $n$  is the sample number, and the sampling rate is given by  $F_s$ . The sampled data signal  $s(n)$  is converted to an analog signal  $s(t)$  by an  $a$  digital to analog converter 112. The analog signal  $s(t)$  is processed by a transmit filter 114 to remove unwanted components from the analog signal  $s(t)$ . The analog signal  $s(t)$  is then amplified by a line driver 116 and transmitted onto communications channel 106. It should be noted that the transmitted analog signal  $s(t)$  is not strictly a continuous time representation of the sampled data signal  $s(n)$  since transmit filter 114 modifies the signal, but is represented as such herein for the purposes of explanation.

The transmitted analog signal  $s(t)$  passes through communications channel 106, which has an impulse response of  $h(t)$  and corresponding transfer function  $H(f)$ . The output of communications channel 106  $x(t)$  is the convolution of the transmitted analog signal  $s(t)$  and the channel impulse response  $h(t)$ , given by

$$x(t)=s(t)*h(t) \quad (1)$$

The signal received by receiver 104  $y(t)$  is the sum of the output of communications channel 106  $x(t)$  and an additive noise signal  $w(t)$ , given by

$$y(t)=x(t)+w(t) \quad (2)$$

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where the additive noise signal  $w(t)$  consists of any form of interference introduced by communications channel **106**, for example crosstalk, and an additive white Gaussian noise component.

A differential amplifier and receive filters **118** process the received signal  $y(t)$  to generate an amplified and filtered (to remove undesired components) signal  $y(t)$ . An analog-to-digital converter **120** samples the amplified and filtered signal  $y(t)$  to generate a digital signal  $y(n)$  which at this point is still modulated. It should be pointed out that  $y(n)$  is not strictly a sampled version of  $y(t)$  due to the processing of receive filters **120** which modify the signal, but is represented as such herein for the purposes of explanation.

A time domain equalizer **122** processes the digital signal  $y(n)$  in the time domain to remove ISI and recover the transmitted modulated data  $z(n)$ . The modulated data  $z(n)$  is processed by a serial to parallel converter **124** and then by a demodulator **126**, e.g., via an FFT, to generate recovered source data **130**, which ideally very closely approximates source data **108**. Transmitter **102** and receiver **104** may include other components not included here for purposes of explanation, for example, an encoder and decoder, respectively.

## III. Mathematical Overview

For purposes of explanation, the process of modulation, transmission and demodulation may be described mathematically as follows. Let the  $l^{\text{th}}$  transmitted symbol be  $c_1$ . This is a vector consisting of  $N$  complex numbers:  $c_1 = (c_{1k})_{k=0}^{N-1}$ . For ADSL applications,  $N=256$  downstream and  $N=32$  upstream. Each component  $C_{1k}$  is a point in a constellation of  $2^{b_k}$  points. From  $c_1$ , a vector  $d_1 = (d_{1k})_{k=0}^{M-1}$  containing  $M=2N$  points is created having Hermitian symmetry. That is,  $d_{10}$  and  $d_{1N}$  are real numbers, and  $d_{1,M-k} = d_{1k}^*$  for  $1 \leq k \leq N-1$ . In ADSL applications,  $d_{10} = d_{1N} = 0$  and  $d_{1k} = g_k c_{1k}$  for  $1 \leq k \leq N-1$ . The real factor  $g_k$  is a fine gain factor used to equalize bit error rates between tones. Modulation is achieved by taking the inverse FFT of  $d_1$ . This creates the real vector  $x_1$  with components

$$x_{1r} = \frac{1}{M} \sum_{k=0}^{M-1} d_k \exp\left(\frac{2\pi i r k}{M}\right)$$

Note that  $x_{1r}$  is periodic in  $r$ , with period  $M$ . The transmitted analog signal is created by passing the vector  $(x_{1r})_{r=-c}^{M-1}$  through a parallel to serial converter and digital to analogue converter:

$$x(t) = \sum_l \sum_{r=-c}^{M-1} x_{1r} p(t - (r+c)T - (M+c)lT)$$

Through the periodicity of  $x_{1r}$ , the first  $c$  samples in a block are equal to the last  $C$  an effect known as a cyclic prefix. For ADSL applications,  $c=32$  downstream and  $c=4$  upstream. Also,  $1/T=2.208$  MHz upstream and  $1/T=276$  kHz downstream. The transmitted pulse shape  $p(t)$  includes the effect of any transmit filters.

According to the ADSL standard, data is grouped into frames. Each frame is encoded into a DMT symbol, modulated and transmitted. A synchronization symbol is transmitted after every 68 frames of data. The combination of the 68

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frames of data and the synchronization symbol is known as a superframe. The synchronization symbol  $(s_k)_{k=0}^{N-1}$  is obtained from a pseudo-random sequence and does not change from one superframe to the next superframe. The fine gain factors are not applied to the synchronization symbol.

At the receiver, the received signal is filtered and sampled to create the discrete time signal  $y_r$ , which is filtered by the time domain equalizer (TDEQ) to produce  $z_r$ . If the TDEQ coefficients are  $\phi_0 \dots \phi_p$  then

$$z_t = \sum_{n=1}^p \phi_n y_{t-n}$$

The signal  $z_r$  is split into blocks of length  $M+c$ , possibly with a timing offset  $\theta$ :  $Z_{ms} = Z_{\theta+(s+c)+(M+c)m}$  for  $-c \leq s \leq M$ . The first  $c$  samples are removed and the remainder demodulated by taking an FFT:

$$Z_{mk} = \sum_{s=0}^{M-1} Z_{ms} \exp\left(-\frac{2\pi i k s}{M}\right)$$

The frequency domain equalizer (FDEQ) re-maps this demodulated symbol by multiplying  $Z_{mk}$  by a complex number  $F_k$ . The result is input to a decoder to retrieve the transmitted data.

## IV. Dynamically Updating the Time Domain and Frequency Domain Equalizers

As briefly indicated herein, the approach generally involves updating a time domain equalizer and a frequency domain equalizer based upon communications channel performance data. The communications channel performance data is described hereinafter in accordance with an embodiment of the invention in the context of using synchronization symbols to evaluate the performance of the communications channel. The synchronization symbols may be used with "real" data, or with training data, depending upon the requirements of a particular application. The invention, however, is not limited to the use of synchronization symbols and any approach may be used to determine the performance of the communications channel and then adjust the time domain and frequency domain equalizers based upon the performance of the communications channel.

A synchronization symbol is data that is known to the receiver and transmitted over the communications channel with other data. Since the synchronization symbol is known to the receiver, the receiver can determine the performance of the communications channel.

FIG. 2 is a flow diagram **200** of an approach for updating time domain and frequency domain equalizers according to an embodiment of the invention. In step **202**, an average of the synchronization symbols is determined. In step **204**, an updated frequency domain equalizer **128** is determined based upon the average of the synchronization symbols. In step **206**, the equalization matrix for time domain equalizer **122** is determined. In step **208**, new coefficients are determined for time domain equalizer **122**. In step **210**, the signal to noise ratios are determined **210**. In step **212**, a determination is made which equalizer is to be used, e.g., staying with the existing equalizer coefficients, or implementing the new equalizer coefficients. In step **214**, the time domain and fre-



quency domain equalizers are updated if appropriate. In step 216, any necessary bit swaps and fine gain adjustments are made as appropriate. Each of these steps is described in more detail hereinafter.

#### A. Determine Average of Synchronization Symbols

For a block of  $n$  consecutive superframes, let  $U_{mk}$  be the  $k^{\text{th}}$  tone in the  $m^{\text{th}}$  demodulated synchronization symbol (prior to frequency domain equalization) in this block. The synchronization symbols are averaged to obtain

$$\bar{U}_k = \frac{1}{n} \sum_{m=1}^n U_{mk}$$

#### B. Determine Updated Frequency Domain Equalizer

Let  $S_k$  be the transmitted point on the  $k^{\text{th}}$  tone in the synchronization symbol. Let  $F_k = S_k / \bar{U}_k$  and multiply by  $F_k$  on the  $k^{\text{th}}$  tone to perform the frequency domain equalization. It may be necessary to perform further scaling before decoding to compensate for the fine gain factors and any other constellation scaling performed by transmitter 102.

#### C. Calculate the Time Domain Equalization Matrix

Let the number of time domain equalizer taps be  $p+1$ . Let  $\phi_{10}, \dots, \phi_{1p}$  be the current set of coefficients for time domain equalizer 122 and let  $(\Phi_{1k})_{k=0}^{M-1}$  be the frequency response of time domain equalizer 122, where  $(\Phi_{1k})_{k=0}^{M-1}$  is the  $M$  point FFT of  $\phi_{10}, \dots, \phi_{1p}$ .  $A_k = \bar{U}_k / \Phi_{1k}$  and the inverse FFT are determined to obtain the real sequence

$$a_t = (1/M) \sum_{k=0}^{M-1} A_k \exp(2\pi i t k / M).$$

Let  $(y_{mt})_{t=-c}^{M-1}$  be the  $m^{\text{th}}$  synchronization symbol prior to equalization. Thus

$$u_{mt} = \sum_{j=0}^p \phi_{1j} y_{t-j}$$

is the equalized  $m^{\text{th}}$  synchronization symbol prior to demodulation, and  $U_{mk}$  is the FFT of  $u_{mt}$ . The  $(p+1) \times (p+1)$  equalization matrix is determined with entries

$$G_{kl} = \frac{1}{n} \sum_{m=1}^n \sum_{t=0}^{M-1} (y_{m,t-k} - a_{t-k})(y_{m,t-l} - a_{t-l}), \quad 0 \leq k, l \leq p$$

#### D. Determine New Time Domain Equalizer

The new equalizer coefficients are computed to minimize the quadratic form  $\phi' G \phi$  under the constraint  $\phi_0 = 1$ . If the matrix  $G$  is partitioned as

$$G = \begin{bmatrix} r & f' \\ f & F \end{bmatrix}$$

where  $r$  is a scalar,  $f$  is  $p \times 1$  and  $F$  is  $p \times p$ , then the new equalizer is  $\phi_2 = (1, \zeta)'$  where  $\zeta$  solves  $F\zeta = -f$ . The time domain equalizer coefficients may also be re-scaled by multiplying by a constant. This does not affect the signal to noise ratios.

#### E. Determine the Signal to Noise Ratios When Using the Current and New Time Domain Equalizers

According to one embodiment of the invention, the SNRs are determined when using the current and new time domain equalizers so that a determination can be made whether to continue using the current set of time domain equalizer coefficients or switch to the new set of coefficients. This involves determining the SNR when using a time domain equalizer with the current coefficients and determining the SNR when using a time domain equalizer with the new coefficients. Consider a second block of  $n$  consecutive superframes. Using the current equalizer  $\phi_1$ , let  $U_{mk}$  be the  $k^{\text{th}}$  tone on the  $m^{\text{th}}$  demodulated synchronization symbol as before. Calculate

$$\bar{U}_k = \frac{1}{n} \sum_{m=1}^n U_{mk}$$

and

$$V_k = \frac{1}{n} \sum_{m=1}^n |U_{mk} - \bar{U}_k|^2.$$

The signal to noise ratios under the current equalizer are  $\text{SNR}_{1k} = |\bar{U}_k|^2 / V_k$ . To calculate the signal to noise ratios under the new equalizer, let  $(y_{mt})_{t=-c}^{M-1}$  be the  $m^{\text{th}}$  synchronization symbol prior to equalization. Filter  $y_{mt}$  by  $\phi_2$  to obtain

$$w_{mt} = \sum_{j=0}^p \phi_{2j} y_{m,t-j}.$$

Then demodulate  $w_{mt}$  to obtain

$$W_{mk} = \sum_{t=0}^{M-1} w_{mt} \exp(-2\pi i k t / M).$$

Calculate

$$\bar{W}_k = \frac{1}{n} \sum_{m=1}^n W_{mk}$$

and

$$X_k = \frac{1}{n} \sum_{m=1}^n |W_{mk} - \bar{W}_k|^2.$$

The signal to noise ratios under the new equalizer are  $\text{SNR}_{2k} = |\bar{W}_k|^2 / X_k$ .

#### F. Determine Whether to Use the New Equalizer

Once the SNRs have been determined for the current and new sets of equalizer coefficients, a determination is made whether to use the new set of equalizer coefficients or continue to use the existing set of equalizer coefficients. Let  $T$  be the set of tones on which the fine gain factor is non-zero. According to one embodiment of the invention, this set of tones is defined during initialization and is not changed by any bit swap routines. Let  $K$  be the number of tones in  $T$ . To decide which equalizer to use, count the number of tones within  $T$  on which  $\text{SNR}_{2k} > \text{SNR}_{1k}$ , i.e. compute  $N = \sum_{k \in T} I(\text{SNR}_{2k} > \text{SNR}_{1k})$ , where  $I(\text{condition})$  equals 1 if the condition is true, and zero otherwise. Then use the new equalizer in

preference to the old if N satisfies a threshold value. According to one embodiment of the invention, the new set of equalizer coefficients is used in place of the current set of equalizer coefficients if

$$N > \frac{K}{2} + \frac{z}{2}\sqrt{K}$$

where z is the upper (1- $\alpha$ ) percentile of the normal distribution. For a 5% confidence level test z=1.64. For a 10% confidence level test z=1.28. The reasoning behind this test, which is a simple sign test, is as follows. Under the null hypothesis that the new equalizer coefficients is no better than the current equalizer coefficients, and assuming independence of signal to noise ratios between tones, N will have a binomial distribution with probability of success  $\frac{1}{2}$ , so E(N)=K/2 and V(N)=K/4. We reject this null hypothesis in favor of the alternative, i.e., that the new equalizer coefficients are better than the current set of equalizer coefficients, if N is sufficiently large. According to one embodiment of the invention, the threshold level is determined by the normal approximation:  $(N-K/2)/\sqrt{K/4} \sim N(0,1)$ , so the null hypothesis is rejected if  $(N-K/2)/\sqrt{K/4} > z$  where  $P(Z > z) = \alpha$  if  $Z \sim N(0,1)$ . This provides the necessary test.

#### G. Update the Time Domain and Frequency Domain Equalizers

Based upon the foregoing test, a determination is made whether to continue to use the current set of time domain equalizer coefficients or switch to the new set of time domain equalizer coefficients. The frequency domain equalizer is updated if decision is made to use the new set of time domain equalizer coefficients. The frequency domain equalizer may also be updated if a determination is made to continue using the current set of time domain equalizer coefficients. If a decision is made to continue to use the existing time domain equalizer coefficients, then set  $F_k = S_k / U_k$ . If the new time domain equalizer coefficients are implemented, then set  $[F_k = S_k / W_k]$   $F_k = S_k / W_k$ .

#### H. Perform Any Necessary Bit Swaps and Gain Adjustments

Once the time domain and frequency domain equalizers have been updated, any necessary bit swaps and gain adjustments are made in accordance with the current signal to noise ratios SNR<sub>1k</sub> or SNR<sub>2k</sub> as appropriate. According to one embodiment of the invention, the bit swaps and/or gain adjustments are made using an appropriate DSL bit swap mechanism to reallocate bits among the tones.

#### V. Implementation Mechanisms

The approach described herein for updating time domain and frequency domain equalizers may be implemented in a receiver, such as receiver 104, or may be implemented into a stand-alone mechanism. Furthermore, the approach may be implemented in computer software, in hardware circuitry, or as a combination of computer software and hardware circuitry. Accordingly the invention is not limited to a particular implementation. For example, FIG. 3A is a block diagram that depicts a communications receiver arrangement 300 on which embodiments of the invention may be implemented. In arrangement 302, a transmitter communicates with a receiver 304 via a communications channel 306. Receiver 304 includes a frequency domain equalizer 308, a time domain equalizer 310 and an update mechanism 312. According to one embodiment of the invention, update mechanism 312 is configured to update frequency domain equalizer 308, time domain equalizer 310, or both, using the approach described

herein. As depicted in FIG. 3A update mechanism 312 is integrated into receiver 304. As depicted in FIG. 3B, in communications receiver arrangement 350, update mechanism 312 is implemented as a separate mechanism that is communicatively coupled to receiver 304.

The timing of when time domain and frequency domain equalizers are updated according the approach described herein may be based upon a variety of factors, depending upon the requirements of a particular application, and the invention is not limited to any particular approach. For example, the approach may be implemented so that the analysis and updates are performed on a periodic basis. It may be advantageous to implement the approach during a time when a communications system is known to be inactive to reduce the adverse effects on performance attributable to updating the time domain and frequency domain equalizers. As another example, the approach may be initiated based upon external factors that indicate that there has been a change in a condition of a transmission line or a change in an interference source.

FIG. 4 is a block diagram that illustrates a computer system 400 upon which an embodiment of the invention may be implemented. Computer system 400 includes a bus 402 or other communication mechanism for communicating information, and a processor 404 coupled with bus 402 for processing information. Computer system 400 also includes a main memory 406, such as a random access memory (RAM) or other dynamic storage device, coupled to bus 402 for storing information and instructions to be executed by processor 404. Main memory 406 also may be used for storing temporary variables or other intermediate information during execution of instructions to be executed by processor 404. Computer system 400 further includes a read only memory (ROM) 408 or other static storage device coupled to bus 402 for storing static information and instructions for processor 404. A storage device 410, such as a magnetic disk or optical disk, is provided and coupled to bus 402 for storing information and instructions.

Computer system 400 may be coupled via bus 402 to a display 412, such as a cathode ray tube (CRT), for displaying information to a computer user. An input device 414, including alphanumeric and other keys, is coupled to bus 402 for communicating information and command selections to processor 404. Another type of user input device is cursor control 416, such as a mouse, a trackball, or cursor direction keys for communicating direction information and command selections to processor 404 and for controlling cursor movement on display 412. This input device typically has two degrees of freedom in two axes, a first axis (e.g., x) and a second axis (e.g., y), that allows the device to specify positions in a plane.

The invention is related to the use of computer system 400 for dynamically updating time domain and frequency domain equalizers in a communications receiver based upon performance data that indicates the performance of a communications channel from which the communications receiver receives data. According to one embodiment of the invention, the dynamic updating time domain and frequency domain equalizers in a communications receiver based upon performance data that indicates the performance of a communications channel from which the communications receiver receives data is performed by computer system 400 in response to processor 404 executing one or more sequences of one or more instructions contained in main memory 406. Such instructions may be read into main memory 406 from another computer-readable medium, such as storage device 410. Execution of the sequences of instructions contained in main memory 406 causes processor 404 to perform the pro-

cess steps described herein. One or more processors in a multi-processing arrangement may also be employed to execute the sequences of instructions contained in main memory 406. In alternative embodiments, hardwired circuitry may be used in place of or in combination with software instructions to implement the invention. Thus, embodiments of the invention are not limited to any specific combination of hardware circuitry and software.

The term "computer-readable medium" as used herein refers to any medium that participates in providing instructions to processor 404 for execution. Such a medium may take many forms, including but not limited to, non-volatile media, volatile media, and transmission media. Non-volatile media includes, for example, optical or magnetic disks, such as storage device 410. Volatile media includes dynamic memory, such as main memory 406. Transmission media includes coaxial cables, copper wire and fiber optics, including the wires that comprise bus 402. Transmission media can also take the form of acoustic or light waves, such as those generated during radio wave and infrared data communications.

Common forms of computer-readable media include, for example, a floppy disk, a flexible disk, hard disk, magnetic tape, or any other magnetic medium, a CD-ROM, any other optical medium, punch cards, paper tape, any other physical medium with patterns of holes, a RAM, a PROM, and EPROM, a FLASH-EPROM, any other memory chip or cartridge, a carrier wave as described hereinafter, or any other medium from which a computer can read.

Various forms of computer readable media may be involved in carrying one or more sequences of one or more instructions to processor 404 for execution. For example, the instructions may initially be carried on a magnetic disk of a remote computer. The remote computer can load the instructions into its dynamic memory and send the instructions over a telephone line using a modem. A modem local to computer system 400 can receive the data on the telephone line and use an infrared transmitter to convert the data to an infrared signal. An infrared detector coupled to bus 402 can receive the data carried in the infrared signal and place the data on bus 402. Bus 402 carries the data to main memory 406, from which processor 404 retrieves and executes the instructions. The instructions received by main memory 406 may optionally be stored on storage device 410 either before or after execution by processor 404.

Computer system 400 also includes a communication interface 418 coupled to bus 402. Communication interface 418 provides a two-way data communication coupling to a network link 420 that is connected to a local network 422. For example, communication interface 418 may be an integrated services digital network (ISDN) card or a modem to provide a data communication connection to a corresponding type of telephone line. As another example, communication interface 418 may be a local area network (LAN) card to provide a data communication connection to a compatible LAN. Wireless links may also be implemented. In any such implementation, communication interface 418 sends and receives electrical, electromagnetic or optical signals that carry digital data streams representing various types of information.

Network link 420 typically provides data communication through one or more networks to other data devices. For example, network link 420 may provide a connection through local network 422 to a host computer 424 or to data equipment operated by an Internet Service Provider (ISP) 426. ISP 426 in turn provides data communication services through the worldwide packet data communication network now commonly referred to as the "Internet" 428. Local network 422

and Internet 428 both use electrical, electromagnetic or optical signals that carry digital data streams. The signals through the various networks and the signals on network link 420 and through communication interface 418, which carry the digital data to and from computer system 400, are exemplary forms of carrier waves transporting the information.

Computer system 400 can send messages and receive data, including program code, through the network(s), network link 420 and communication interface 418. In the Internet example, a server 430 might transmit a requested code for an application program through Internet 428, ISP 426, local network 422 and communication interface 418. In accordance with the invention, one such downloaded application provides for dynamically updating time domain and frequency domain equalizers in a communications receiver based upon performance data that indicates the performance of a communications channel from which the communications receiver receives data as described herein.

The received code may be executed by processor 404 as it is received, and/or stored in storage device 410, or other non-volatile storage for later execution. In this manner, computer system 400 may obtain application code in the form of a carrier wave.

In the foregoing specification, particular embodiments have been described. It will, however, be evident that various modifications and changes may be made thereto without departing from the broader spirit and scope of the invention. The specification and drawings are, accordingly, to be regarded in an illustrative rather than a restrictive sense.

What is claimed is:

1. A communications receiver comprising:

a time domain equalizer;

a frequency domain equalizer; and

an update mechanism configured to update both the time domain equalizer and the frequency domain equalizer based upon performance of a communications channel from which the communications receiver receives data, wherein updating the time domain equalizer includes:

causing the time domain equalizer to operate with a first set of coefficients,

determining first performance data that reflects performance of the time domain equalizer [when the time domain equalizer is] operating with the first set of coefficients,

changing [the] coefficients used by the time domain equalizer so that the time domain equalizer operates with a second set of coefficients,

determining second performance data that reflects performance of the time domain equalizer [when the time domain equalizer is] operating with the second set of coefficients,

selecting for use by the time domain equalizer, based upon the first performance data and the second performance data, either the first set of coefficients or the second set of coefficients,

if the first set of coefficients is selected for use, then changing the coefficients used by the time domain equalizer so that the time domain equalizer operates with the first set of coefficients, and

if the second set of coefficients is selected for use, then causing the time domain equalizer to continue to operate with the second set of coefficients.

2. The communications receiver as recited in claim 1, wherein the update mechanism is further configured to determine the first and second performance data based upon synchronization symbols received by the communications receiver.

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3. The communications receiver as recited in claim 1, wherein the communications receiver is a digital subscriber line communications receiver.

4. The communications receiver as recited in claim 1, wherein the communications receiver is part of a discrete multitone communications system.

5. The communications receiver as recited in claim 1, wherein:

the first performance data reflects one or more signal to noise ratios of data received by the communications receiver when the time domain equalizer is operating with the first set of coefficients; and

the second performance data reflects one or more signal to noise ratios of data received by the communications receiver when the time domain equalizer is operating with the second set of coefficients.

6. The communications receiver as recited in claim 1, wherein:

the communications receiver uses an initial bit allocation; and

the update mechanism is further configured to determine an updated bit allocation based upon the first and second performance data, and cause the communications receiver to use the updated bit allocation.

7. The communications receiver as recited in claim 1, wherein the update mechanism is further configured to perform gain adjustments on tones based upon the first and second performance data.

8. An update mechanism configured to:

update both a time domain equalizer and a frequency domain equalizer in a communications receiver based upon performance of a communications channel from which the communications receiver receives data, wherein updating the time domain equalizer includes: causing the time domain equalizer to operate with a first set of coefficients,

determining first performance data that reflects performance of the time domain equalizer [when the time domain equalizer is] operating with the first set of coefficients,

changing [the] coefficients used by the time domain equalizer so that the time domain equalizer operates with a second set of coefficients,

determining second performance data that reflects performance of the time domain equalizer [when the time domain equalizer is] operating with the second set of coefficients,

selecting for use by the time domain equalizer, based upon the first performance data and the second performance data, either the first set of coefficients or the second set of coefficients,

if the first set of coefficients is selected for use, then changing the coefficients used by the time domain equalizer so that the time domain equalizer operates with the first set of coefficients, and

if the second set of coefficients is selected for use, then causing the time domain equalizer to continue to operate with the second set of coefficients.

9. The update mechanism as recited in claim 8, wherein the update mechanism is further configured to determine the first and second performance data based upon synchronization symbols received by the communications receiver.

10. The update mechanism as recited in claim 8, wherein the communications receiver is a digital subscriber line communications receiver.

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11. The update mechanism as recited in claim 8, wherein the communications receiver is part of a discrete multitone communications system.

12. The update mechanism as recited in claim 8, wherein: the first performance data reflects one or more signal to noise ratios of data received by the communications receiver when the time domain equalizer is operating with the first set of coefficients; and

the second performance data reflects one or more signal to noise ratios of data received by the communications receiver when the time domain equalizer is operating with the second set of coefficients.

13. The update mechanism as recited in claim 8, wherein: the communications receiver uses an initial bit allocation; and

the update mechanism is further configured to determine an updated bit allocation based upon the first and second performance data, and cause the communications receiver to use the updated bit allocation.

14. The update mechanism as recited in claim 8, wherein the update mechanism is further configured to perform gain adjustments on tones based upon the first and second performance data.

15. A method for configuring a communications receiver comprising:

updating both a time domain equalizer and a frequency domain equalizer contained in the communications receiver based upon performance of a communications channel from which the communications receiver receives data, wherein updating the time domain equalizer includes:

causing the time domain equalizer to operate with a first set of coefficients,

determining first performance data that reflects performance of the time domain equalizer [when the time domain equalizer is] operating with the first set of coefficients,

changing [the] coefficients used by the time domain equalizer so that the time domain equalizer operates with a second set of coefficients,

determining second performance data that reflects performance of the time domain equalizer [when the time domain equalizer is] operating with the second set of coefficients,

selecting for use by the time domain equalizer, based upon the first performance data and the second performance data, either the first set of coefficients or the second set of coefficients,

if the first set of coefficients is selected for use, then changing the coefficients used by the time domain equalizer so that the time domain equalizer operates with the first set of coefficients, and

if the second set of coefficients is selected for use, then causing the time domain equalizer to continue to operate with the second set of coefficients.

16. The method as recited in claim 15, further comprising determining the first and second performance data based upon synchronization symbols received by the communications receiver.

17. The method as recited in claim 15, wherein the communications receiver is a digital subscriber line communications receiver.

18. The method as recited in claim 15, wherein the communications receiver is part of a discrete multitone communications system.

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19. The method as recited in claim 15, wherein:  
the first performance data reflects one or more signal to  
noise ratios of data received by the communications  
receiver when the time domain equalizer is operating  
with the first set of coefficients; and  
the second performance data reflects one or more signal to  
noise ratios of data received by the communications  
receiver when the time domain equalizer is operating  
with the second set of coefficients.

20. The method as recited in claim 15, wherein:  
the communications receiver uses an initial bit allocation;  
and  
the method further comprises  
determining an updated bit allocation based upon the  
first and second performance data, and  
causing the communications receiver to use the updated  
bit allocation.

21. The method as recited in claim 15, further comprising  
performing gain adjustments on tones based upon the first and  
second performance data.

22. A *non-transitory* computer-readable medium *having*  
*computer-readable instructions* for configuring a communi-  
cations receiver *stored thereon, wherein the instructions are*  
*configured to be executed by a processor of a computing*  
*device, and wherein the [computer-readable medium carry-*  
*ing] instructions [which, when processed by one or more*  
*processors, cause] comprise:*

[updating] *instructions to update* both a time domain  
equalizer and a frequency domain equalizer contained in  
the communications receiver based upon performance  
of a communications channel from which the commu-  
nications receiver receives data, wherein [updating]  
*instructions to update* the time domain equalizer  
[includes] *comprise:*

[causing] *instructions to cause* the time domain equal-  
izer to operate with a first set of coefficients[.];

[determining] *instructions to determine* first perfor-  
mance data that reflects performance of the time  
domain equalizer [when the time domain equalizer is]  
operating with the first set of coefficients[.];

[changing the] *instructions to change* coefficients used  
by the time domain equalizer so that the time domain  
equalizer operates with a second set of coefficients[.];

[determining] *instructions to determine* second perfor-  
mance data that reflects performance of the time  
domain equalizer [when the time domain equalizer is]  
operating with the second set of coefficients[.];

[selecting] *instructions to select* for use by the time  
domain equalizer, based upon the first performance  
data and the second performance data, either the first  
set of coefficients or the second set of coefficients[.];

if the first set of coefficients is selected for use, then  
[changing] *instructions to change* the coefficients  
used by the time domain equalizer so that the time  
domain equalizer operates with the first set of coeffi-  
cients[.]; and

if the second set of coefficients is selected for use, then  
[causing] *instructions to cause* the time domain  
equalizer to continue to operate with the second set of  
coefficients.

23. The computer-readable medium as recited in claim 22,  
*wherein the computer-readable instructions further [com-*  
*prising one or more additional] comprise instructions [which,*  
*when processed by the one or more processors, cause deter-*  
*mining] to determine* the first and second performance data  
based upon synchronization symbols received by the com-  
munications receiver.

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24. The computer-readable medium as recited in claim 22,  
*wherein the communications receiver is a digital subscriber*  
*line communications receiver.*

25. The computer-readable medium as recited in claim 22,  
*wherein the communications receiver is part of a discrete*  
*multitone communications system.*

26. The computer-readable medium as recited in claim 22,  
*wherein:*

the first performance data reflects one or more signal to  
noise ratios of data received by the communications  
receiver when the time domain equalizer is operating  
with the first set of coefficients; and  
the second performance data reflects one or more signal to  
noise ratios of data received by the communications  
receiver when the time domain equalizer is operating  
with the second set of coefficients.

27. The computer-readable medium as recited in claim 22,  
*wherein:*

the communications receiver uses an initial bit allocation;  
and

the computer-readable [medium further comprises one or  
more additional instructions which, when processed by  
the one or more processors, causes] *instructions further*  
*comprise:*

[determining] *instructions to determine* an updated bit  
allocation based upon the first and second perfor-  
mance data, and

[causing] *instructions to cause* the communications  
receiver to use the updated bit allocation.

28. The computer-readable medium as recited in claim 22,  
*wherein the computer-readable instructions further [com-*  
*prises one or more additional] comprise instructions [which,*  
*when processed by the one or more processors, causes per-*  
*forming] to perform* gain adjustments on tones based upon  
the first and second performance data.

29. A *communications receiver comprising:*

a time domain equalizer;

a frequency domain equalizer; and

*an update mechanism configured to update both the time*  
*domain equalizer and the frequency domain equalizer*  
*based upon performance of a communications channel*  
*from which the communications receiver receives data,*  
*wherein the update mechanism is further configured to:*  
*cause the frequency domain equalizer to operate with*  
*synchronization symbols;*

*determine an average of the synchronization symbols;*  
*determine an updated frequency domain equalizer*  
*based upon the average of the synchronization sym-*  
*bols; and*

*determine whether a new set of time domain equalizer*  
*coefficients are used for the determined updated fre-*  
*quency domain equalizer and, if so, update the fre-*  
*quency domain equalizer.*

30. The communications receiver as recited in claim 29,  
*wherein the update mechanism is further configured to per-*  
*form gain adjustments on tones based upon the new set of time*  
*domain equalizer coefficients.*

31. The communications receiver as recited in claim 29,  
*wherein determining whether the new set of time domain*  
*equalizer coefficients are used comprises conducting a*  
*threshold test.*

32. The communications receiver as recited in claim 31,  
*wherein the threshold test is based on a normal approxima-*  
*tion.*

33. The communications receiver as recited in claim 31,  
*wherein the threshold test is based on signal to noise ratio*  
*determinations.*

34. The communications receiver as recited in claim 29, wherein updating the frequency domain equalizer further includes determining whether a current set of time domain equalizer coefficients are used and, if so, updating the frequency domain equalizer.

35. The communications receiver as recited in claim 29, wherein the communications receiver is a digital subscriber line communications receiver.

36. The communications receiver as recited in claim 29, wherein the communications receiver is part of a discrete multitone communications system.

37. A method for configuring a communications receiver comprising:

updating both a time domain equalizer and a frequency domain equalizer contained in the communications receiver based upon performance of a communications channel from which the communications receiver receives data, wherein updating the frequency domain equalizer includes:

causing the frequency domain equalizer to operate with synchronization symbols;

determining an average of the synchronization symbols;

determining an updated frequency domain equalizer based upon the average of the synchronization symbols; and

determining whether a new set of time domain equalizer coefficients are used for the determined updated frequency domain equalizer and, if so, updating the frequency domain equalizer.

38. The method as recited in claim 37, wherein updating the frequency domain equalizer further includes performing gain adjustments on tones based upon the new set of time domain equalizer coefficients.

39. The method as recited in claim 37, wherein determining whether the new set of time domain equalizer coefficients are used comprises conducting a threshold test.

40. The method as recited in claim 39, wherein the threshold test is based on a normal approximation.

41. The method as recited in claim 39, wherein the threshold test is based on signal to noise ratio determinations.

42. The method as recited in claim 37, wherein updating the frequency domain equalizer further includes determining whether a current set of time domain equalizer coefficients are used and, if so, updating the frequency domain equalizer.

43. A non-transitory computer-readable medium having computer-readable instructions for configuring a communications receiver stored thereon, wherein the instructions are configured to be executed by a processor of a computing device, and wherein the instructions comprise:

instructions to update both a time domain equalizer and a frequency domain equalizer contained in the communications receiver based upon performance of a communications channel from which the communications receiver receives data, wherein instructions to update the frequency domain equalizer comprise:

instructions to cause the frequency domain equalizer to operate with synchronization symbols;

instructions to determine an average of the synchronization symbols;

instructions to determine an updated frequency domain equalizer based upon the average of the synchronization symbols; and

instructions to determine whether a new set of time domain equalizer coefficients are used for the determined updated frequency domain equalizer and, if so, updating the frequency domain equalizer.

44. The computer-readable medium as recited in claim 43, wherein updating the frequency domain equalizer further comprises determining whether a current set of time domain equalizer coefficients are used and, if so, updating the frequency domain equalizer.

\* \* \* \* \*

UNITED STATES PATENT AND TRADEMARK OFFICE  
**CERTIFICATE OF CORRECTION**

PATENT NO. : RE43,402 E  
APPLICATION NO. : 12/562529  
DATED : May 22, 2012  
INVENTOR(S) : Storm et al.

Page 1 of 2

It is certified that error appears in the above-identified patent and that said Letters Patent is hereby corrected as shown below:

In Column 5, Line 30, delete “  $c_{1\cdot}$  ” and insert --  $c_l$  --, therefor.

In Column 5, Line 31, delete “  $c_1 = (c_{1k})_{k=0}^{N-1}$  ” and insert --  $c_l = (c_{lk})_{k=0}^{N-1}$  --, therefor.

In Column 5, Line 33, delete “  $C_{1k}$  ” and insert --  $c_{lk}$  --, therefor.

In Column 5, Line 34, delete “  $C_1$  ” and insert --  $c_l$  --, therefor.

In Column 5, Line 34, delete “  $d_1 = (d_{1k})_{k=0}^{M-1}$  ” and insert --  $d_l = (d_{lk})_{k=0}^{M-1}$  --, therefor.

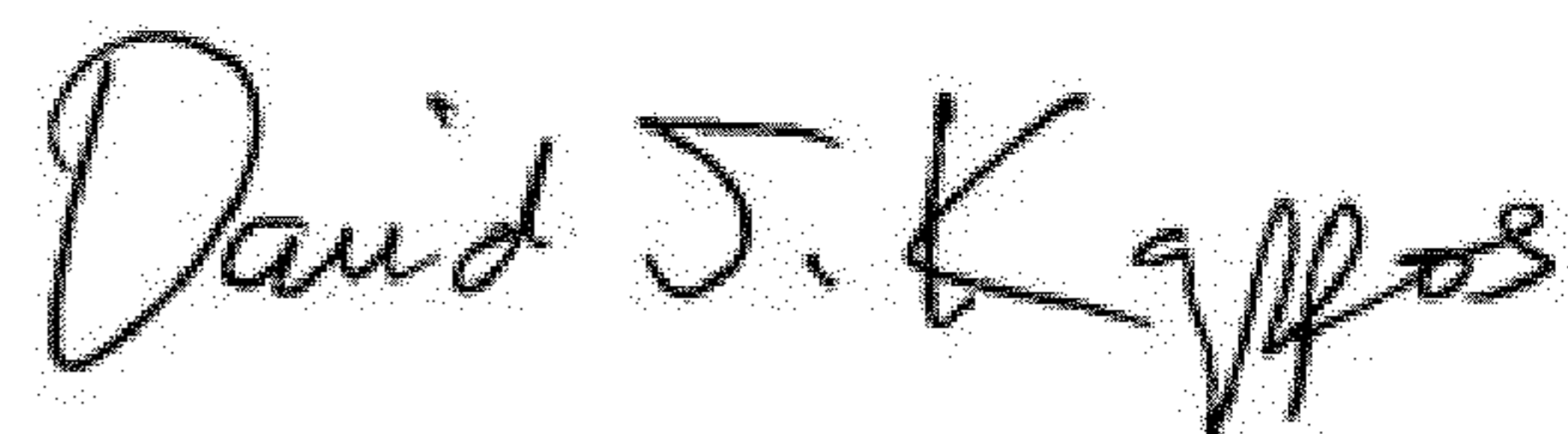
In Column 5, Line 35, delete “  $d_{10}$  and ” and insert --  $d_{l0}$  and --, therefor.

In Column 5, Line 36, delete “  $d_{1N}$  ” and insert --  $d_{lN}$  --, therefor.

In Column 5, Line 36, delete “  $d_{1,M-k} = d_{1k}^*$  ” and insert --  $d_{l,M-k} = d_{lk}^*$  --, therefor.

In Column 5, Line 37, delete “  $d_{10} = d_{1N} = 0$  ” and insert --  $d_{l0} = d_{lN} = 0$  --, therefor.

Signed and Sealed this  
Thirteenth Day of November, 2012



David J. Kappos  
Director of the United States Patent and Trademark Office

In Column 5, Line 37, delete “ $d_{1k} = g_k c_{1k}$ ” and insert --  $d_{lk} = g_k c_{lk}$  --, therefor.

In Column 5, Line 40, delete “ $d_{1\cdot}$ ” and insert --  $d_{l\cdot}$  --, therefor.

In Column 5, Line 40, delete “ $X_{1\cdot}$ ” and insert --  $x_{l\cdot}$  --, therefor.

In Column 5, Line 48, delete “ $X_{1r}$ ” and insert --  $x_{lr}$  --, therefor.

In Column 5, Line 49, delete “ $(X_{1r})_{r=-c}^{M-1}$ ” and insert --  $(x_{lr})_{r=-c}^{M-1}$  --, therefor.

In Column 5, Line 58, delete “ $X_{1r}$ ” and insert --  $x_{lr}$  --, therefor.

In Column 5, Line 59, delete “C an” and insert -- c an --, therefor.

In Column 7, Line 25, delete “ $\phi_{10}, \dots, \phi_{1p}$ ” and insert --  $\varphi_{10}, \dots, \varphi_{1p}$  --, therefor.

In Column 7, Line 28, delete “ $\phi_{10}, \dots, \phi_{1p} \cdot A_{k=\bar{U}_k} / \Phi_{1k}$ ” and

insert --  $\varphi_{10}, \dots, \varphi_{1p} \cdot A_k = \bar{U}_k / \Phi_{1k}$  --, therefor.

In Column 7, Line 55, delete “ $\phi_0 = 1$ ” and insert --  $\varphi_0 = 1$  --, therefor.

In Column 11, Line 4, delete “handwired” and insert -- hand-wired --, therefor.

In Column 12, Lines 53-54, in Claim 1, delete “secondperformance” and insert -- second performance --, therefor.