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MULTIPLIER, DIVIDER AND WATTMETER [54] USING A SWITCHING CIRCUIT AND A PULSE-WIDTH AND FREQUENCY MODULATOR

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307/229; 324/142

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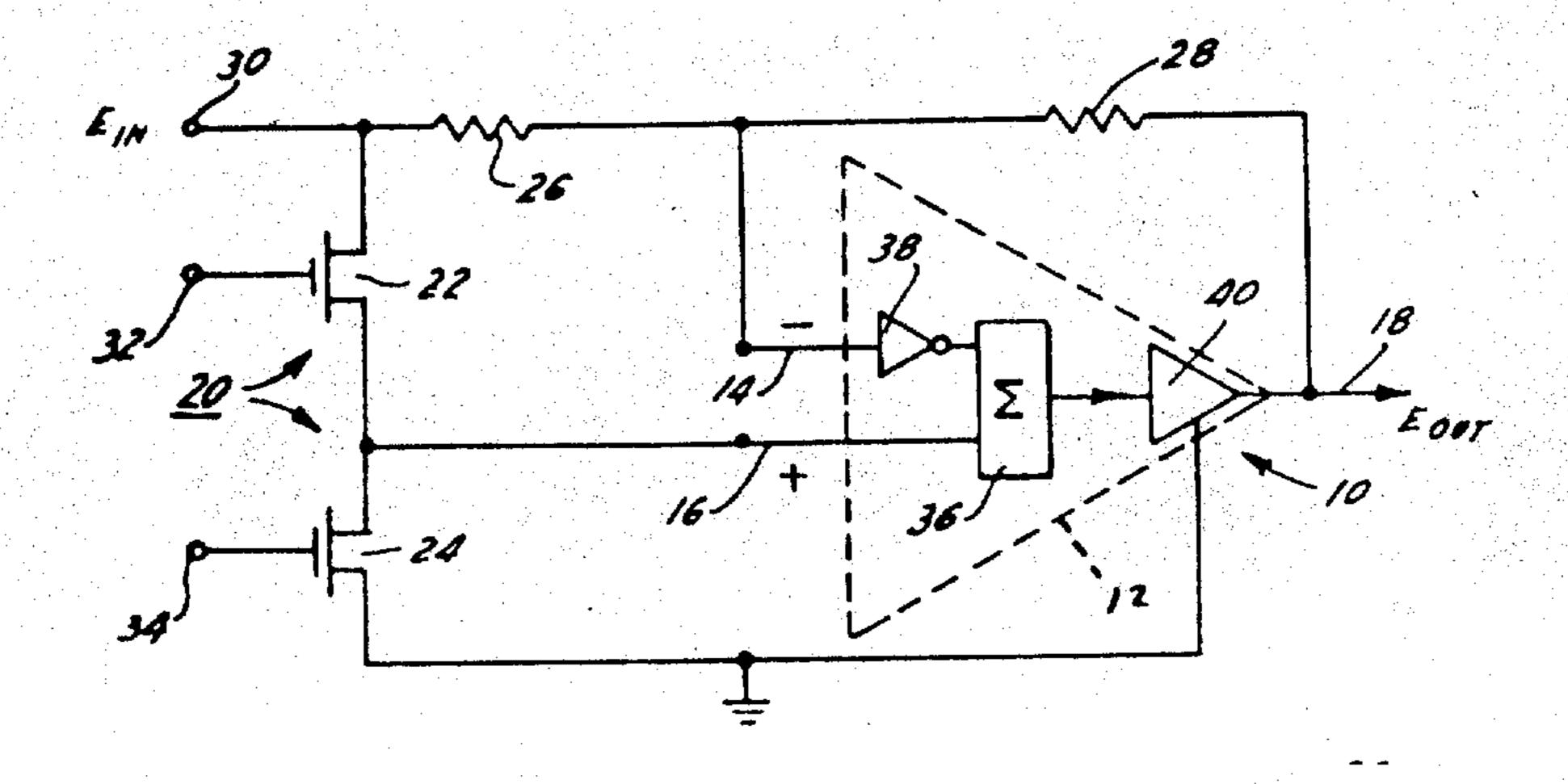
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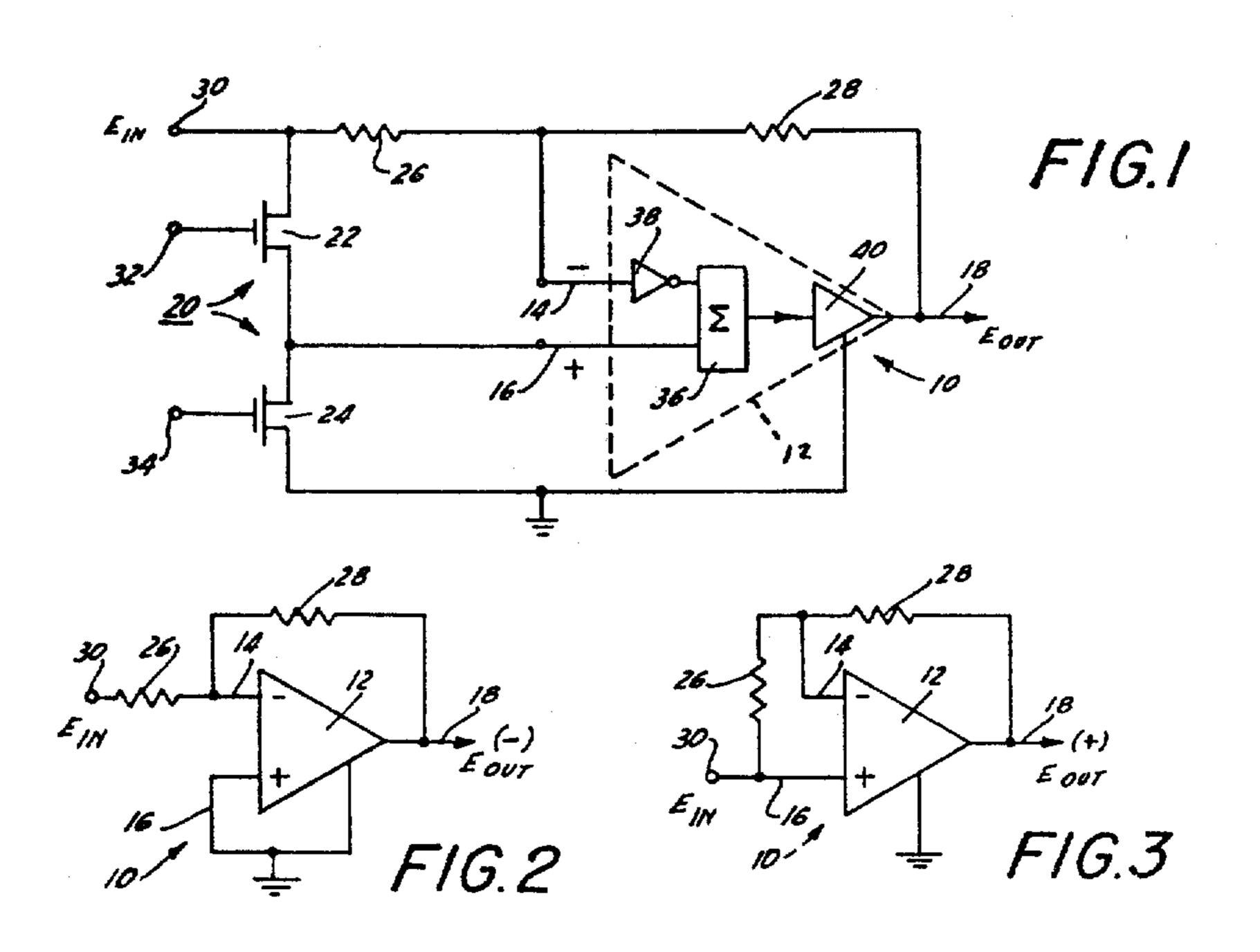
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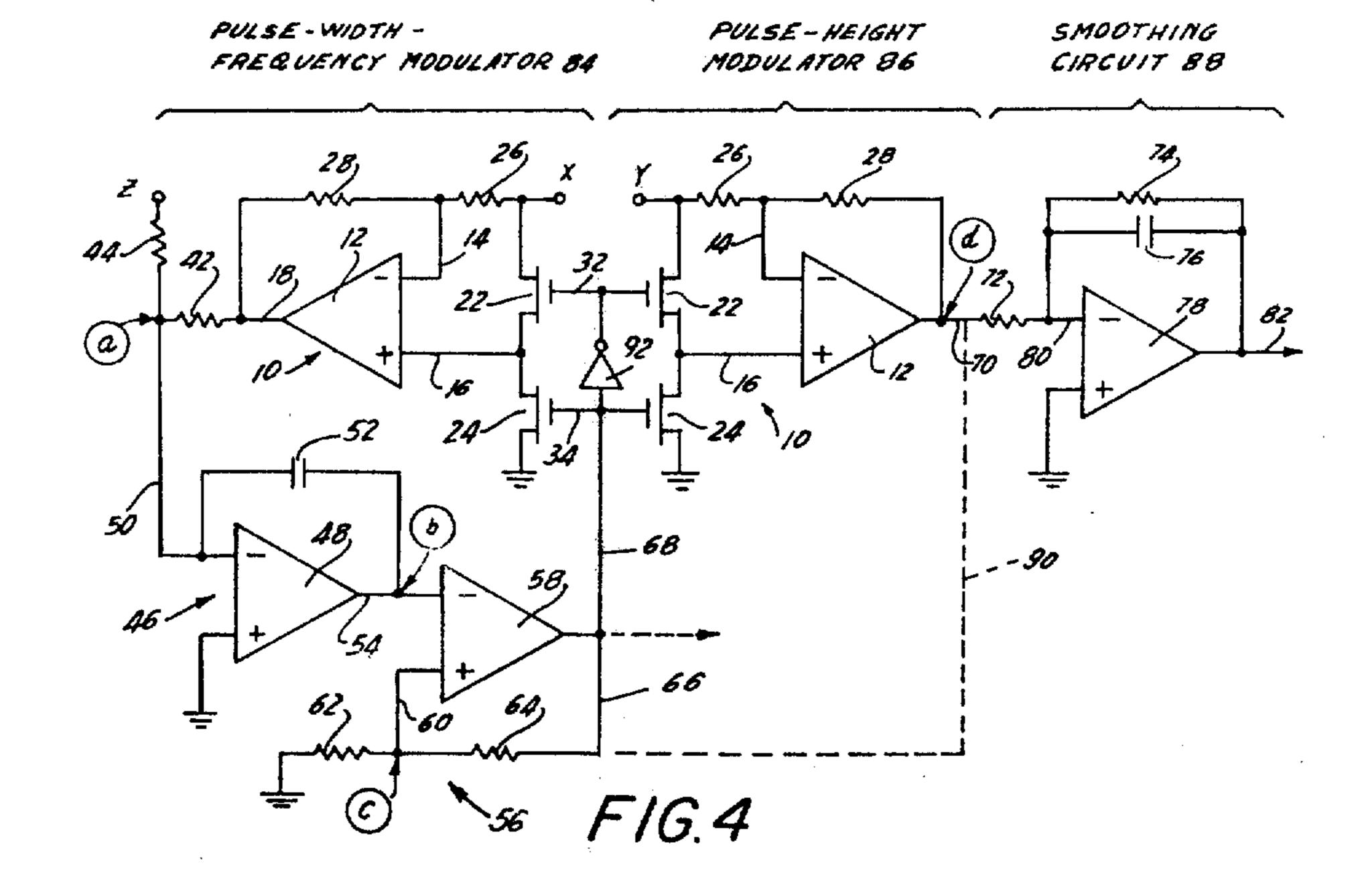
ABSTRACT [57]

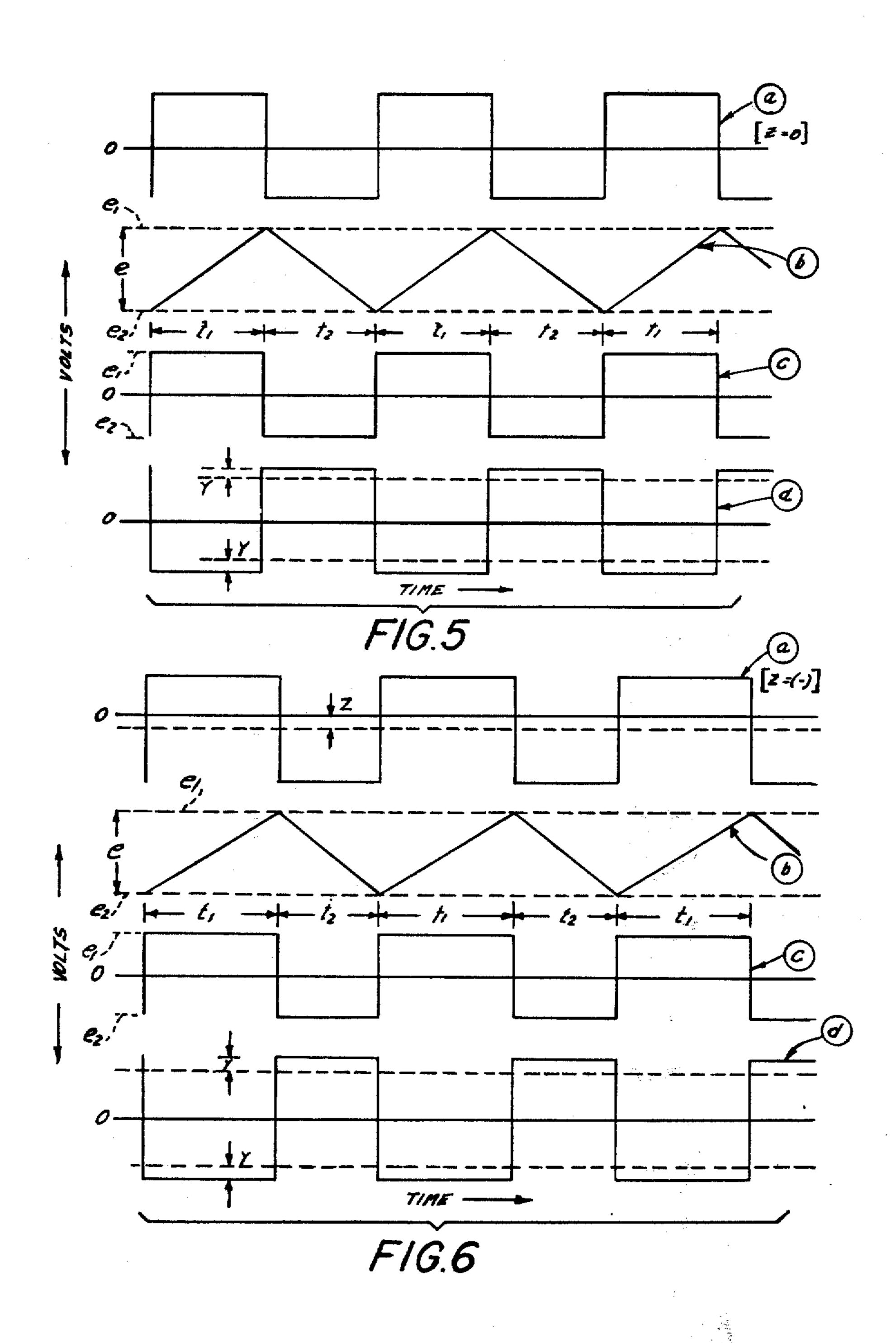
In the switching circuit, the output signal from a highgain differential amplifier is caused to reverse polarity by means of the switching of a pair of field-effect transistors. A resistor is connected in series with the inverting lead of the differential amplifier, and a signal source is connected to the resistor at an input terminal. The field-effect transistors are used to connect the noninverting lead of the differential amplifier alternately between the input terminal and ground. This causes the amplifier output to reverse polarity, and thus provides a very accurate, low-offset means of switching the output of the amplifier. The switching circuit is used in a modulator which permits output pulses to be modulated in width and/or frequency. The reversible output signals of the switching circuit are added to the modulating input signal and the sum is integrated. A level detector changes the polarity of its output voltage whenever the output of the integrator reaches either of two fixed levels. The output of the level detector is used to drive the field-effect transistors to control the switching of the output of the differential amplifier. Such a pulse-width modulator is used to drive a pulseheight modulator so as to form a multiplier. The multiplier can be used to great advantage as a wattmeter by supplying load current and voltage signals to the pulsewidth and pulse-height modulators so as to multiply the load current and voltage by one another. The same device can be used as a divider by holding either one input to the pulse-width modulator or the input to the pulse-height modulator constant, while allowing the other of those inputs to be variable and the input to the switching circuit to be the other variable.

24 Claims, 6 Drawing Figures









MULTIPLIER, DIVIDER AND WATTMETER USING A SWITCHING CIRCUIT AND A PULSE-WIDTH AND FREQUENCY MODULATOR

Matter enclosed in heavy brackets [] appears in the original patent but forms no part of this reissue specification; matter printed in italics indicates the additions made by reissue.

This application is a continuation-in-part of U.S. patent applications Ser. No. 184,026 filed Sept. 27, 1971, now U.S. Pat. No. 3,745,557, issued July 10, 1973 and Ser. No. 6,075, filed Jan. 27, 1970, now U.S. Pat. No. 153,626,292, issued Dec. 7, 1971, said application Ser. No. 184,026 being a division of said application Ser. No. 6,075. The disclosure of both parent applications hereby is incorporated herein by reference.

This invention relates to switching devices, and par-20 ticularly to devices for providing rapid output switching of an amplifier, particularly a differential amplifier. The invention further relates to pulse-width modulators, pulse-height modulators, and to multipliers, divid-

ers and wattmeters and methods.

There are many needs for rapid switching devices, and particularly for devices which reverse the polarity of a signal rapidly. It is an object of the invention to produce such a device which is simple and relatively inexpensive, and yet is highly accurate for use in preci- 30 sion instrumentation.

There also is a need for very accurate, stable and reliable multipliers, dividers and wattmeters. In particular, the lack of wattmeters having sufficient accuracy is believed to cost the electrical power distribution 35 industry in the United States a very substantial amount of money each year. Although many uses easily could be found for electronic wattmeters, the lack of sufficient accuracy, the high cost, and other factors have limited the usefulness of prior electronic wattmeters. 40

In accordance with the foregoing, it is another object of the present invention to provide a highly accurate and relatively inexpensive and reliable multiplier, divider, and wattmeter, and methods of use. It also is desired to provide unique modulators which are capable of pulse-width, pulse-height and frequency modulation. It is another object to provide such devices which make advantageous use of the switching circuit of the invention.

In accordance with the present invention, the foregoing objects are met by the provision of a switching device including a high-gain differential amplifier with first and second input terminals. An impedance is connected between one of the input terminals and a third terminal for receiving the input signal to the circuit. 55 Switching means are provided for switching the second terminal back and forth between the third terminal and a fourth terminal to change the polarity of the difference between the input signals on the input terminals to the differential amplifier, thus causing the output of the 60 device to switch.

The modulator of the invention includes source means (preferably a switching device as described above) for producing an output signal alternating between two levels in response to input switching signals. 65 The output of the source means is added to the modulating input signal, and that sum is integrated by an integrator. A level detector detects the output of the

integrator and produces an output signal change whenever the integrator output reaches either of two different pre-determined levels. The output of the level detector is fed back as an input signal to the source means. This modulator provides pulse-width modulation, as well as frequency modulation. A pulse-height modulator (preferably using another of the switching devices) operates on the output of the pulse-width modulator to provide a multiplier device. By making 10 one of the input signals to the multiplier proportional to a load current, and the other input proportional to a load voltage, the device can be used as a high-precision electronic wattmeter. The same device also can be used as a divider by maintaining one of the inputs constant while allowing the source level to vary, together with one of the inputs. The same device can be used as another type of frequency modulator by connecting the output of the pulse-height modulator back to the level detector to modify the levels at which integrator output signals are detected.

The foregoing and other objects and advantages of the invention will be pointed out or apparent from the following description and drawings.

In the drawings:

FIG. 1 is a schematic circuit diagram of a switching device constructed in accordance with the present invention;

Each of FIGS. 2 and 3 is an equivalent circuit diagram of the circuit shown in FIG. 1, each in a different switching condition;

FIG. 4 is a schematic circuit diagram of a pulse-width-frequency modulator, pulse-height modulator, multiplier, divider and wattmeter device constructed in accordance with the present invention;

FIG. 5 shows waveform diagrams for various electrical signals in the circuit shown in FIG. 4; and

FIG. 6 shows the same waveforms as in FIG. 5, except that certain input signal parameters have been changed.

SWITCHING CIRCUIT DEVICE

FIG. 1 shows a switching circuit device 10 constructed in accordance with the present invention. The device 10 includes an integrated operational differential amplifier 12, which is shown in dashed outline. The differential operational amplifier 12 has a non-inverting input lead 16, an inverting lead 14, and an output lead 18. A negative feed-back resistor 28 is connected between the output lead 18 and the inverting input lead 14 to form a high-gain differential amplifier.

Connected to the inverting lead 14 is a resistor 26 whose other end is connected to an input terminal 30. A gating circuit 20 is provided. The gating circuit includes two field-effect transistors ("FETs") 22 and 24. The source-drain path of the FET 22 is connected between the input terminal 30 and the non-inverting lead 16 of of the differential amplifier. The source-drain path of the FET 24 is connected between the non-inverting lead 16 and the common point in the circuit, e.g., ground. The gate leads 32 and 34 of the two FETs are connected to one or more switching sources (not shown in FIG. 1) which will operate to turn on one of the FETs while the other FET is turned off.

The operational differential amplifier 12 is of the conentional integrated type. This amplifier 12 includes a summing circuit 36 which receives the input from the non-inverting lead 16. The summing circuit 36 also

receives an inverted input signal from the inverting lead 14 through an inverter 38. The output of the summing circuit is delivered to an amplifier 40 whose output lead is the output lead 18 of the differential amplifier.

FIG. 2 shows the effective connection of the FIG. 1 5 circut in the "inverting" mode; that is, in the mode in which FET 24 is turned on and FET 22 is turned off. The non-inverting lead 16 is connected to ground, and the inverting lead 14 is connected through resistor 26 to the input terminal 30. The relationship between the 10 input voltage E_{IN} applied at terminal 30 and the output voltage E_{OUT} at terminal 18 is given by the following equation:

$$E_{OUT} = -E_{IN}(R_{28}/R_{26}) \tag{1}$$

in which R_{28} is the resistance of resistor 28, and R_{26} is the resistance of resistor 26.

It can be seen from the foregoing equation that when R_{26} equals R_{28} , the circuit 10 merely is a unity-gain inverter.

FIG. 3 shows the circuit 10 in the other of its two switching modes, the non-inverting mode. In this mode, the FET 22 is turned on and FET 24 is turned off. Thus, the non-inverting terminal 16 now is connected to the 25 input signal terminal 30 instead of to ground. The full input voltage is applied to the non-inverting terminal 16, while the voltage drop across the resistor 26 is applied (together with a negative feed-back signal through resistor 28) to the inverting terminal 14 of the differential amplifier. The result of this connection is that initially a differential voltage appears between the terminals 14 and 16 due to the operation of the gating circuit 20. This differential lasts only during the very brief switching time of the amplifier output. In the ensuing steady-state condition, the signal on terminal 16 equals that on terminal 14 when the output 18 is equal to input 30 and no current flows through resistors 26 and 28. Therefore, the output of FIG. 1 is noninverted; that is, if the input signal is positive, the output signal is positive and equal, and vice versa. Thus, the polarity of the output signals has reversed.

The circuit shown in FIG. 3 is merely a follower type of circuit. Thus, the relationship between the output and input voltages is independent of resistors 26 and 28, and is expressed by the following equation:

$$\mathbf{E}_{otr} = \mathbf{E}_{tN} \tag{2}$$

In the FIG. 3 circuit, the values of resistors 26 and 28 can vary widely without changing the output signal in 50 any significant degree because the amplifier inputs 14 and 16 have a relatively high impedance, and no current flows through the resistors.

The switching circuit shown in FIG. 1 has a number of advantages, particularly in electrical instrumenta- 55 tion. The device 10 provides an output signal which switches as fast as the operational amplifier 12 will permit, and this can be in the order of a few nanoseconds. The switching also can be accurate because of the circuitry does little if anything to degrade the inherently good accuracy of the operational amplifier. For example, the series "on" resistances of the FETs 22 and 24 do not adversely affect the accuracy because the "off" resistance is millions of times greater than the "on" resistance.

The circuit device 10 has a substantial cost advantage over circuits previously used in instrumentation for similar purposes. For example, a typical prior art de-

vice is shown in the article entitled "An Electronic Multiplier for Accurate Power Measurements," by Tomota, Sugiyama and Yamaguchi, in "IEEE Transactions on Instrumentation and Measurement," Volume IM-17, No. 4, Dec., 1968. In such a prior art device, two separate precision voltage sources are used, with the outputs of each being sampled alternately by series FETs. An output operational differential amplifier is used as a buffer. As compared with applicant's circuit, such a prior circuit requires one additional precise voltage source that is not required by applicant's device. Thus, applicant's device is less expensive. Furthermore, the series "on" resistances of the FETs and the extra voltage source are sources of potential error which are not present in the present invention.

Other prior circuits have used a single precise voltage source, series FETs for switching, an inverting operational amplifier, and an output operational amplifier as a buffer. Such circuits also are inherently more costly and error-prone than applicant's circuit because they require one additional precise operational amplifier. Furthermore, the series FETs and extra operational amplifier are extra sources of error.

MODULATORS

FIG. 4 simultaneously shows a number of different circuits using the switching device 10. The first of these circuits is a modulator 84 which is shown in the left hand portion of FIG. 4. The modulator 84 provides an alternating output pulse signal whose width and frequency are moduated by an input signal applied at an input terminal Z or, at a terminal X corresponding to the terminal 30 shown in FIGS. 1, 2 and 3.

The modulator 84 includes the switching device 10 which delivers its output through a resistor 42 to a node (a). An input signal is applied at terminal Z to the node (a) through a resistor 44, and is added to the output of the switching circuit. The sum of the two signals is supplied over a lead 50 to an integrating circuit 46. The output of the integrator 46 is delivered to a comparator or level-detector circuit 56. The output of circuit 56 is delivered through a lead 68 to the gate leads 32 and 34 of FETs 22 and 24 of the switching device 10.

The operation of the modulator 84 now will be ex-45 plained with the assistance of FIGS. 5 and 6. FIG. 5 shows the waveforms of the signals appearing at points (a), (b), and (c) of FIG. 4, for the condition in which the signal applied to terminal Z is zero, and the signal applied to terminal X is a steady DC signal. The rectangular wave (a) shown in FIG. 5 is integrated by the integrator 46, which produces an output whose waveform is substantially triangular, as is shown at (b) in FIG. 5. The reason for this is that as the output voltage from the integrator reaches a first level e₁, the comparator causes its output voltage (c) to reverse polarity abruptly. This causes the switching circuit 10 to operate to reverse the polarity of its output and start the output of integrator 46 declining in a linear fashion. When the integrator output reaches a second level e2, which is equal but opposite in polarity to the level e₁, the comparator again causes its output voltage to reverse polarity, again causing the switching circuit 10 to operate, thus starting the cycle once again.

FIG. 6 illustrates how the above-described wave65 forms are changed when a negative DC signal is applied
to terminal Z. First, the whole waveform (a) is shifted
downwardly, meaning that the positive portions are of
lower magnitude and the negative portions are of

35

greater magnitude. Thus, the slope of the positivegoing portions of the waveform (b) is lower. However, since the negative portions of the waveform (a) are of greater magnitude than before, the slope of the negative-going portions of the waveform (b) is steeper. 5 Since the separation e between comparator levels e_1 and e_2 is constant, the time duration t_1 of the positivegoing portions of the integrator output is greater than the time duration t_2 of the negative-going portions of the wave. This is in contrast to the waveform (b) shown 10 in FIG. 5, in which t_1 is equal to t_2 .

The waveform (c) in FIG. 6 shows the result of this change on the output of the modulator. The positive pulses have the time duration t_1 , and the negative pulses have the duration t_2 . Thus, the widths of the 15 pulses have been modulated by the signal applied to terminal Z. In addition, as will be more readily apparent from the equations set forth below, the frequency of the waveform (b) also has been changed by the application of the input signal. Thus, the modulator 84 20 is both a pulse-width modulator and a frequency modulator.

The following equations describe the operation of the modulator:

$$t_{1}=eC/(I_{z}+I_{x})$$

$$t_{2}=-eC/(I_{z}-I_{x})$$

$$I_{z}/I_{x}=(t_{1}-t_{2}) (R_{28}/R_{26})/(t_{1}+t_{2})$$

$$I_{z}/I_{x}=(t_{1}-t_{2})/(t_{1}+t_{2}) (if R_{28}=R_{26})$$

$$I_{z}/I_{x}=t_{1}/(t_{1}+t_{2}) (if R_{26}=is infinitely large)$$

$$f=I/(t_{1}+t_{2})=(I_{x}^{2}-I_{z}^{2})/2I_{x}eC$$
(8)

In which: t₁, t₂ and e are defined above and in FIGS. 5 and 6.

C is the integrating capacitance, the capacitance of capacitor 52.

 \hat{I}_z is the current input to terminal Z.

I_x is the current input to terminal X.

 R_{26} and R_{28} are the resistances to resistors 26 and 28, 40 respectively.

f is the frequency of the output of the modulator; i.e., the frequency of the waveform (b) in FIGS. 5 and 6.

The modulator 84 also can be considered to be a free-running oscillator whose output frequency f varies 45 inversely with the magnitude of the Z signal, and directly with the X signal. Equation (8) shows this variation.

The foregoing equations also demonstrate the effect of the input signal on terminal X in the width modula- 50 tion of the output signal. Thus, both the X and Z inputs are capable of modulating the pulse width and frequency of the output.

Integrator 46 is a conventional high-precision integrator circuit. It includes an operational differential 55 amplifier 48 and a negative feedback capacitor 52. The input signal is applied to the inverting terminal, and the non-inverting terminal is grounded. The value of the capacitance C of the capacitor 52 is set so that, in accordance with equation (8) above, the frequency will 60 be within the desired range. For steady DC inputs, the frequency f is not particularly critical. However, for AC or fluctuating DC inputs, the frequency f should be high enough to ensure that a very small sample of the input wave is taken during each cycle of modulator 65 operation. For precision measurements, f should be over 100 times the frequency of the input wave, and preferably is at least 200 times the latter frequency.

The comparator circuit 56 consists of another operational differential amplifier 58 with a resistor network connected to the non-inverting input lead 60 of the amplifier 58. The inverting input lead receives the output from the integrator circuit 46. The resistor network consists of resistors 62 and 64 and is connected with the non-inverting input 60 and the output of the amplifier 58 in order to provide positive feedback.

In operation, the comparator produces a large (e.g., 10 volts) DC output signal of one polarity when the signals applied to the input terminals are different from one another in one direction, and a DC signal of equal magnitude but the opposite polarity when the input voltage difference is in the opposite direction. The ratio of the resistance of resistor 64 to the resistance of resistor 62 sets the value of the voltages or "break points" e₁ and e₂ (FIGS. 5 and 6) at which the signals on the input leads of the amplifier 58 equal one another. In essence, the circuit 56 can be thought of as a level detector, since it produces a change in the output signal whenever the input reaches a predetermined, fixed level. In this context, the circuit 56 also can be thought of as a level-sensitive flip-flop circuit.

In applying the output of the comparator 56 to the FETs 22 and 24, an inverter 92 is connected between the leads 32 and 34. This is because the FETs 22 and 24 are of the same channel type; that is, both FETs are of either the N-channel or P-channel type. The inverter 92 ensures that the gate of one FET always will receive a signal of a polarity opposite to that received by the other gate. If FETs of opposite channel type are used, that is, if one FET is of the N-channel type and the other of the P-channel type, the inverter can be omitted and replaced by an ordinary conductor.

PULSE-HEIGHT MODULATOR

The middle portion 86 of the circuit shown in FIG. 4 is a pulse-height modulator. Actually, the modulator is nothing more than a switching device 10 as is shown in FIGS. 1, 2 and 3, with an input terminal Y corresponding to the terminal 30 to receive the modulating input. Pulses from the comparator 56 are applied to the input gate leads 32 and 34 in the manner described above. The curve (d) in each of FIGS. 5 and 6 shows how the height of the pulses has been increased by the signal Y. It is readily apparent from equations (1) and (2) how this pulse-height modulation is accomplished.

MULTIPLE-INPUT FREQUENCY MODULATOR

A different frequency modulator can be produced by simply re-arranging the circuitry of the modulators 84 and 86 already described. In this embodiment of the invention, the connection 66 coupling the resistor 64 to the output of amplifier 58 is removed. Also, the output lead 70 of the pulse-height modulator 86 is connected by means of a lead 90 (shown as a dashed line) to the right end of resistor 64. Further, there is no input on the Z terminal.

With this arrangement, the output of the pulse-height modulator 86 is used to vary the separation e of the breakpoints e_1 and e_2 of the comparator circuit 56. This provides a further means of changing the frequency f of the output signal at point (c) in FIG. 4. Equation (8) above shows the effect of such a change of the quantity e on the frequency f of the output wave. The following equation describes the simple relationship between the inputs at terminals X and Y in the circuit here being described:

 $f=I_x/I_y$

In which I_y is the current input at terminal Y, and I_x is defined above.

It is readily evident that this circuit has valuable utility as a voltage-to-frequency convertor, and specifically as a dividing circuit with an output in terms of frequency. This can be very valuable in analog-to-digital conversion.

MULTIPLIER, WATTMETER AND DIVIDER CIRCUIT

The entire circuit shown in FIG. 4 forms a high-precision multiplier, wattmeter and divider circuit. The circuit includes the modulators 84 and 86 described 15 above, together with a smoothing circuit 88 which smooths the output from the pulse-height modulator 86 to produce a DC signal on the output lead 82 which is proportional to the product or division of two input signals.

The smoothing circuit 88 includes an input resistor 72, a differential operational amplifier 78, and a negative feed-back network consisting of parallel-connected resistor 74 and capacitor 76. The input signal is received on the inverting terminal 80. In essence, the 25 smoothing circuit 88 in a low-pass filter.

MULTIPLICATION, POWER MEASUREMENT AND DIVIDING METHODS

The mode of operation of the circuit shown in FIG. 4 30 depends upon the method by means of which it is used. The general equation of the circuit is set forth below:

$$\mathbf{I}_{OUT} = \mathbf{I}_{z} \mathbf{I}_{y} / \mathbf{I}_{z} \tag{10}$$

In which I_{OUT} is the output current from terminal 82, and the other quantities have been defined above.

The circuit can be used as a multiplier by holding the signal input at terminal X constant, and allowing the signals applied to terminals Y and Z to be the input 40 variables. When this is done, it can be seen from equation (10) that the output is simply the product of the inputs at terminals Z and Y.

When the device is used as a wattmeter, it is used as a multiplier; one of the input signals at Y or Z is made 45 equal to the load current of the power being measured, and the other input is made equal to the load voltage. The output then is the product of the load current and voltage; i.e., the load power.

In use as a dividing circuit, either of the output sig- 50 nals on terminals Z and Y is held constant while the signals on terminals X and the other one of the terminals Z and Y is allowed to vary. It can be seen from equation (10) above that the output is proportional to the result of dividing either the Z or the Y signal by the 55 out departing from the spirit or scope of the invention. X signal.

The circuits described above have a number of advantages, particularly in precision instrumentation. It is believed that the invention provides a multiplier, divider and wattmeter which, in practical use, is signifi- 60 cantly more accurate than devices previously available. Furthermore, this greater accuracy is obtained with a substantial reduction in the number of components, and particularly in the number of expensive precision operational amplifiers required in previous circuits. 65 Also, the device can be fast-operating (depending upon the cost one is willing to pay for the operational amplifiers), and is very reliable in operation.

The invention is particularly useful and unexpectedly accurate when operating with time-varying input signals whose waveform is relatively steep at low voltages and relatively flat at higher voltages. Typical of such a waveform is a sine wave, which has the greatest slope at zero volts and the lowest slope at its peaks. As noted above, typically, the frequency f at which the circuit operates is at least 100, and preferably 200 times the frequency of the Z input signal. The reason for the 10 desirability of such frequent sampling is that it ensures that each portion of the input wave most closely approximates a DC signal. Since accuracy is reduced if this approximation is not carefully maintained, it is desired to have a relatively high operating frequency in areas in which the waveform has a steep slope and tends to degrade the approximation, whereas, a lower operating frequency will be sufficient where the waveform is relatively level and is close to being DC anyway. In the present invention, the frequency f of the operation of the circuit automatically is highest with the lowest input signals. In fact, when the input signal Z is zero, the highest frequency f occurs, and when the highest input signal occurs (Z must be lower than X), the lowest frequency results. Thus, it is an unexpected advantage of the circuit that is automatically operates with such waveforms in the most precise manner.

Following is a list of components which have been used in a circuit as shown in FIG. 4 which has been built and successfully operated. It should be noted that the components for the switching circuit 10 are included in this list.

Component 35 Operational Amplifiers 12, 48, 53 and 78 and Inverter 92

FETs 22 and 24

Capacitor 76

Amplifier sold by Analog Devices Corp.; a fast-setting operational differential amplifier. Switching time: 100 nanoseconds. Amplitude precision: 2 parts per million. 3N153 (N-channel depletion type) 2,000 ohms, (matched precisely)

The Model 44 Operational

Resistors 26 and 28 Resistor 42 Resistors 62, 64 and 72 Resistor 44 Resistor 74 Capacitor 52

2,000 ohms 5,000 ohms 0.02 microfarads (precise for voltage/frequency converter) 2 microfarads

The above description of the invention is intended to be illustrative and not limiting. Various changes or modifications in the embodiments described may occur to those skilled in the art and these can be made with-

1,000 ohms

10,000 ohms

I claim:

1. A multiplier device comprising a pulse-width modulator controlled by a first input signal, and a pulse height modulator controlled by a second input signal. and connected to said pulse-width modulator, said pulse-width modulator comprising source means for producing an output signal alternating between two levels in response to input switching signals, means for modifying at least one of said levels in response to a modulating signal, integrator means for integrating the output of said source means so modified, said source means including a switching device, said device including a high-gain differential amplifier having first and second input terminals, an impedance connected between the first of said input terminals and a third terminal for receiving an input signal, and gating means for connecting the second of said input terminals alternately between said third terminal and a fourth terminal for receiving a signal of a magnitude lower than the input signal applied to said third terminal, level detector means for detecting the output of said integrator means and producing an output signal change whenever the integrator output reaches either of two different pre-determined levels, and means for transmitting the output of said level detector means to said source means as input gating signals to said source means.

2. A multiplier as in claim 1 including a smoothing 15 circuit for the output of said pulse-height modulator.

- 3. A wattmeter including a multiplier as in claim 1 with means for receiving inputs proportional to the load voltage and the load current of the power being measured.
- 4. A device as in claim 1 in which said switching means includes a pair of field effect transistors, one connected between said second and third terminals, and the other connected between said second and fourth terminals.
- 5. A device as in claim 1 in which said differential amplifier is an operational amplifier with a feed-back impedance connected between its output terminal and said first input terminal.

6. A device as in claim 1 in which said first terminal 30 is an inverting terminal, and said second terminal is a

non-inverting terminal.

7. A device as in claim 4 in which one of said transistors is of the P-channel type, and the other is of the N-channel type, a common connection to the gate 35 leads of the transistors, the output of said level detector means being connected to said common connection.

- 8. A device as in claim 4 in which said transistors are of the same channel type, means connecting the output of said level detector means directly to the gate lead of 40 one of said transistors, and an inverter connecting said output of said level detector means to the gate lead of the other transistor.
- 9. A device as in claim 1 in which said impedance is a resistor.
- 10. A device as in claim 1 in which said fourth terminal is common to the input and output of said source means.
- 11. A mathematical circuit device comprising a pulse-width modulator controlled by a first input signal, 50 and a pulse height modulator controlled by a second input signal and connected to said pulse-width modulator, said pulse-width modulator including a circuit device for producing an output signal alternating between two levels in response to input switching signals, said 55 circuit device including a high-gain differential amplifier having two input terminals and an output terminal, and means for reversing the polarity of the output signal on said output terminal in response to a change in polarity of the difference between the input signals on 60 said input terminals, an impedance connected at one end to a first one of said input terminals and at the other end to a third signal-receiving terminal, a fourth terminal for receiving a reference signal, and switching means for connecting the second of said input termi- 65 nals alternately to said third terminal and said fourth terminal, integrator means for integrating said output signal, level detector means for detecting the output of

said integrator means and producing an output signal change whenever the integrator output reaches either of two different pre-determined levels, and means for transmitting the output of said level detector means as input gating signals to said circuit device.

12. A device as in claim 11 including a smoothing circuit for the output of said pulse-height modulator.

- 13. A wattmeter including a device as in claim 11 with means for receiving inputs proportional to the load voltage and the load current of the power being measured.
- 14. A device as in claim 11 in which said differential amplifier includes a summing circuit, an inverter connected between one of said input terminals and said summing circuit, said other input terminal being connected to deliver an uninverted input signal to said summing circuit, a signal amplifier connected to receive the output of said summing circuit, and a feedback impedance connected between said one input terminal and the output of said signal amplifier.
- 15. A device as in claim 11 in which said differential amplifier is an integrated operational amplifier and said feed-back impedance is an external resistor, the first-named impedance is a resistor, said switching means comprises a first field-effect transistor with its source-drain path connected between said second and third terminals, a second field-effect transistor with its source-drain path connected between said second terminal and ground, said transmitting means comprising means for connecting the output of said level detector means to the gate leads of said transistors and turning one of said transistors on while turning the other off.

16. A device as in claim 1 in which the frequency of the output of said level dtector means is at least 100 times the frequency of said modulating signal.

17. A device as in claim 1 in which said levels of said level detector means are fixed and independent from said modulating input signal.

18. A device as in claim 1 in which said level detector
40 means comprises an operational amplifier having inverting and non-inverting input terminals, the integrator output being connected to said inverting terminal, a bias impedance connected to said non-inverting terminal, and a positive feed-back impedance connected
45 between said non-inverting terminal and the output of said operational amplifier.

19. A device as in claim 1 including a node at the output of said source means and the input of said integrator means, and means for conducting said modulat-

ing input signal into said node.

20. A device as in claim 1 including means for changing at least one of said levels of said level detector means in response to another modulating input signal, thereby changing the frequency of the output of said level detector as a function of said other modulating input signal.

- 21. A device as in claim 11 in which said level detector means includes an operational amplifier having inverting and non-inverting input terminals, the integrator output being connected to said inverting terminal, a bias resistor connected to the non-inverting terminal of said operational amplifier, and a feed-back resistor connected between the non-inverting terminal and the output of said operational amplifier.
- 22. An electrical device comprising, in combination, an integrator, level sense means responsive to the output of said integrator for producing a first signal of alternating polarity in which each polarity reversal

occurs when said integrator output reaches one of two different signal levels, means for conducting said first alternating signal to the input of said integrator, first modulating means for changing the magitude of said first signal in accordance with a first input variable X, 5 second modulating means for changing the magnitude of said signal in accordance with a second input variable Y and thus forming a second signal, and means for controlling the separation between said different signal levels in accordance with said second signal.

23. A device as in claim 22 in which at least one of said first and second modulating means comprises a circuit device for producing an output signal alternating between two levels in response to input switching ential amplifier having two input terminals and an output terminal, and means for reversing the polarity of

the output signal on said output terminal in response to a change in polarity of the difference between the input signals on said input terminals, an impedance connected at one end to a first one of said input terminals and at the other end to a third signal-receiving terminal, a fourth terminal for receiving an input variable signal, and switching means for connecting the second of said input terminals alternately to said third terminal and said fourth terminal.

24. A device as in claim 22 in which said level sensor means comprises an operational amplifier having inverting and non-inverting input terminals, the integrator output being connected to said inverting terminal, a bias impedance connected to said non-inverting termisignals, said circuit device including a high-gain differ- 15 nal, and means for conducting said second signal to

said non-inverting terminal.