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Eliaz

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- (54) **DECISION FEEDBACK EQUALIZER WITH MULTIPLE CORES FOR HIGHLY-SPECTRALLY-EFFICIENT COMMUNICATIONS**
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Related U.S. Application Data

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- (51) **Int. Cl.**
H03H 7/30 (2006.01)
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- (52) **U.S. Cl.**
CPC *H04L 25/03949* (2013.01)
USPC *375/232; 375/229; 375/230*
- (58) **Field of Classification Search**
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See application file for complete search history.

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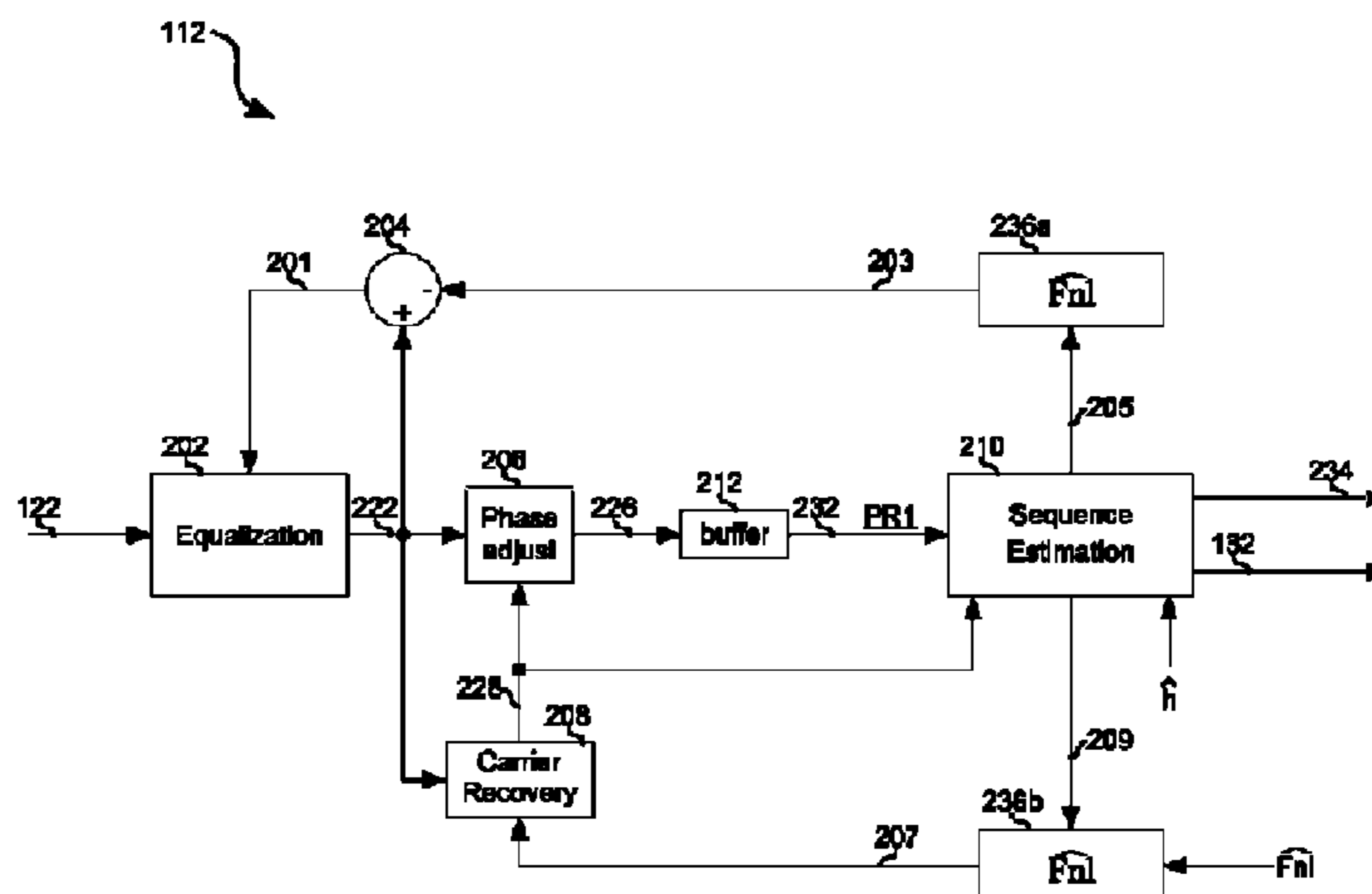
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(57) **ABSTRACT**

One or more embodiments describe a decision feedback equalizer with multiple cores for highly spectrally efficient communications. An equalization circuit in a receiver may include a decision feedback equalizer circuit having a first plurality of tap coefficients that are determined based on a cost function that receives as input an error signal that is an inter-symbol-correlated (ISC) signal. The decision feedback equalizer circuit may further include a second plurality of tap coefficients that are determined based on a filter with an ISC response. The cost function may determine the mean square of the error signal. The cost function is constrained or unconstrained. The error signal may represent error caused by a channel. In some embodiments, the ISC signal may be a partial response signal, and the filter with an ISC response may be a partial response filter.

22 Claims, 12 Drawing Sheets



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Fig. 1

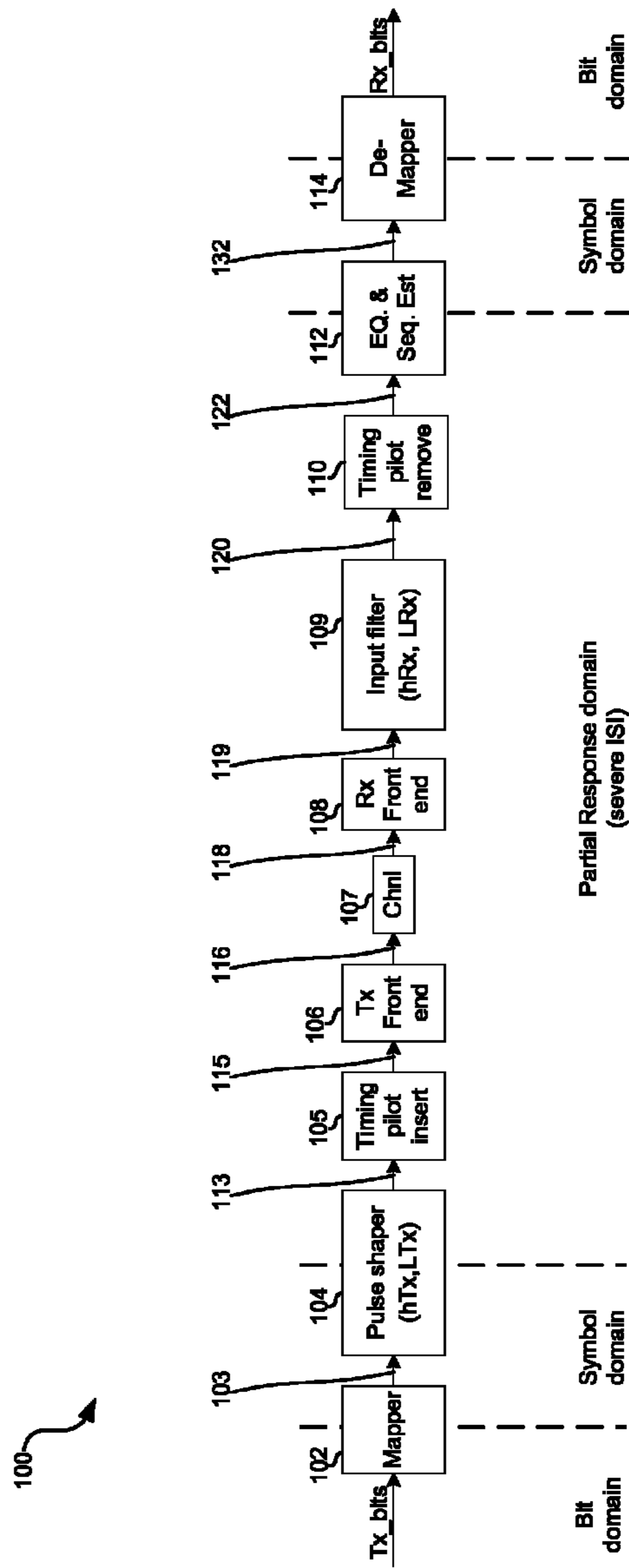


Fig. 2

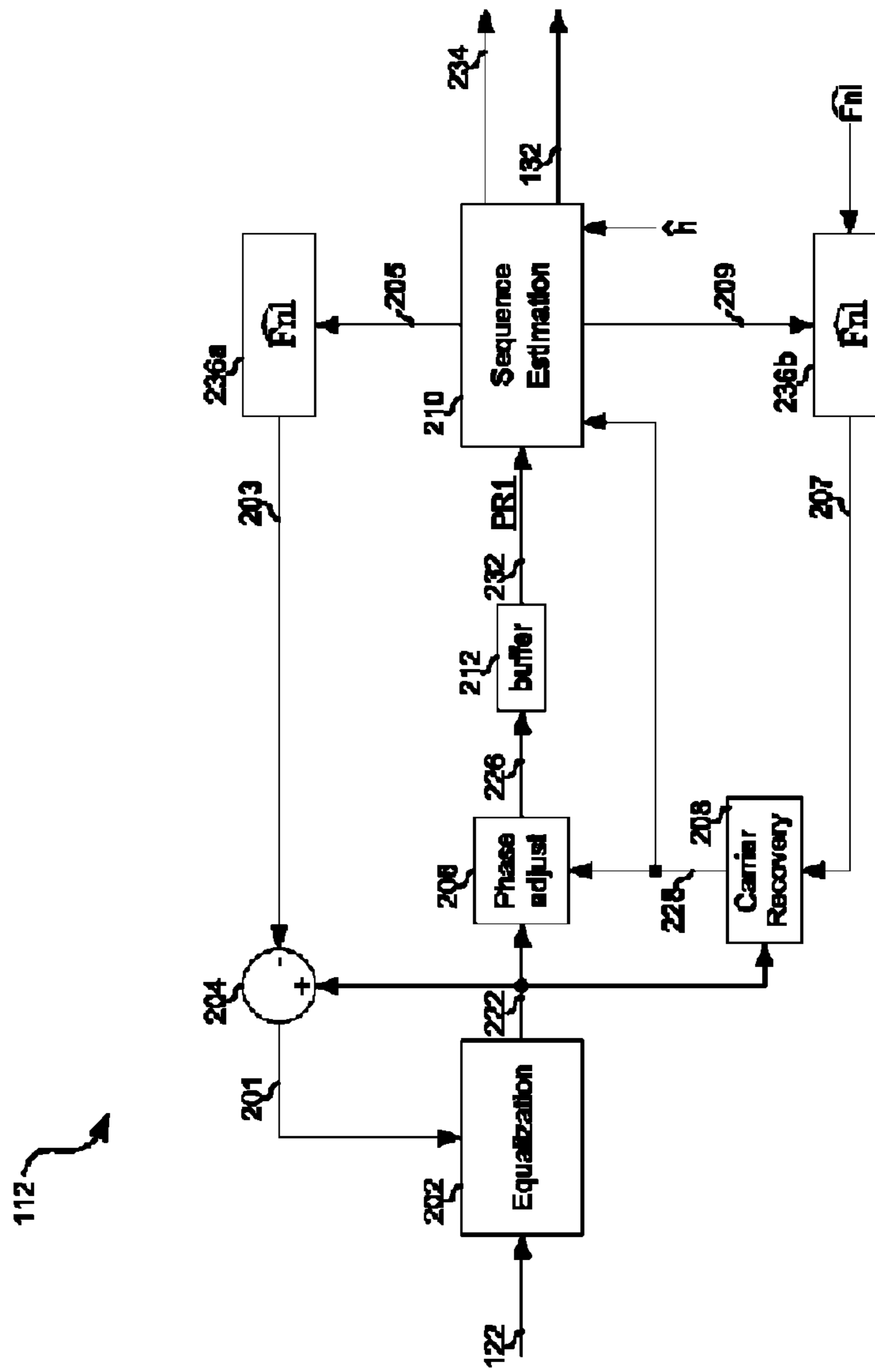


Fig. 3

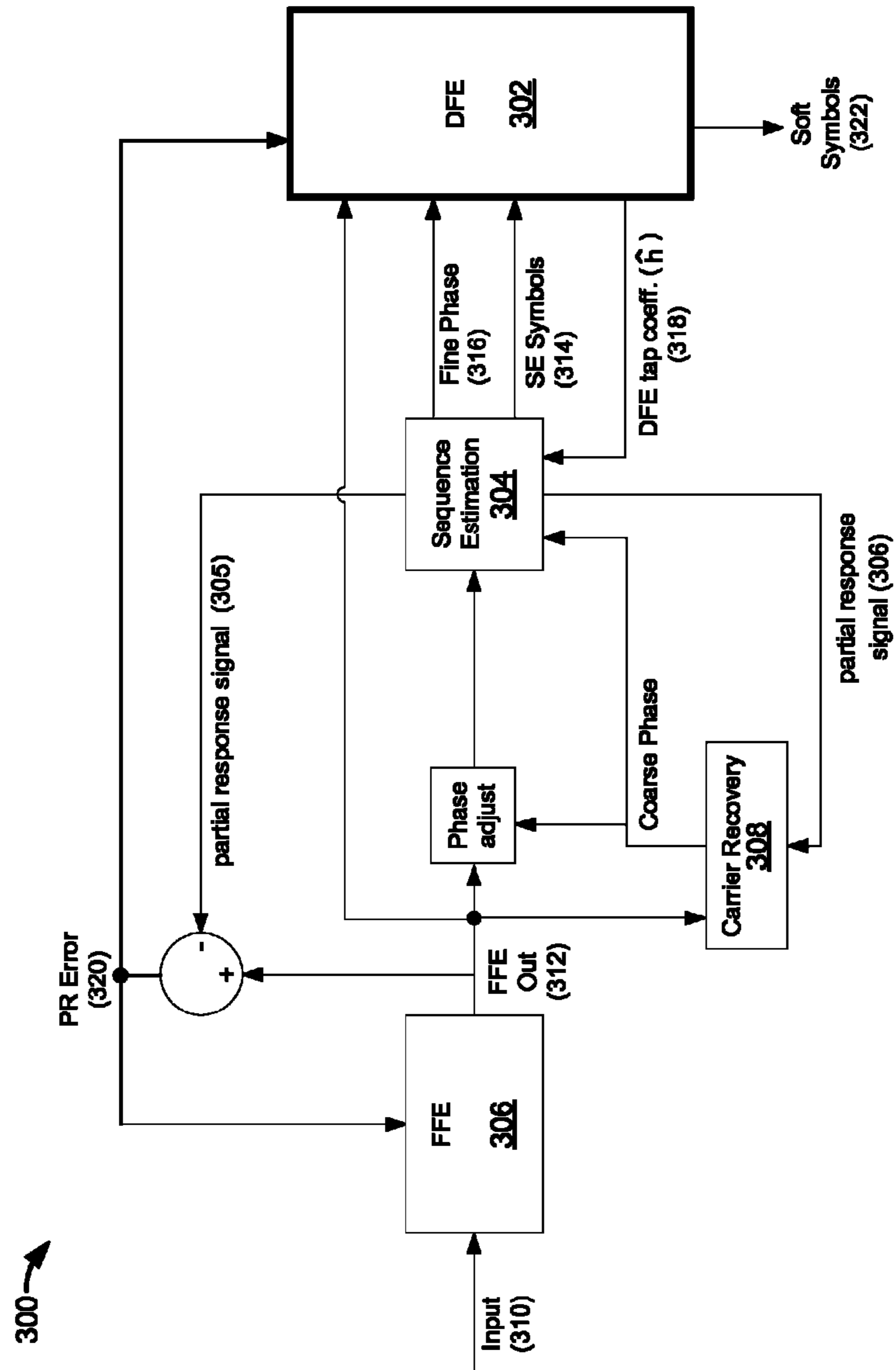


Fig. 4

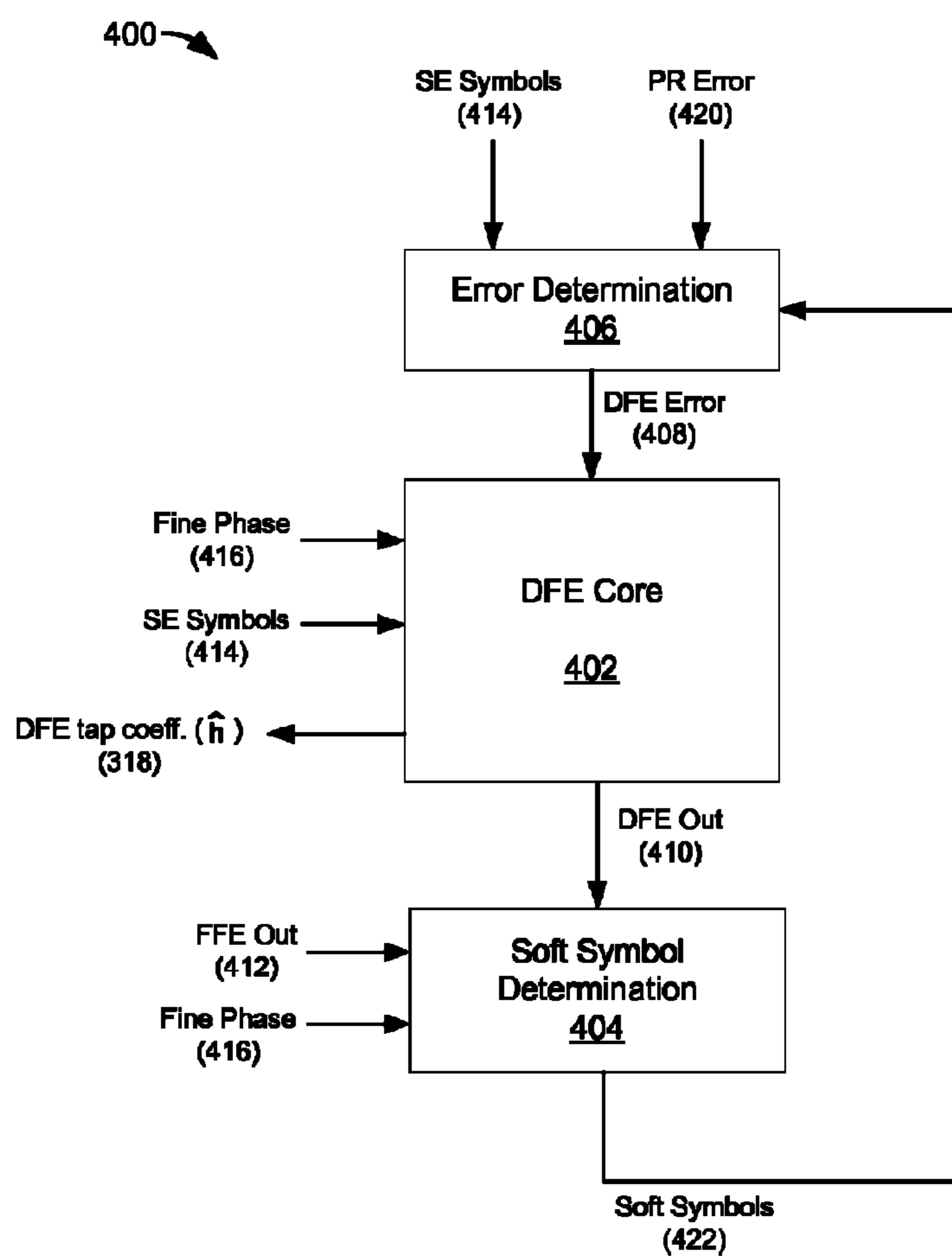


Fig. 5

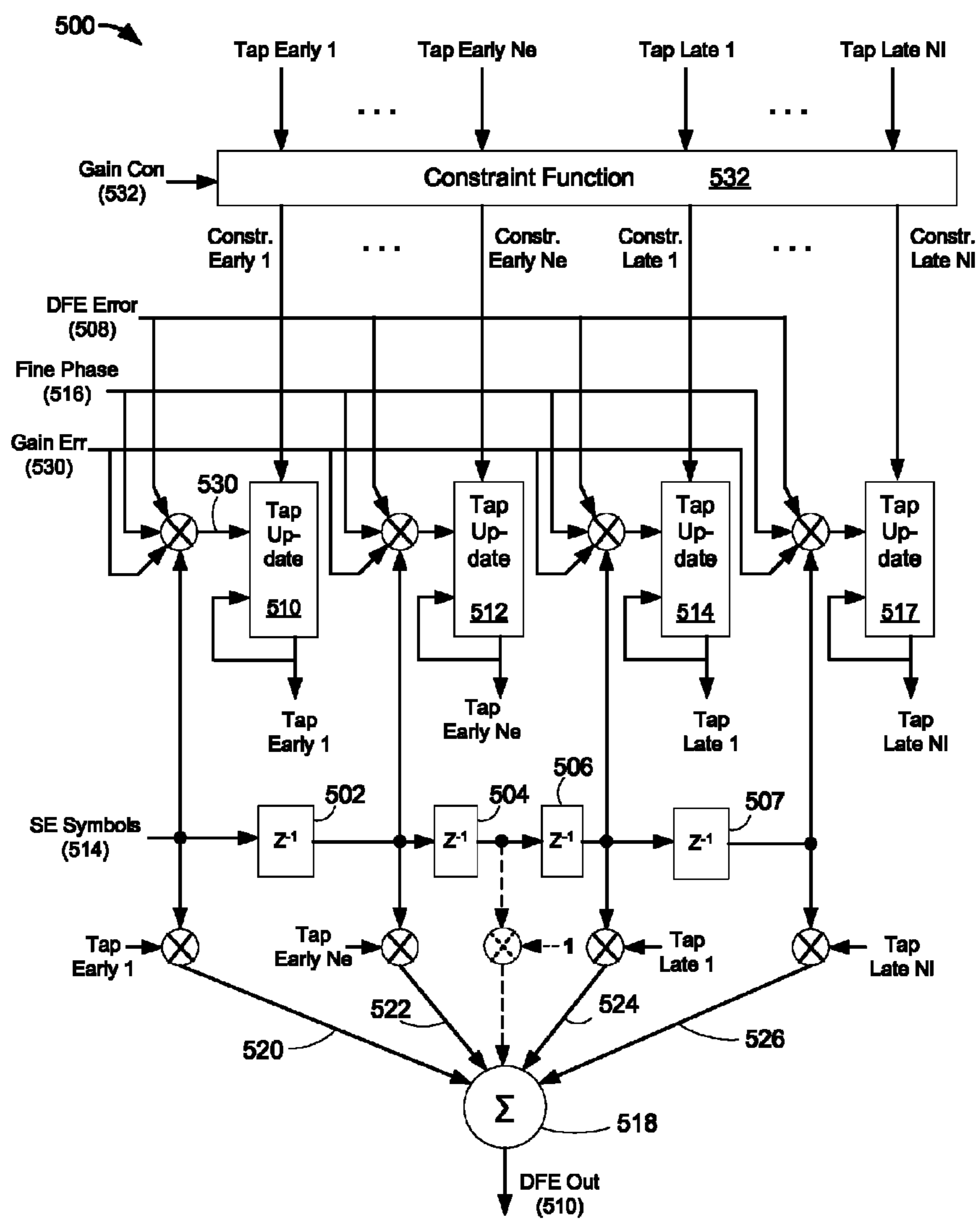


Fig. 6

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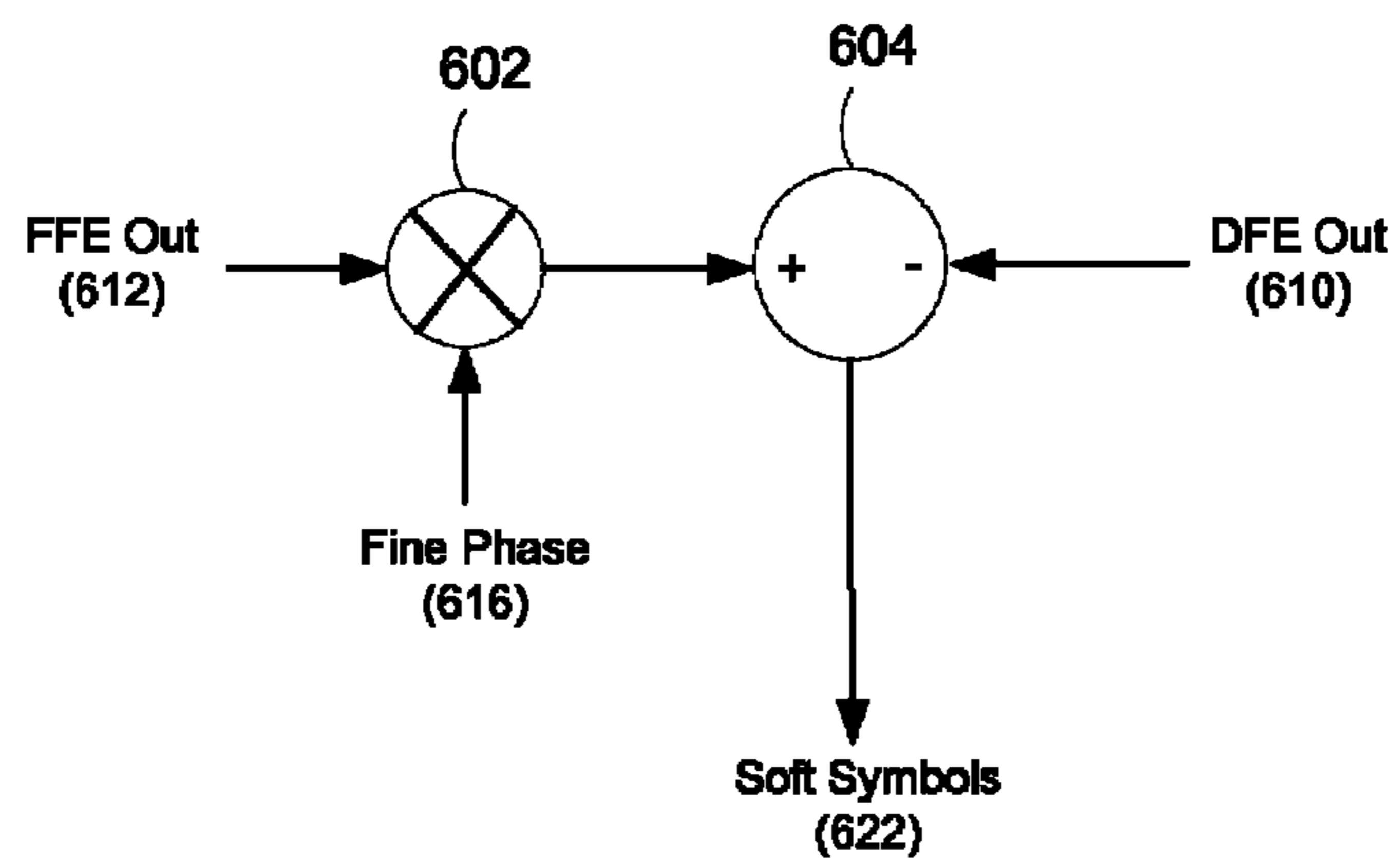


Fig. 7

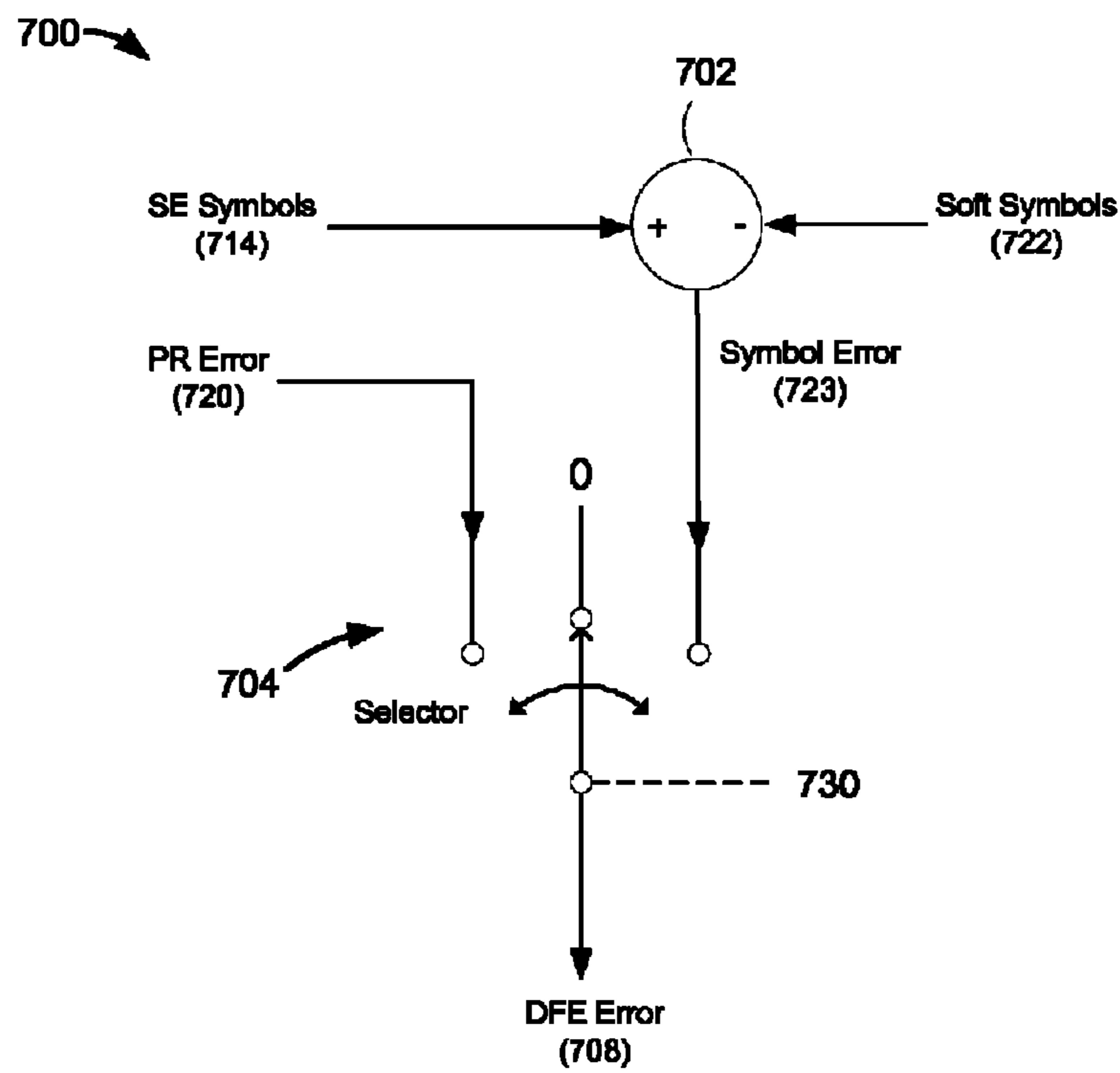


Fig. 8

800 →

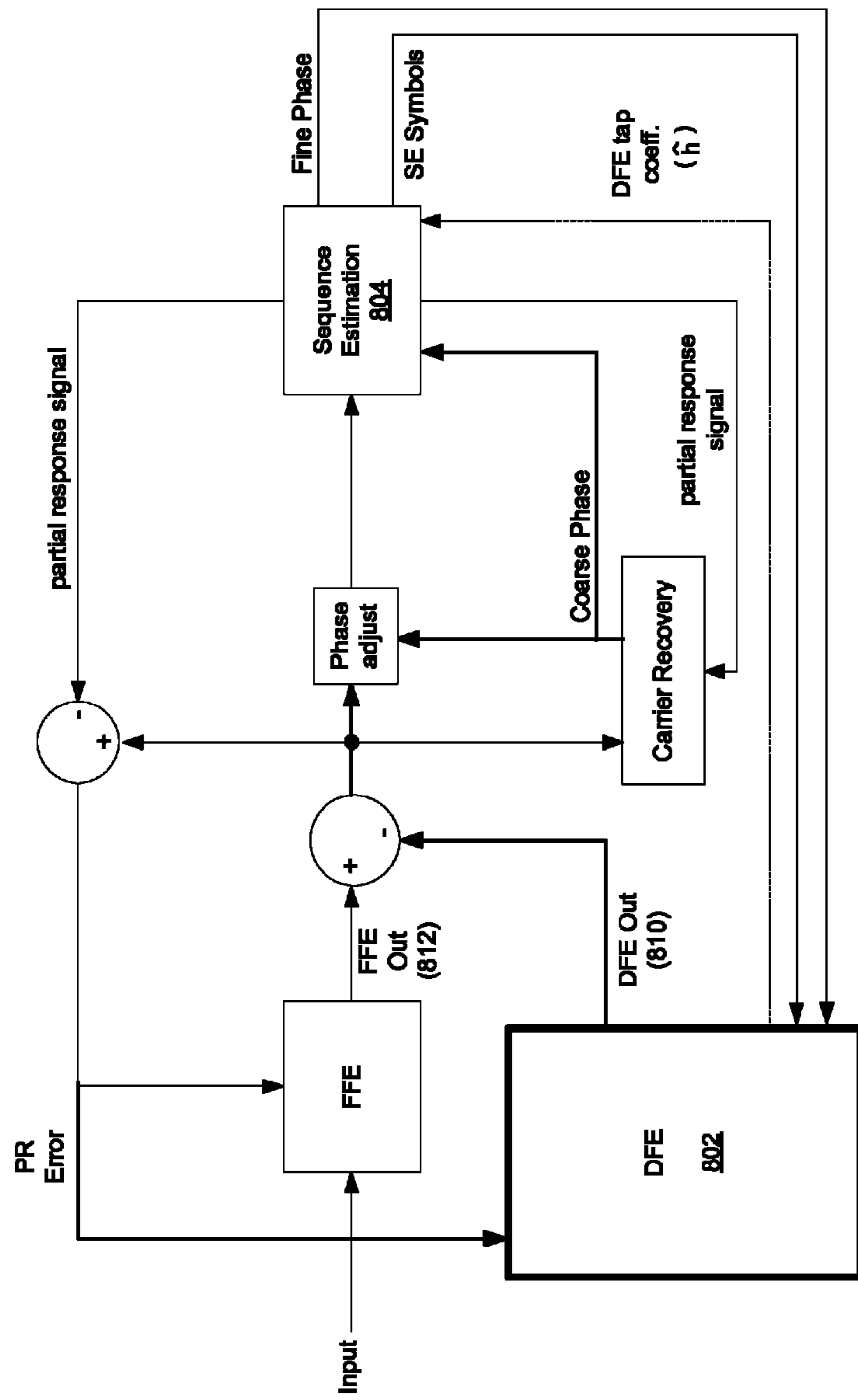


Fig. 9

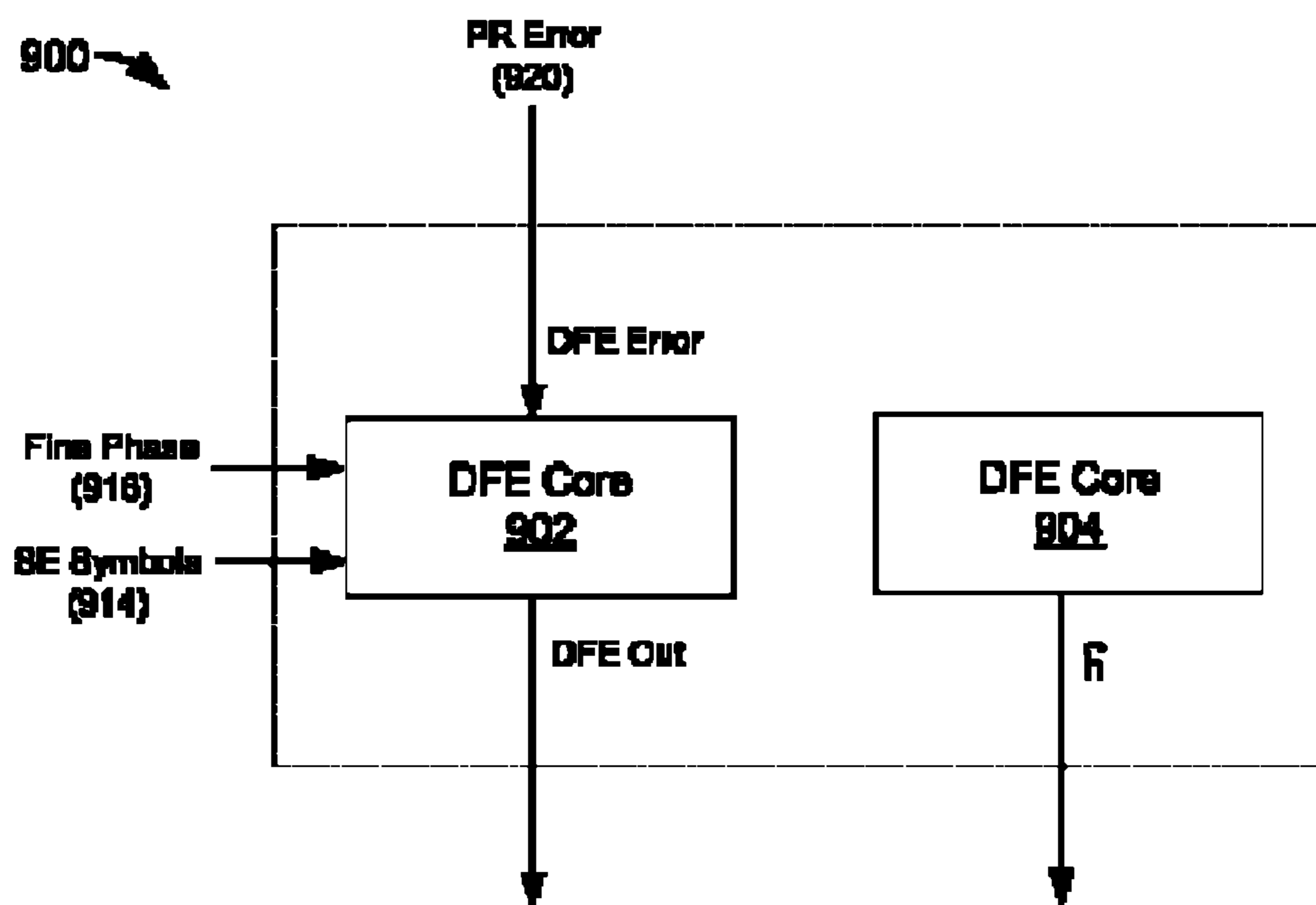


Fig. 10

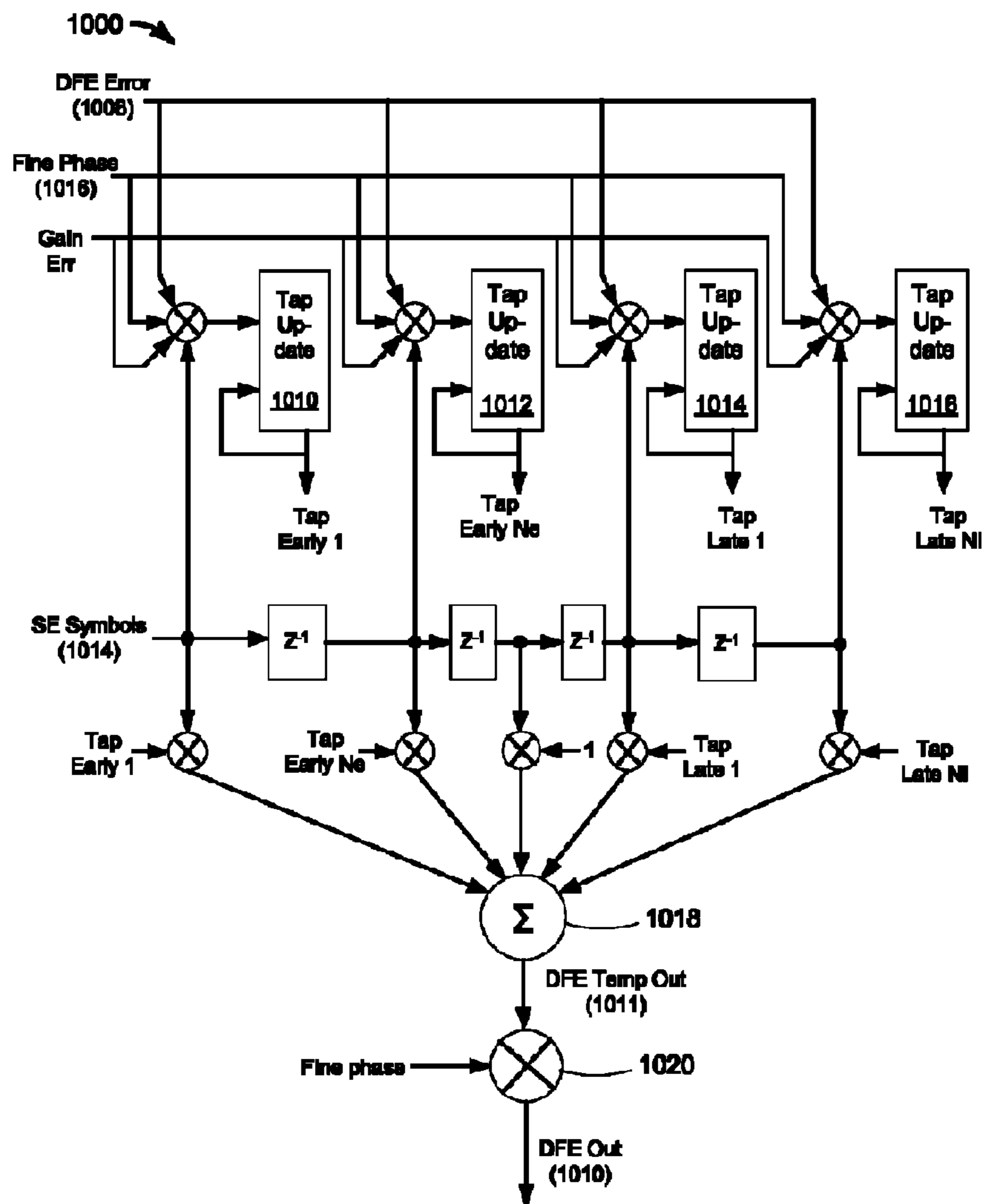


Fig. 11

1100 →

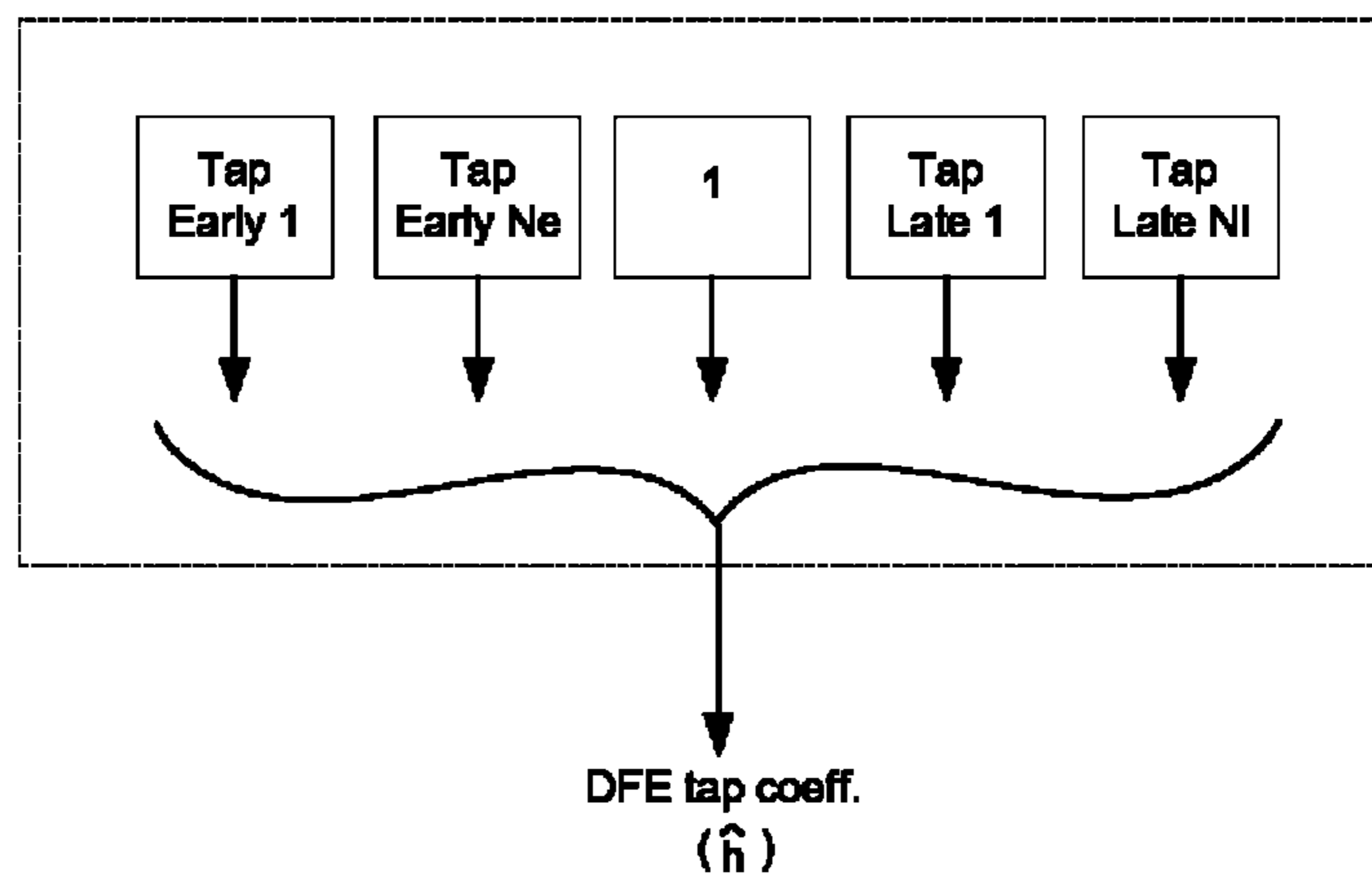
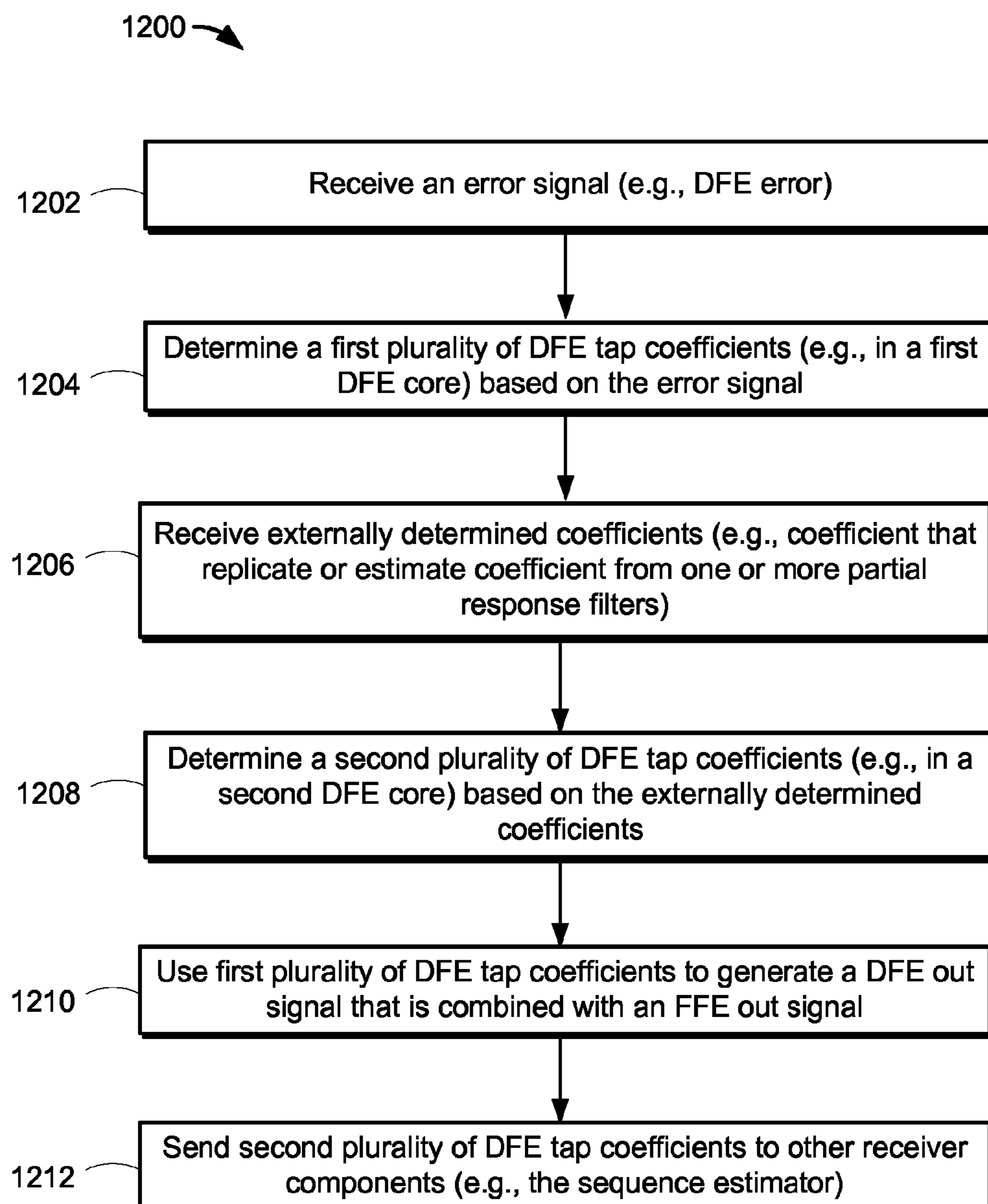


Fig. 12

**DECISION FEEDBACK EQUALIZER WITH
MULTIPLE CORES FOR
HIGHLY-SPECTRALLY-EFFICIENT
COMMUNICATIONS**

CLAIM OF PRIORITY

This patent application makes reference to, claims priority to and claims benefit from:

U.S. Provisional Patent Application Ser. No. 61/662,085
entitled "Apparatus and Method for Efficient Utilization of
Bandwidth" and filed on Jun. 20, 2012;

U.S. Provisional Patent Application Ser. No. 61/726,099
entitled "Modulation Scheme Based on Partial Response"
and filed on Nov. 14, 2012;

U.S. Provisional Patent Application Ser. No. 61/729,774
entitled "Modulation Scheme Based on Partial Response"
and filed on Nov. 26, 2012; and

U.S. Provisional Patent Application Ser. No. 61/747,132
entitled "Modulation Scheme Based on Partial Response"
and filed on Dec. 28, 2012.

Each of the above stated applications is hereby incorporated herein by reference in its entirety.

INCORPORATION BY REFERENCE

This patent application makes reference to:

U.S. patent application Ser. No. 13/754,964, titled "Low-Complexity, Highly-Spectrally-Efficient Communications," and filed on Jan. 31, 2013 (now U.S. Pat. No. 8,582,637);

U.S. patent application Ser. No. 13/754,998, titled "Design and Optimization of Partial Response Pulse Shape Filter," and filed on Jan. 31, 2013;

U.S. patent application Ser. No. 13/755,001, titled "Constellation Map Optimization For Highly Spectrally Efficient Communications," and filed on Jan. 31, 2013;

U.S. patent application Ser. No. 13/755,008, titled "Dynamic Filter Adjustment for Highly-Spectrally-Efficient Communications," and filed on Jan. 31, 2013 (now U.S. Pat. No. 8,571,131);

U.S. patent application Ser. No. 13/755,011, titled "Timing Synchronization for Reception of Highly-Spectrally-Efficient Communications," and filed on Jan. 31, 2013 (now U.S. Pat. No. 8,559,494);

U.S. patent application Ser. No. 13/755,014, titled "Signal Reception Using Non-linearity-compensated, partial response feedback," and filed on Jan. 31, 2013 (now U.S. Pat. No. 8,559,496);

U.S. patent application Ser. No. 13/755,018, titled "Feed Forward Equalization for Highly-Spectrally-Efficient Communications," and filed on Jan. 31, 2013;

U.S. patent application Ser. No. 13/755,021, titled "Decision Feedback Equalizer for Highly-Spectrally-Efficient Communications," and filed on Jan. 31, 2013;

U.S. patent application Ser. No. 13/755,026, titled "Decision Feedback Equalizer Utilizing Symbol Error Rate Biased Adaptation Function for Highly-Spectrally-Efficient Communications," and filed on Jan. 31, 2013 (now U.S. Pat. No. 8,559,498);

U.S. patent application Ser. No. 13/755,028, titled "Coarse Phase Estimation for Highly-Spectrally-Efficient Communications," and filed on Jan. 31, 2013 (now U.S. Pat. No. 8,548,097);

U.S. patent application Ser. No. 13/755,039, titled "Fine Phase Estimation for Highly Spectrally Efficient Communications," and filed on Jan. 31, 2013 (now U.S. Pat. No. 8,565,363); and

U.S. patent application Ser. No. 13/755,043, titled "Joint Sequence Estimation of Symbol and Phase with High Tolerance of Nonlinearity," and filed on Jan. 31, 2013.

Each of the above stated applications is hereby incorporated herein by reference in its entirety.

TECHNICAL FIELD

Aspects of the present application relate to electronic communications.

BACKGROUND

Existing communications methods and systems are overly power hungry and/or spectrally inefficient. Further limitations and disadvantages of conventional and traditional approaches will become apparent to one of skill in the art, through comparison of such approaches with some aspects of the present method and system set forth in the remainder of this disclosure with reference to the drawings.

BRIEF SUMMARY

Methods and systems are provided for low-complexity, highly-spectrally efficient communications, substantially as illustrated by and/or described in connection with at least one of the figures, as set forth more completely in the claims.

BRIEF DESCRIPTION OF THE DRAWINGS

FIG. 1 is a block diagram depicting an example system configured for highly-spectrally-efficient communications.

FIG. 2 is a block diagram depicting an example equalization and sequence estimation circuit for use in a system configured for highly-spectrally-efficient communications.

FIG. 3 is a block diagram depicting an example equalization and sequence estimation circuit for use in a system configured for highly-spectrally-efficient communications.

FIG. 4 is a block diagram depicting an example decision feedback equalizer for use in a system configured for highly-spectrally-efficient communications.

FIG. 5 is a block diagram depicting an example decision feedback equalizer core circuit.

FIG. 6 is a block diagram depicting an example soft symbol determination circuit.

FIG. 7 is a block diagram depicting an example error determination circuit.

FIG. 8 is a block diagram depicting an example equalization and sequence estimation circuit for use in a system configured for highly-spectrally-efficient communications.

FIG. 9 is a block diagram depicting an example decision feedback equalizer for use in a system configured for highly-spectrally-efficient communications.

FIG. 10 is a block diagram depicting an example decision feedback equalizer core circuit.

FIG. 11 is a block diagram depicting an example decision feedback equalizer core circuit.

FIG. 12 is a flow diagram depicting example steps in a method for a decision feedback equalizer with multiple cores for highly-spectrally-efficient communications.

DETAILED DESCRIPTION

The present disclosure describes a decision feedback equalizer with multiple cores for highly-spectrally-efficient communications. As utilized herein the terms "circuits" and "circuitry" refer to physical electronic components (i.e. hard-

ware) and any software and/or firmware (“code”) which may configure the hardware, be executed by the hardware, and or otherwise be associated with the hardware. As used herein, for example, a particular processor and memory may comprise a first “circuit” when executing a first one or more lines of code and may comprise a second “circuit” when executing a second one or more lines of code. As utilized herein, “and/or” means any one or more of the items in the list joined by “and/or”. As an example, “x and/or y” means any element of the three-element set $\{(x), (y), (x, y)\}$. As another example, “x, y, and/or z” means any element of the seven-element set $\{(x), (y), (z), (x, y), (x, z), (y, z), (x, y, z)\}$. As utilized herein, the term “exemplary” means serving as a non-limiting example, instance, or illustration. As utilized herein, the terms “e.g.,” and “for example” set off lists of one or more non-limiting examples, instances, or illustrations. As utilized herein, circuitry is “operable” to perform a function whenever the circuitry comprises the necessary hardware and code (if any is necessary) to perform the function, regardless of whether performance of the function is disabled, or not enabled, by some user-configurable setting. As utilized herein the terms “dynamic,” “dynamically,” “adaptive,” “adaptively” and the like may refer to values, parameters and/or the like that may be set, configured or updated during run-time of the transmitter and/or receiver (e.g., in, or near, real-time) based, for example, on recently received signals/values and/or signals/values currently being received.

FIG. 1 is a block diagram depicting an example system configured for low-complexity, highly-spectrally-efficient communications. The system 100 comprises a mapper circuit 102, a pulse shaping filter circuit 104, a timing pilot insertion circuit 105, a transmitter front-end circuit 106, a channel 107, a receiver front-end 108, a filter circuit 109, a timing pilot removal circuit 110, an equalization and sequence estimation circuit 112, and a de-mapping circuit 114. The components 102, 104, 105, and 106 may be part of a transmitter (e.g., a base station or access point, a router, a gateway, a mobile device, a server, a computer, a computer peripheral device, a table, a modem, a set-top box, etc.), the components 108, 109, 110, 112, and 114 may be part of a receiver (e.g., a base station or access point, a router, a gateway, a mobile device, a server, a computer, a computer peripheral device, a table, a modem, a set-top box, etc.), and the transmitter and receiver may communicate via the channel 107.

The mapper 102 may be operable to map bits of the Tx_bitstream to be transmitted to symbols according to a selected modulation scheme. The symbols may be output via signal 103. For example, for an quadrature amplitude modulation scheme having a symbol alphabet of N (N-QAM), the mapper may map each $\text{Log}_2(N)$ bits of the Tx_bitstream to single symbol represented as a complex number and/or as in-phase (I) and quadrature-phase (Q) components. Although N-QAM is used for illustration in this disclosure, aspects of this disclosure are applicable to any modulation scheme (e.g., amplitude shift keying (ASK), phase shift keying (PSK), frequency shift keying (FSK), etc.). Additionally, points of the N-QAM constellation may be regularly spaced (“on-grid”) or irregularly spaced (“off-grid”). Furthermore, the symbol constellation used by the mapper may be optimized for best bit-error rate performance that is related to log-likelihood ratio (LLR) and to optimizing mean mutual information bit (MMIB). The Tx_bitstream may, for example, be the result of bits of data passing through a forward error correction (FEC) encoder and/or an interleaver. Additionally, or alternatively, the symbols out of the mapper 102 may pass through an interleaver.

The pulse shaper 104 may be operable to adjust the waveform of the signal 103 such that the waveform of the resulting

signal 113 complies with the spectral requirements of the channel over which the signal 113 is to be transmitted. The spectral requirements may be referred to as the “spectral mask” and may be established by a regulatory body (e.g., the Federal Communications Commission in the United States or the European Telecommunications Standards Institute) and/or a standards body (e.g., Third Generation Partnership Project) that governs the communication channel(s) and/or standard(s) in use. The pulse shaper 104 may comprise, for example, an infinite impulse response (IIR) and/or a finite impulse response (FIR) filter. The number of taps, or “length,” of the pulse shaper 104 is denoted herein as LTx, which is an integer. The impulse response of the pulse shaper 104 is denoted herein as hTx. The pulse shaper 104 may be configured such that its output signal 113 intentionally has a substantial amount of inter-symbol interference (ISI). Accordingly, the pulse shaper 104 may be referred to as a partial response pulse shaping filter, and the signal 113 may be referred to as a partial response signal or as residing in the partial response domain, whereas the signal 103 may be referred to as residing in the symbol domain. The number of taps and/or the values of the tap coefficients of the pulse shaper 104 may be designed such that the pulse shaper 104 is intentionally non-optimal for additive white Gaussian noise (AWGN) in order to improve tolerance of non-linearity in the signal path. In this regard, the pulse shaper 104 may offer superior performance in the presence of non-linearity as compared to, for example, a conventional near zero positive ISI pulse shaping filter (e.g., root raised cosine (RRC) pulse shaping filter). The pulse shaper 104 may be designed as described in one or more of: the United States patent application titled “Design and Optimization of Partial Response Pulse Shape Filter,” the United States patent application titled “Constellation Map Optimization For Highly Spectrally Efficient Communications,” and the United States patent application titled “Dynamic Filter Adjustment For Highly-Spectrally-Efficient Communications,” each of which is incorporated herein by reference, as set forth above.

It should be noted that a partial response signal (or signals in the “partial response domain”) is just one example of a type of signal for which there is correlation among symbols of the signal (referred to herein as “inter-symbol-correlated (ISC) signals”). Such ISC signals are in contrast to zero (or near-zero) ISI signals generated by, for example, raised-cosine (RC) or root-raised-cosine (RRC) filtering. For simplicity of illustration, this disclosure focuses on partial response signals generated via partial response filtering. Nevertheless, aspects of this disclosure are applicable to other ISC signals such as, for example, signals generated via matrix multiplication (e.g., lattice coding), and signals generated via decimation below the Nyquist frequency such that aliasing creates correlation between symbols.

The timing pilot insertion circuit 105 may insert a pilot signal which may be utilized by the receiver for timing synchronization. The output signal 115 of the timing pilot insertion circuit 105 may thus comprise the signal 113 plus an inserted pilot signal (e.g., a sine wave at $\frac{1}{4} \times \text{fbaud}$, where fbaud is the symbol rate). An example implementation of the pilot insertion circuit 105 is described in the United States patent application titled “Timing Synchronization for Reception of Highly-Spectrally-Efficient Communications,” which is incorporated herein by reference, as set forth above.

The transmitter front-end 106 may be operable to amplify and/or upconvert the signal 115 to generate the signal 116. Thus, the transmitter front-end 106 may comprise, for example, a power amplifier and/or a mixer. The front-end may introduce non-linear distortion and/or phase noise (and/or

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other non-idealities) to the signal **116**. The non-linearity of the circuit **106** may be represented as FnlTx which may be, for example, a polynomial, or an exponential (e.g., Rapp model). The non-linearity may incorporate memory (e.g., Volterra series).

The channel **107** may comprise a wired, wireless, and/or optical communication medium. The signal **116** may propagate through the channel **107** and arrive at the receive front-end **108** as signal **118**. Signal **118** may be noisier than signal **116** (e.g., as a result of thermal noise in the channel) and may have higher or different ISI than signal **116** (e.g., as a result of multi-path).

The receiver front-end **108** may be operable to amplify and/or downconvert the signal **118** to generate the signal **119**. Thus, the receiver front-end may comprise, for example, a low-noise amplifier and/or a mixer. The receiver front-end may introduce non-linear distortion and/or phase noise to the signal **119**. The non-linearity of the circuit **108** may be represented as FnlRx which may be, for example, a polynomial, or an exponential (e.g., Rapp model). The non-linearity may incorporate memory (e.g., Volterra series).

The timing pilot recovery and removal circuit **110** may be operable to lock to the timing pilot signal inserted by the pilot insertion circuit **105** in order to recover the symbol timing of the received signal. The output **122** may thus comprise the signal **120** minus (i.e., without) the timing pilot signal. An example implementation of the timing pilot recovery and removal circuit **110** is described in the United States patent application titled "Timing Synchronization for Reception of Highly-Spectrally-Efficient Communications," which is incorporated herein by reference, as set forth above.

The input filter **109** may be operable to adjust the waveform of the partial response signal **119** to generate partial response signal **120**. The input filter **109** may comprise, for example, an infinite impulse response (IIR) and/or a finite impulse response (FIR) filter. The number of taps, or "length," of the input filter **109** is denoted herein as LRx, an integer. The impulse response of the input filter **109** is denoted herein as hRx. The number of taps, and/or tap coefficients of the input filter **109** may be configured based on: a non-linearity model, \widehat{Fnl} , signal-to-noise ratio (SNR) of signal **120**, the number of taps and/or tap coefficients of the Tx partial response filter **104**, and/or other parameters. The number of taps and/or the values of the tap coefficients of the input filter **109** may be configured such that noise rejection is intentionally compromised (relative to a perfect match filter) in order to improve performance in the presence of non-linearity. As a result, the input filter **109** may offer superior performance in the presence of non-linearity as compared to, for example, a conventional near zero positive ISI matching filter (e.g., root raised cosine (RRC) matched filter). The input filter **109** may be designed as described in one or more of: the United States patent application titled "Design and Optimization of Partial Response Pulse Shape Filter," the United States patent application titled "Constellation Map Optimization For Highly Spectrally Efficient Communications," and the United States patent application titled "Dynamic Filter Adjustment For Highly-Spectrally-Efficient Communications," each of which is incorporated herein by reference, as set forth above.

As utilized herein, the "total partial response (h)" may be equal to the convolution of hTx and hRx, and, thus, the "total partial response length (L)" may be equal to LTx+LRx-1. L may, however, be chosen to be less than LTx+LRx-1 where, for example, one or more taps of the Tx pulse shaper **104** and/or the Rx input filter **109** are below a determined level.

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Reducing L may reduce decoding complexity of the sequence estimation. This tradeoff may be optimized during the design of the system **100**.

The equalizer and sequence estimator **112** may be operable to perform an equalization process and a sequence estimation process. Details of an example implementation of the equalizer and sequence estimator **112** are described below with respect to FIG. 2. The output signal **132** of the equalizer and sequence estimator **112** may be in the symbol domain and may carry estimated values of corresponding transmitted symbols (and/or estimated values of the corresponding transmitted information bits of the Tx_bitstream) of signal **103**. Although not depicted, the signal **132** may pass through an interleaver en route to the de-mapper **114**. The estimated values may comprise soft-decision estimates, hard-decision estimates, or both.

The de-mapper **114** may be operable to map symbols to bit sequences according to a selected modulation scheme. For example, for an N-QAM modulation scheme, the mapper may map each symbol to $\text{Log}_2(N)$ bits of the Rx_bitstream. The Rx_bitstream may, for example, be output to a de-interleaver and/or an FEC decoder. Alternatively, or additionally, the de-mapper **114** may generate a soft output for each bit, referred as LLR (Log-Likelihood Ratio). The soft output bits may be used by a soft-decoding forward error corrector (e.g. a low-density parity check (LDPC) decoder). The soft output bits may be generated using, for example, a Soft Output Viterbi Algorithm (SOVA) or similar. Such algorithms may use additional information of the sequence decoding process including metrics levels of dropped paths and/or estimated bit probabilities for generating the LLR, where

$$LLR(b) = \log\left(\frac{P_b}{1 - P_b}\right),$$

where P_b is the probability that bit $b=1$.

In an example implementation, components of the system upstream of the pulse shaper **104** in the transmitter and downstream of the equalizer and sequence estimator **112** in the receiver may be as found in a conventional N-QAM system. Thus, through modification of the transmit side physical layer and the receive side physical layer, aspects of the invention may be implemented in an otherwise conventional N-QAM system in order to improve performance of the system in the presence of non-linearity as compared, for example, to use of RRC filters and an N-QAM slicer.

FIG. 2 is a block diagram depicting an example equalization and sequence estimation circuit for use in a system configured for low-complexity, highly-spectrally-efficient communications. Shown are an equalizer circuit **202**, a signal combiner circuit **204**, a phase adjust circuit **206**, a sequence estimation circuit **210**, and non-linearity modeling circuits **236a** and **236b**.

The equalizer **202** may be operable to process the signal **122** to reduce ISI caused by the channel **107**. The output **222** of the equalizer **202** is a partial response domain signal. The ISI of the signal **222** is primarily the result of the pulse shaper **104** and the input filter **109** (there may be some residual ISI from multipath, for example, due to use of the least means square (LMS) approach in the equalizer **202**). The error signal, **201**, fed back to the equalizer **202** is also in the partial response domain. The signal **201** is the difference, calculated by combiner **204**, between **222** and a partial response signal **203** that is output by non-linearity modeling circuit **236a**. An example implementation of the equalizer is described in the

United States patent application titled “Feed Forward Equalization for Highly-Spectrally-Efficient Communications,” which is incorporated herein by reference, as set forth above.

The carrier recovery circuit **208** may be operable to generate a signal **228** based on a phase difference between the signal **222** and a partial response signal **207** output by the non-linearity modeling circuit **236b**. The carrier recovery circuit **208** may be as described in the United States patent application titled “Coarse Phase Estimation for Highly-Spectrally-Efficient Communications,” which is incorporated herein by reference, as set forth above.

The phase adjust circuit **206** may be operable to adjust the phase of the signal **222** to generate the signal **226**. The amount and direction of the phase adjustment may be determined by the signal **228** output by the carrier recovery circuit **208**. The signal **226** is a partial response signal that approximates (up to an equalization error caused by finite length of the equalizer **202**, a residual phase error not corrected by the phase adjust circuit **206**, non-linearities, and/or other non-idealities) the total partial response signal resulting from corresponding symbols of signal **103** passing through pulse shaper **104** and input filter **109**.

The buffer **212** buffers samples of the signal **226** and outputs a plurality of samples of the signal **226** via signal **232**. The signal **232** is denoted PR1, where the underlining indicates that it is a vector (in this case each element of the vector corresponds to a sample of a partial response signal). In an example implementation, the length of the vector PR1 may be Q samples.

Input to the sequence estimation circuit **210** are the signal **232**, the signal **228**, and a response \hat{h} . Response \hat{h} is based on h (the total partial response, discussed above). For example, response \hat{h} may represent a compromise between h (described above) and a filter response that compensates for channel non-idealities such as multi-path. The response \hat{h} may be conveyed and/or stored in the form of $L_{Tx}+L_{Rx}-1$ tap coefficients resulting from convolution of the L_{Tx} tap coefficients of the pulse shaper **104** and the L_{Rx} tap coefficients of the input filter **109**. Alternatively, response \hat{h} may be conveyed and/or stored in the form of fewer than $L_{Tx}+L_{Rx}-1$ tap coefficients—for example, where one or more taps of the L_{Tx} and L_{Rx} is ignored due to being below a determined threshold. The sequence estimation circuit **210** may output partial response signals **205** and **209**, a signal **234** that corresponds to the finely determined phase error of the signal **120**, and signal **132** (which carries hard and/or soft estimates of transmitted symbols and/or transmitted bits). One or more example implementations of the sequence estimation circuit **210** and/or various other aspects/embodiments of the sequence estimation circuit may be described in one or more of: the United States patent application titled “Low-Complexity, Highly-Spectrally-Efficient Communications,” the United States patent application titled “Signal Reception Using Non-linearity-compensated, partial response feedback,” the United States patent application titled “Fine Phase Estimation for Highly Spectrally Efficient Communications,” and the United States patent application titled “Joint Sequence Estimation of Symbol and Phase with High Tolerance of Nonlinearity,” each of which is incorporated by reference herein, as set forth above.

The non-linear modeling circuit **236a** may apply a non-linearity function \widehat{Fnl} (a model of the non-linearity seen by the received signal en route to the circuit **210**) to the signal **205** resulting in the signal **203**. Similarly, the non-linear modeling circuit **236b** may apply the non-linearity function \widehat{Fnl} to the signal **209** resulting in the signal **207**. \widehat{Fnl} may

be, for example, a third-order or fifth-order polynomial. Increased accuracy resulting from the use of a higher-order polynomial for \widehat{Fnl} may tradeoff with increased complexity of implementing a higher-order polynomial. Where Fnl_{Tx} is the dominant non-linearity of the communication system **100**, \widehat{Fnl} modeling only Fnl_{Tx} may be sufficient. Where degradation in receiver performance is above a threshold due to other non-linearities in the system (e.g., non-linearity of the receiver front-end **108**) the model \widehat{Fnl} may take into account such other non-linearities.

FIG. **3** is a block diagram depicting an example equalization and sequence estimation (ESE) circuit **300** for use in a system configured for highly-spectrally-efficient communications. ESE **300** may be similar to the equalization and sequence estimation circuit **112** of FIG. **2**, for example. As can be seen by comparing FIG. **2** and FIG. **3**, several components and connections are similar (e.g., sequence estimation, carrier recovery, etc.). FIG. **3** may be simplified from FIG. **2** in various respects in order to focus on particular example implementations of FIG. **3** and subsequent figures. For example, FIG. **3** does not show a buffer at the output of the phase adjust (e.g., buffer **212** of FIG. **2**) nor non-linear modeling circuits (e.g., circuits **236a** and **236b** of FIG. **2**). These components may be excluded in FIG. **3** to focus on additional features of particular example implementations of equalization and sequence estimation circuits that follow. However, it should be understood that the embodiments of FIG. **3** and other embodiments described herein may include these components (e.g., buffer, non-linear modeling circuits, etc.). To the extent that similar components and connections are labeled differently in FIG. **3** than they are in FIG. **2**, the labels of FIG. **3** will generally be used going forward, as the descriptions that follow may refer to particular example implementations of (and additions to) the general circuit described in FIG. **2**.

ESE circuit **300** may include a DFE (Decision Feedback Equalizer) **302**. As shown in FIG. **3**, DFE **302** may be in communication with a sequence estimation circuit **304**, with an FFE (Feed Forward Equalizer) **306** (or other type of equalizer) and/or optionally with other circuits. Sequence estimation circuit **304** may be similar to sequence estimation circuit **210** of FIG. **2**, and FFE **306** may be similar to equalizer **202** of FIG. **2**, for example. It should be understood that although various descriptions herein may refer to FFE **306** as one example type of equalizer, other equalizers may be used, and descriptions herein that use FFE may be expanded to use any type of equalizer. FFE **306** may receive an input signal **310** (e.g., similar to signal **122** of FIG. **2**) and may generate an FFE out signal **312** (e.g., similar to signal **222** of FIG. **2**).

DFE **302** may receive symbols (e.g., SE symbols **314**) from the sequence estimation circuit **304**. SE symbols **314** may be a similar signal to signal **132** of FIG. **2**, for example. These SE symbols **314** may be the same symbols that are output by the ESE circuit **300** (e.g., sent to de-mapper **114** of FIG. **1**). DFE **302** may receive a fine phase signal **316** from the sequence estimation circuit **304**. Fine phase **316** may be a similar signal to signal **234** of FIG. **2**, for example. Signals received by the receiver may have been phase distorted, for example, by the channel. Fine phase **316** may represent the overall phase compensation/correction required to compensate for phase error in a received input signal (e.g., input to FFE **306**). Throughout this disclosure, a received signal (e.g., at the input **310** to FFE **306**) will be considered to have zero phase error, and unless otherwise specified, the terms “performing or applying a phase rotation,” “performing or applying a

phase correction,” “phase rotated,” “phase corrected” or the like will refer to correcting a signal or a corrected signal.

DFE **302** may send a number of tap coefficients (e.g., DFE taps coefficients . . . , **318**, or generally represented by \hat{h}) to the sequence estimation circuit **304**. The \hat{h} signal of FIG. **3** may be similar to the \hat{h} signal of FIG. **2**, for example. The DFE taps may be used by the sequence estimation circuits to perform various aspects (routines, calculations, algorithms, etc.) of the symbol estimation process. Various aspects (e.g., symbol detection routines, calculations and/or algorithms) of one or more sequence estimation process may be described in one or more of: the United States patent application titled “Low-Complexity, Highly-Spectrally-Efficient Communications,” the United States patent application titled “Signal Reception Using Non-linearity-compensated, partial response feedback,” the United States patent application titled “Fine Phase Estimation for Highly Spectrally Efficient Communications,” and the United States patent application titled “Joint Sequence Estimation of Symbol and Phase with High Tolerance of Nonlinearity,” each of which is incorporated by reference herein, as set forth above.

DFE **302** may receive an FFE out signal **312** from FFE **306**. FFE **306** may have equalized the input signal **310** to compensate for channel response (e.g., multipath and/or other channel distortions). DFE **302** may receive a PR error signal **320** as shown in FIG. **3**. PR error **320** may be similar to signal **201** of FIG. **2**, for example. The PR error signal **320** may be based on a partial response signal **305** generated by the sequence estimation circuit **304**. Signal **305** may be a signal that estimates the total partial response signal (e.g., the transmitted symbols after they pass through a Tx partial response filter and/or an Rx filter). Signal **305** may be similar to signal **205** and/or **203** of FIG. **2**. DFE **302** may use the PR error signal **320** to generate and/or determine an error signal (e.g., DFE Error **408**) for use in the DFE equalizer adaptation.

DFE **302** may output a soft symbols signal **322**, as shown in FIG. **3**. The soft symbols signal may be comprised of estimated symbols, for example, estimates of symbols that were sent by the transmitter, received by the receiver and/or output (e.g., SE symbols **314**) by the ESE circuit **300**. The soft symbols signal **322** may be used as a quality indication of the symbols output by the ESE circuit **300**. The soft symbols signal **322** may be used by the DFE to generate and/or determine an error signal (e.g., DFE Error **408**) for use in the DFE equalizer adaptation.

DFE (Decision Feedback Equalizer) **302** may perform equalization, for example, additional equalization beyond the equalization performed by the FFE **306**. For example, the DFE **302** may converge to the overall response at the output of the FFE **306**, which may include the response of the Tx partial response pulse shape filter, the response of the Rx filter, any residual channel response (e.g., channel response not corrected by the FFE) and/or the response of the FFE **306**. In some embodiments, DFE **302** may perform biased equalization. For example, DFE **302** may compromise between optimally equalizing to correct for signal distortions (e.g., distortions due to multipath, etc.) one the one hand and achieving other optimization goals on the other hand. Examples of other optimization goals include, for example, improving sequence estimation (e.g., in sequence estimation circuit **304**), improving the minimum distance and SER. The concept of minimum distance is further explained below in the description related to error vectors. The concept of compromising between optimization goals is also further explained below.

FIG. **4** is a block diagram depicting an example decision feedback equalizer (DFE) **400** for use in a system configured for highly-spectrally-efficient communications. DFE **400**

may be similar to DFE **302** of FIG. **3**, for example. DFE **400** may include a DFE core circuit **402**, which is explained in more detail below (e.g., with regard to FIG. **5**). DFE **400** may include a soft symbol determination circuit **404**. The soft symbol determination unit **404** may receive as inputs DFE out **410**, FFE out **412** and, optionally, fine phase **416**. The soft symbol determination circuit **404** may output a soft symbols signal **422**. The soft symbols signal may comprise estimated symbols, for example, estimates of symbols (e.g., SE symbols) that are output by the ESE circuit **300**. The soft symbols signal **422** may be used as a quality indication of the symbols output by the ESE circuit **300**. The soft symbols signal **422** may be used by the DFE to generate and/or determine an error signal (e.g., DFE Error **408**) for use in the DFE equalizer adaptation. DFE **400** may include an error determination circuit **406**. The error determination circuit **406** may generate an error output (e.g., DFE error **408**), where the error output may be based on PR error **420**, or based on SE symbols **414** and soft symbols **422**, or based on other inputs. In some embodiments, the error determination circuit **406** may simply pass the PR error **420** through to the DFE error signal **408**. In other embodiments, the error determination circuit may be excluded from the DFE **400**, and the PR error signal **420** may be connected directly to the DFE error signal **408**.

DFE core circuit **402** may perform equalization, for example, with the goal of cancelling ISI (inter-symbol interference) that exists in the input signal to the ESE circuit. DFE core circuit **402** may receive SE symbols **414** (e.g., from the sequence estimation circuit **304**) as input. DFE core circuit **402** may include a number of taps and delays. DFE core circuit **402** may calculate or determine a number of tap coefficients (e.g., DFE tap coefficients **318**), for example, to send to the sequence estimation circuit **304**. DFE core circuit **402** may include one or more tap update/adaptation units that update the tap coefficients based on various inputs (e.g., based on DFE error **408**). DFE core circuit **402** may receive fine phase **416** as an input, and may use fine phase to phase rotate/correct an error signal (e.g., DFE error **408**). Fine phase **416** may be generated by the sequence estimation circuit and may represent the overall phase compensation/correction required to compensate for phase error in a received input signal (e.g., input to FFE **306**).

FIG. **5** is a block diagram depicting an example DFE core circuit **500**. DFE core circuit **500** may be similar to DFE core circuit **402** of FIG. **4**, for example. DFE core circuit **500** may receive SE symbols **514** as an input and may include a number of delay elements (e.g., Z transform delay elements **502**, **504**, **506**, **507**), where each delay element may constitute one symbol delay. DFE core circuit **500** may generate a number of tap coefficients, for example, tap coefficients Tap Early 1, Tap Early Ne, Tap Late 1, Tap Late Nl. The tap coefficients may be generated at the outputs of tap update units, for example, tap update units **510**, **512**, **514**, **517**. DFE core circuit **500** may output a DFE out signal **510**, which may be generated by an accumulator **518**. The accumulator **518** may receive as inputs a number of signals (e.g., signals **520**, **522**, **524**, **526**), where each signal may be the product of a symbol (e.g., the current input SE symbol **514** or previous input SE symbol that has been delayed by one or more symbol times) and a tap. As one example, input signal **522** may be the product of the delayed symbol between delay units **502** and **504** and the tap coefficient Tap Early Ne. In some embodiments, the aggregator **518** may receive, as an input, a signal based on the ‘1’ tap (or center tap) as shown in FIG. **5**. In other embodiments, the aggregator may not receive any input based on the ‘1’ tap (or center tap).

DFE core circuit **500** may generate a number of tap coefficients. In an example implementation, the tap coefficients may be categorized into early tap coefficients (e.g., Tap Early 1, Tap Early Ne) and late tap coefficients (e.g., Tap Late 1, Tap Late NI), a ‘1’ (or “center”) tap coefficient (associated with a tap in the middle of the early taps and the late taps). The 1 tap (or center tap) may relate to the reference symbol. The term reference symbol may refer generally to the symbol being equalized by the DFE at any given time. As one example, the reference symbol in FIG. **5** may refer to the delayed symbol between delay elements **504** and **506**. The reference symbol may also be referred to as the “cursor”. From the standpoint of an equalizer (e.g., the DFE core circuit **500**), the early taps may compensate for pre-cursor interference, and the late taps may compensate for post-cursor interference. It should be understood that although FIG. **5** shows four taps in the DFE core circuit, the DFE core circuit **500** may include more or less taps (e.g., more taps between the tap associated with coefficient Tap Early 1 and the tap associated with coefficient Tap Early Ne, and/or more taps between the tap associated with Tap Late 1 and the tap associated with coefficient Tap Late NI). For each tap of the DFE core circuit **500**, the DFE core circuit may include an associated delay element and tap update unit. It may be said that the delay element and current tap coefficient associated with a particular tap constitutes a part of the DFE response, where all of the response parts together constitute the total response of the DFE core **500** or the DFE (e.g., DFE **400**).

In some embodiments, the response (e.g., \hat{h}) of the DFE (e.g., DFE **400**) may replicate or approximate the total partial response (h) of the system. The “total partial response” (h) may include the response of the Tx partial response filter and/or the response of the Rx filter. The phrase “partial response filter” used generally without reference to the Tx or Rx may refer to the Tx partial response filter and/or the Rx filter. In other words, the total partial response (h) may be split between the Tx partial response filter and the Rx filter, where h_{Tx} may represent the component associated with the Tx partial response filter and h_{Rx} may represent the component associated with the Rx filter. More specifically, the total partial response (h) may be the convolution of the Tx partial response (h_{Tx}) and the Rx partial response (h_{Rx}), i.e., $h = \text{conv}(h_{Tx}, h_{Rx})$. Then, \hat{h} may be an estimate of h . If \hat{h} were to exactly match h , the number of taps (L) in the DFE core **500** would be equal to $L_{Tx} + L_{Rx} - 1$, where L_{Tx} is the number of taps of the Tx partial response filter and L_{Rx} is the number of taps of the Rx filter. Similarly, if \hat{h} were to exactly match h , the tap coefficients in the DFE would be precisely determined by $\hat{h} = \text{conv}(h_{Tx}, h_{Rx})$. In some embodiments, one or more of the taps of the Tx partial response filter may be ignored or omitted in determining the number and/or coefficients of taps in the DFE (in such an embodiment L would be less than $L_{Tx} + L_{Rx} - 1$). For example, in some embodiments, certain taps of the Tx partial response filter and/or of the Rx filter that have coefficients that are below a determined threshold (and, consequently, that will have little effect on the decoding performance of the ESE) may be ignored when determining the number and/or coefficients of taps in the DFE. In some examples, pre-cursor taps may be ignored so that the amplitude of first pre-cursor tap in the DFE will be large enough to assure reliable sequence estimation.

In some embodiments, the sequence estimation circuit (e.g., **304**) may use the DFE tap coefficients (i.e., the tap coefficients corresponding to \hat{h}). The sequence estimation circuit may assume that \hat{h} is close to the total partial response of the system (e.g., h). In order to configure or initialize the DFE (e.g., **302**) to replicate or approximate the total partial

response, the tap coefficients of the DFE may be set or initialized to be equal to (or close to) tap coefficients corresponding to the total partial response (i.e., the tap coefficients of the Tx partial response filter convolved with the tap coefficients of the Rx partial response filter). In this respect, it may be said that the DFE taps to the left of the reference symbol (the “early taps”) represent the leading part (e.g., pre-cursor) of the total partial response, and the DFE taps to the right of the reference symbol (the “late taps”) represent the trailing part (e.g., post-cursor) of the total partial response. It may be the case that the total partial response (e.g., the response of the Tx partial response pulse shape filter convolved with the response of the Rx filter) varies (e.g., is not static). For example, the Tx and Rx filters may be optimized during run-time based on various factors. For example, the Tx and Rx filters may be optimized using error vectors, for example, in a similar manner to the way some implementations of the DFE core **500** may use error vectors, as explained below. If the total partial response (h) varies, the taps coefficients in DFE core **500** may vary as well such that, for example, DFE core **500** continues to replicate or approximate the total partial response (e.g., within a determined error bound). In some embodiments, the input symbols to the DFE may be decimated down, or up sampled, to the symbol rate, for example, by passing the input symbols through an up/down sampler at the input of the DFE core circuit. This decimation may be required, for example, if the Tx partial response filter and the Rx filter are sampling at higher or lower rates than the symbol rate.

In some embodiments, the DFE core **500** may be initialized or calibrated to replicate or approximate the total partial response of the system (h). This initialization/calibration may occur at various times, for example, at one or more of: power up of the system, upon receiving each packet of data (e.g., at the beginning of the packet), at the beginning of a packet stream, upon receiving a preamble, on every X (an integer) iterations of the sequence estimation process performed by the sequence estimation circuit. After initialization, the DFE core **500** may adapt/update to achieve a compromise between multiple optimization goals. In other words, the DFE core may perform biased equalization. The adaptation of the DFE core may compromise between optimally equalizing to correct for signal distortions (e.g., distortions due to multipath, AWGN, etc.) on one hand, and achieving other optimization goals on the other hand. Examples of other optimization goals include, improving sequence estimation (e.g., in sequence estimation circuit **304**) and improving the minimum distance and/or SER. The concept of minimum distance is explained below along with the description of error vectors. In an example implementation, if the channel was perfect (e.g., without distortion, multipath, ISI, etc.), then the DFE core’s primary purpose would be to replicate or approximate the total partial response of the system (h) such that, for example, the sequence estimation circuit may perform optimal symbol detection. This situation may be thought of as one extreme of the compromise. On the other hand, if the DFE was not concerned with providing \hat{h} (an approximation of h) to the sequence estimation unit, then the DFE core’s primary purpose would be to equalize for channel distortions (e.g., multipath, ISI from the channel, etc.). This situation may be thought of as the other extreme of the compromise. In some embodiments, the DFE core (e.g., DFE core **500** shown in FIG. **5**) may compromise between the two extremes, for example, by equalizing to correct for channel distortions while biasing that equalization goal to also provide an acceptable approximation of h . This biasing may also be thought of as biasing or pushing the pure LMS solution/adaptation (ex-

plained more below) to also achieve acceptable symbol detection. Biasing or pushing the LMS solution may be done by using the constraint function **532**, or alternatively, by using an SER (symbol error rate) to bias the adaptation (explained more below). Alternatively, as further explained below (e.g., with respect to FIG. **9**), the DFE may include multiple DFE cores/sets of DFE tap coefficients, where each DFE core seeks to achieve a different optimization goal.

The tap coefficients in the DFE core circuit **500** may be scaled or normalized. For example, the tap coefficients may be normalized to a maximum value of 1, where the maximum tap coefficient of 1 may be associated with the center '1' tap. It should be understood that the taps may be normalized to any value, for example, values other than 1. Additionally, the '1' tap (or center tap) as shown in FIG. **5** may be any other value, for example, a static or fixed value. The normalization value may be determined by the power level at the input to the DFE. In some embodiments, the taps may be scaled such that some or all of the "early" taps have at least a minimum amplitude (e.g., a defined value), which may improve performance of the sequence estimation circuit. As explained above, the '1' tap (or center tap) may be associated with the reference symbol, and it may be located between the early taps of the DFE and the late taps of the DFE. In some embodiments, the '1' tap (or center tap) may not be used to generate the DFE out signal (e.g., used for ISI cancellation and/or determining soft symbols). As one example, by excluding the 1' tap (or center tap) and the reference symbol from the aggregator **518** and the DFE out **510** calculation, the DFE out signal **510** may be comprised of solely, or mainly, ISI components. In this respect the DFE out signal may be used to cancel ISI from a related signal that includes ISI components.

The DFE tap coefficients (e.g., Tap Early 1, Tap Early Ne, the '1' Tap, Tap Late 1, Tap Late Ni) may be sent to the sequence estimation circuit (e.g., **304**). The sequence estimation circuit may perform one or more routines, algorithms or the like that assume that \hat{h} (the DFE tap coefficients) is close to the total partial response (i.e., h) of the system (e.g., the Tx partial response filter and the Rx filter). Referring to the embodiment of FIG. **5**, the DFE tap coefficients may be initialized to total partial response, but the DFE core circuit **500** may update the taps to achieve (e.g., to a certain extent) various optimization goals (e.g., to compensate for channel distortion such as multipath). In perfect channel conditions the DFE tap coefficients values would correspond to the total partial response (h). In non-perfect channel conditions the DFE core circuit **500** may adapt to achieve various optimization goals (e.g., to compensate for multipath distortion).

DFE core circuit **500** may include one or more tap update/adaptation units (e.g., tap update units **510**, **512**, **514**) that update the tap coefficients based on various inputs, for example, DFE error **508**, fine phase **516** and/or Gain Err **530**. It should be understood that in some embodiments, the tap update units may be combined into a single unit instead of being configured as discrete units as shown in FIG. **5**. The DFE Error signal **508** may be determined by an error determination circuit (e.g., circuit **406** of FIG. **4**) or DFE error **508** may be the PR error **320** as shown in FIG. **3**. If the DFE error **508** is the PR error, the DFE core circuit **500** may phase correct/rotate the DFE error signal (e.g., using the fine phase signal **516**). This phase correction of DFE error may be required such that DFE error **508** is phase aligned with the SE symbols **514**, which have been generated using a phase corrected signal. This phase correction may be possible because PR error (e.g., **320**) is in the signal domain (as opposed to the symbol domain), which means that PR error includes ISI, as does the signal (e.g., FFE out **312**) used by the carrier recov-

ery circuit and the sequence estimation circuit (e.g., carrier recovery circuit **308** and sequence estimation circuit **304** of FIG. **3**) to calculate the phase correction signal. In some embodiments and/or situations, the DFE error may be a symbol error (i.e., a signal that lives in the symbol domain). In these embodiments and/or situations, the phase correction may not be performed (e.g., the fine phase signal may not be used and/or received).

As explained above, in some situations (e.g., perfect channel conditions) the DFE taps coefficients may hold at the total partial response values. In other situations, the tap coefficients may update or adapt. This updating/adaptation may also be referred to as convergence, for example, if the tap coefficients are approaching optimal values. The DFE core circuit **500** may adapt/converge using one or more adaptation functions (e.g., cost functions such as Least Mean Square-error or "LMS"). As one example, and referring to FIG. **5**, each tap update unit (e.g., tap update units **510**, **512**, **514**, **517**) may include an aggregator or integrator that utilizes an adaptation function (e.g., a cost function such as LMS). As one example, tap update unit **510** may include an "early 1 adaptation function" that receives as inputs the current tap coefficient (e.g., Tap Early 1), an adaptation error signal **530** (e.g., the phase shifted, and optionally, gain adjusted, DFE error signal **508** multiplied by the associated symbol) and, optionally, a constraint (e.g., Constraint Early 1). Generally speaking, the early 1 adaptation function may determine (e.g., dynamically, continuously or repeatedly) an optimal or enhanced Tap Early 1 value such that the adaptation error signal becomes minimized. As shown in FIG. **5**, the tap coefficients (e.g., Tap Early 1) may directly loop back into the adaptation functions (e.g., the early 1 adaptation function). Alternatively or additionally, the tap coefficients may loop back into the adaptation functions by way of their use by the sequence estimation circuit in generating a signal (e.g., signal **305** in FIG. **3**) which is used to generate the PR error (e.g., **320**) signal, which, in turn, is used to generate the DFE error signal (e.g., **408**). In this respect, the error signal used by the DFE core **500** (e.g., DFE error **508**) may be referred to as a "loop error."

The adaptation functions may minimize the square error of the adaptation error signals (e.g., adaptation error signal **530**). If the DFE error **508** is the PR error (e.g., **320**), then the adaptation functions may effectively minimize the PR error. If the DFE error is symbol error (e.g., based on the soft symbols as described herein), then the adaptation functions may effectively minimize symbol error. Therefore, it can be seen that in various embodiments and/or various situations, the adaptation functions (e.g., updating of the taps) may be driven by different error signals. In some situations (e.g., if the DFE error is the PR error), the adaptation function may be driven by PR error (e.g., **320**) which is related to an approximation of the total partial response signal (generated by the sequence estimation circuit). This error signal is in the signal domain (as opposed to the symbol domain). In some situations (e.g., if the DFE error is symbol error), the adaptation function may be driven by symbol error which is related to estimated received symbols (generated by the sequence estimation circuit). This error signal is in the symbol domain (verses the signal domain). As explained above, the DFE core **500** may perform biased equalization to achieve multiple optimization goals. In this respect, the adaptation function may be limited or constrained (e.g., by constraint function **532**) such that the DFE adaptation does not degrade sequence estimation and/or minimum distance. Alternatively, an SER (symbol error rate) biased adaptation function may be used

(explained more below) that considers both the mean square error and the goal of sequence estimation.

In some embodiments, the DFE error **508** may be multiplied by a gain factor (e.g., Gain Err **530** as shown in FIG. **5**), which may improve the convergence time of the adaptation function(s). A properly selected gain factor may adapt the DFE core circuit to converge quickly without producing a noisy or distorted DFE out signal **510**. For example, if the gain factor is too small, the convergence may not happen quickly, and if the gain factor is too large, the convergence may happen quickly, but the output signal (e.g., DFE out) may have undesirable noise or distortions. The gain factor may be adjusted based on an adaptive function, for example, a symbol counter that counts the number of symbols that have entered the DFE core circuit since a reference point. The gain factor may also be adjusted based on other performance metrics/indicators such as SNR and/or multipath (e.g., as indicated by SNR and/or multipath estimators).

As can be seen in FIG. **5**, the tap update units may each receive a constraint, where each constraint may be generated by a constraint function **532**. The constraints (e.g., Constraint Early 1, Constraint Early Ne, Constraint Late 1, Constraint Late NI,) may be used by the tap update units to set limits on the adaptation of the DFE core circuit **500**. As explained above, the DFE (e.g., **302**) may compromise between various optimization goals. If the DFE core circuit **500** were allowed to adapt (e.g., to achieve LMS criteria) without constraint, the DFE out signal **510** may improve from a signal quality standpoint (e.g., corrected for channel distortions), but the tap coefficients (e.g., DFE tap coefficients sent to the sequence estimator) may stray from their initial values (e.g., values corresponding to the total partial response, h). This may reduce the performance of the symbol estimation in the sequence estimator. For example, the minimum distance of the transmitted symbols may degrade, and consequently, symbol error rate may increase. The constraints (e.g., Constraint Early 1, Constraint Early Ne, Constraint Late 1, Constraint Late NI) may set a limit on DFE adaptation to maintain, restore or improve the minimum distance. In other words, the constraints may allow for \hat{h} to stray from h by only a limited amount (e.g., by a determined threshold amount).

The constraint function **532** may control the impact that the constraints have on the adaptation functions (in the tap update units). Alternatively, control of the impact may be performed by each tap update unit. Control of the impact may depend on an input, for example, Gain Con **532** as shown in FIG. **5**. For example, if Gain Con **532** is adjusted in one direction (or set to a first value), the constraints may have less effect on adaptation, and \hat{h} may stray further from h (e.g., \hat{h} may be restricted to within a first determined error bound of h). If Gain Con is adjusted in the other direction (or set to a second value), the constraints may have more of an effect on adaptation, and \hat{h} may remain closer to h e.g., \hat{h} may be restricted to within a second determined error bound of h). Keeping \hat{h} closer to h may limit minimum distance degradation. In some situations, it may be possible to adjust Gain Con to force DFE adaptation to improve the minimum distance in the receiver beyond the minimum distance in the transmitter, but this may result in increased signal distortion (e.g., increased noise, increased values of the PR Error signal, etc.), which may cause less-reliable signal decoding in the equalizer and sequence estimator overall. Gain Con **532** may be static (e.g., configured as part of system calibration) or it may vary dynamically, for example, based on SNR, multipath and/or other system performance metrics/indicators.

As explained above, Gain Con **532** may control the impact of the constraints, and Gain Err **530** may control the impact of

the DFE error (e.g., causing faster convergence). Therefore, the ratio of Gain Err to Gain Con may represent the compromise between unconstrained adaptation of the DFE and sequence estimation performance of the sequence estimation circuit. In some embodiments, there may be an optimum level of Gain Con for any particular level of Gain Err that maximizes sequence estimation performance and reduces symbol error rate. This optimum level may be function of multipath, SNR, SER, BER, a measured indication of phase error, a measure indication of sequence estimation reliability, and/or other system performance metrics or other measure performance indicators.

When considering all the inputs that may affect the adaptation functions (e.g., DFE error, Gain Err, Gain Con, the constraints), the tap coefficients may be updated (e.g., dynamically, continuously or repeatedly) based on several metrics that may be important to the performance of the system (e.g., to reliable decoding of incoming signals). Such metrics may include SNR, phase noise, channel conditions and minimal distance reduction. By considering these various metrics when updating the tap coefficients, the DFE is adapted to be a smart and flexible equalization component.

Referring to FIG. **5**, the constraint function **532** may generate a number of constraints that may be used by the tap update units to maintain the minimum distance and maintain performance of the sequence estimation circuit. The constraint function **532** may accept as inputs the current tap coefficients (e.g., Tap Early 1, Tap Early Ne, Tap Late 1, Tap Late NI). The constraint function **532** may accept various other inputs, such as various error vectors (explained more below with regard to the SER-biased adaptation function). The constraint function **532** may use error vectors to calculate the minimum distance, and may compute constraints (e.g., Constraint Early 1, Constraint Early Ne, Constraint Late 1, Constraint Late NI) that maintain the minimum distance (e.g., within a certain acceptable range). In general, the constraint function may perform computations that are similar to the computations explained below with regard to the SER-biased adaptation function. The constraint function **532** may be adaptive based on other inputs, such as SNR, multipath and/or other system performance metrics. The constraint function may be any type of function. For example, the constraint function may be a linear function, a non-linear function, a limit or constraint calculation, a lookup table or any other type of function.

In some embodiments of the present disclosure, DFE core circuit **500**, and tap update units **510**, **512**, **514** in particular, may utilize an adaptation function that seeks to minimize symbol error rate (SER). Referring to FIG. **5**, as explained above, DFE core circuit **500** may include one or more tap update/adaptation units (e.g., tap update units **510**, **512**, **514**). Additionally, as explained above, the constrain function **532** may generate a number of constraints that may be used to set limits on the adaptation. In some embodiments, instead of having a constraint function and constraints fed into the adaptation function(s), the adaptation function(s) may automatically or naturally adapt the DFE tap coefficients according to similar goals and/or principals as used by the constraint function. For example, it may be the case that the optimal taps for correcting for channel distortion (e.g., multipath, etc.) may degrade the performance of the sequence estimation circuit (e.g., degrade the minimum distance). Therefore, the adaptation function may automatically constrain tap coefficients as part of the adaptation to minimize SER. For example, the tap update/adaptation units may adapt/converge according to a cost function such as Least Mean Square-error or "LMS" (as explained above), but the cost function/LMS function may be

revised or biased to minimize symbol error rate (SER) and/or to maintain or improve minimum distance. The term “SER-biased adaptation function” may refer to an adaptation function used by one or more tap update/adaptation units, as just explained, and as explained further below. In these embodiments, the DFE core (as shown in FIG. 5) may be modified, for example, to exclude the constraint function 532 and to exclude the constraints feeding into the tap update units.

The following describes one example SER-biased adaptation function that may be used by one or more tap update/adaptation units (e.g., tap update units 510, 512, 514). It should be understood that in some embodiments, the tap update units may be combined into a single unit instead of being configured as discrete units as shown in FIG. 5. The following description, and other descriptions herein may refer to an LMS function as one example of an adaptation function, and the various embodiments of the present disclosure may utilize various other adaptation functions. In this respect, the SER-biased adaptation function principals discussed below may be applied to bias other types of adaptation functions, even though LMS is used here to clearly explain one example SER-biased adaptation function.

Various equalizers may utilize an LMS function/algorithm (or LMS criteria) to adapt the equalizer. LMS adaptation may not be optimal in certain situations, for example, in cases of severe multipath. Additionally, LMS adaptation may not seek to achieve minimal symbol error rate (SER) and/or bit error rate (BER). In some embodiments of the present disclosure, DFE core circuit 500 (e.g., in particular tap update units 510, 512, 514) may utilize an SER-biased adaptation function that seeks to minimize SER, and thus may provide optimal performance (perhaps subject to the finite equalizer length). In these embodiments, the SER-biased adaptation function may utilize error vectors in order to determine the probability of symbol error in the system, and thus, a short discussion of error vectors follows.

An error vector (also referred to as an error pattern or error path) indicates the difference between the correct/expected symbol path (e.g., based on correct/expected symbols) and the estimated symbol path (e.g., based on symbols estimated/detected by the receiver). Generally speaking, the correct symbol path corresponds to the transmitted symbol path. Thus, to calculate an error vector, the receiver must know what symbols were transmitted. The receiver may know these symbols because known (or deterministic) symbols are sent and received during initialization or calibration of the system, because preamble symbols are sent periodically through the system, using corrected FEC block that may indicate the errors, or by other methods. The estimated symbol path represents detected or received symbols in the receiver, for example, at the output of the equalization and sequence estimation circuit (e.g., sent to the de-mapper). Received signals from which the estimated symbols are determined may have been convolved by the total partial response, distorted by channel response and then decoded by the equalization and sequence estimation circuit. Both correct symbol paths and the estimated symbol paths are referred to in terms of “paths” or “patterns” or “vectors” because they consist of more than one element (e.g., more than one symbol). The distance between a correct symbol path and an estimated symbol path is an error symbol path or an error vector. Because the partial response shaping (e.g., in the Tx partial response filter and in the Rx filter) increases the number of dimensions in the signal domain, the distance of an error vector is a Euclidian distance. Because partial response is based on convolution and convolution is a linear operation, the error vectors can be used to characterize SER performance of a system, for example,

rather than checking error probability for every symbol pattern which may require huge complexity.

As one example, if the correct symbol path (e.g., a known transmitted value) is $[\alpha\beta\chi\delta]$ and the estimated symbol path is $[\alpha\alpha\alpha\alpha]$, then the error vector would be $[(\alpha-\alpha)(\alpha-\beta)(\alpha-\chi)(\alpha-\delta)]/d_{min}$ (estimated symbol path minus correct symbol path normalized to the minimum between two points in symbol constellation (d_{min})). In other words, one could add the error vector (multiplied by d_{min}) to the correct/transmitted symbol path to get the estimated symbol path. Thus, the error vector may be referred to as an additive vector (up to scaling by d_{min}). The length of an error vector/path may refer to the number of symbols or elements in the error vector/path. It may be the case that an error vector must start and must end with a non-zero element, e.g., the length is 3 in the example above because the first element was not an error. The length of the error vectors/paths may vary based on various factors. The length may be related to the number of taps in the Tx partial response filter, the number of taps in the Rx filter, and/or the size of the symbol constellation (e.g., value of N for an N-QAM constellation), but the length of the error vector may be less than length (L) of the total partial response. It should be understood that there may be as many error vectors as there are possible combinations of correct symbol paths and estimated symbol paths.

Minimum distance in a system is a concept that is closely related to error vectors, in the present disclosure. Symbol distances may refer to the Euclidian distances between different points of the symbol constellation in use. In one or two dimensional modulation schemes which are using symbol slicing (rather than sequence estimation) in the receiver, the minimum distance may refer to the smallest symbol distance that exists in the system. In such schemes minimum distance may be an important metric because it may indicate the likelihood of a symbol slicer arriving at the wrong symbol. In the present system, which uses sequence estimation (rather than symbol slicing), minimum distance may be referenced in terms of multidimensional vector. That is, minimum distance may be measured between paths rather than between constellation points. In the present disclosure, the minimum distance may be defined by one or more particular error vector(s) observed in the system. For example, the minimum distance may be the distance calculated (e.g., via a matrix operation using the filter taps of the of the Tx filter and the Rx filter or via an estimate based on measured SNR) from the error vector that is observed (i.e., occurs) the most frequently in the system. In an example embodiment, the distance of multiple frequently occurring error vectors may be calculated because, for example, there may be some statistical uncertainty regarding whether the most-frequently occurring error vector actually corresponds to the minimum distance.

In some embodiments of the present disclosure, the system may dynamically compute error vectors without the use of known symbols in the receiver. In these embodiments, the “correct” or “expected” symbols may be computed using some sort of error correction scheme, for example, forward error correction (FEC) (e.g., Reed Solomon, or low-density parity check (LDPC)) or another type of correction scheme. The error correction may be performed on the symbols output by the equalization and sequence estimation circuit (e.g., after passing through the de-mapper or following extraction of log likelihood ratios (LLR)). The corrected symbols may then be compared to uncorrected symbols (or corrected symbol paths compared to uncorrected symbol paths) to create error vectors. Thus, various references to “error vectors” herein may refer to error vectors computed dynamically and/or to error vectors computed from known (or deterministic) symbols.

SER-biased dynamic error vectors may update “on the fly” as various symbols/symbol paths are output by the equalization and sequence estimation circuit and corrected by the error correction scheme. As one example, the SER-biased adaptation function may update as these dynamic error vectors change on the fly, as explained more below.

The SER-biased adaptation function may utilize an SER expression that may represent a union bound or upper bound of SER in the partial response system. The SER expression may reliably reflect the actual SER of the partial response system, for example, within an error range (e.g., below 10^{-1}) that provides an adequate margin above the operating zone (e.g., the SER range within which the demodulator or receiver can operate sufficiently without becoming unstable or losing synchronization). The SER expression may drive the loop error used by the tap update units (e.g., similar to how DFE error drives the tap update units in FIG. 5). An example SER expression is provided at Eq. 1 below. The SER expression may represent an upper bound of SER for a system that uses a real pulse shaping filter (e.g., real filter 104 and/or real filter 109) and complex symbols.

$$P_M \leq \sum_{\epsilon_i} Q \left(\sqrt{\frac{d_{min}^2 \delta^2(\epsilon_i)}{2N_0}} \right) 4K_{\epsilon_i} = \sum_{\epsilon_i} Q \left(\sqrt{\frac{E_s D}{N_0 G_b} \delta^2(\epsilon_i)} \right) 4K_{\epsilon_i} \quad (\text{Eq. 1})$$

$$K_{\epsilon} = w_{\epsilon} \prod_{j=0}^{l-L-1} \frac{|\sqrt{M}| - |\epsilon_j|}{|\sqrt{M}|} \quad (\text{Eq. 2})$$

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{u^2}{2}} du \quad (\text{Eq. 3})$$

$$Q'(x) = -\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \quad (\text{Eq. 4})$$

Referring to Eq. 1, ϵ_i represents error vector i from a subset (e.g., the most commonly observed error vectors) of all the observed error vectors. The subset size may be approximately 10 or less, and other subset sizes may be used in other embodiments and/or situations. As explained above, it may be the case that the error vector that is observed most-frequency in the system is the worst-case error vector. The most-frequently-occurring error vectors may be considered by the SER-biased adaptation function. More particularly, error vectors that appear frequently and have larger weights (e.g., a larger number of errors) may have the most significant impact on system performance (e.g., SER performance). Therefore, the subset of error vectors may include the error vectors with the highest “score”, where the score for each error vector considers frequency (e.g., an appearance rate, average appearance rate, or the like) of the error vector and the weight of the error vector. An error vector’s score may reflect the impact of the error vector on system performance, and the N (an integer) error vectors having the highest scores may be selected to compose a subset of error vectors. In some embodiments, the subset is determined at configuration/cali-

bration of the system. In other embodiments, the subset may change dynamically, for example, if error vectors are calculated dynamically as explained above. In these other embodiments, the worst-case errors (e.g., frequencies/appearance rates) may vary during operation (e.g., adaptation) of the system. Likewise, error vector scores may vary. In these embodiments, the worst-case error vectors and/or error vector scores may be monitored dynamically (e.g., on the fly) and the subset of error vectors may be updated accordingly.

Referring again to Eq. 1, $\delta^2(\epsilon_i)$ represents the distance (e.g., Euclidian distance) of error vector ϵ_i . As explained above, the worst-case (e.g., most commonly observed) error vector(s) may relate to the minimum distance, and an adaptation goal may be to maintain, restore and/or improve the minimum distance, for example, to improve sequence estimation and reduce SER. M represents the alphabet size and P_M represents the probability of symbol error in an alphabet of size M . D represents a coefficient based on the modulation order of the system. G_b represents the baud rate gain (e.g., spectral compression) of the total partial response. K_{ϵ} represents a boundary imposed by a particular error vector, and K_{ϵ} may be expanded as shown in Eq. 2 above. w_{ϵ} represents the error weight of the error vector (e.g., the number of errors in the error vector). $l-L$ represents the length of the error vector. ϵ_j represents the j th entry (e.g., symbol) in the error vector. Q represents a Q function or ERF function or other known similar function used in communications. The Q function may be used to quantify the SER, BER, etc. Q may be expanded as shown in Eq. 3 above. The Q function may be derivated to arrive at the Q' function shown in Eq. 4 above.

The SER expression may also be represented as shown below in Eq. 5, where \hat{h} is the adaptive response (e.g., based on total partial response, h). $MSE(\hat{h})$ represents

$$\left(\frac{E_s}{N_0} \right)^{-1},$$

35

for example, factored by equalization noise enhancement.

40

$$\frac{E_s}{N_0}$$

45

is related to signal to noise ratio (SNR). In this respect, $MSE(\hat{h})$ may represent the mean square error of the partial response signal within the signal bandwidth including the noise enhancement penalty of equalization. $A(\hat{\epsilon}_i)$ represents the error vector matrix for error vector $\hat{\epsilon}_i$. H may represent taking the complex conjugate of \hat{h} .

50

$$P_M \leq \sum_{\epsilon_i} Q \left(\sqrt{\frac{D \cdot (\hat{h})^H A(\epsilon_i) \hat{h}}{MSE(\hat{h})}} \right) 4K_{\epsilon_i} \quad (\text{Eq. 5})$$

55

The SER expression of Eq. 5 may be minimized by taking its derivative with respect to \hat{h} (the adaptive response). This derivation results in an SER-biased adaptive gradient shown below in Eq. 6.

60

$$\frac{\partial P_M}{\partial \hat{h}} = \quad (\text{Eq. 6})$$

65

-continued

$$-2(\hat{h})^H \sum_{\epsilon_i} \frac{K_{\epsilon_i} A(\epsilon_i)}{\delta(\epsilon_i)} e^{-\frac{D\delta^2(\epsilon_i)}{2MSE}} + \frac{\partial MSE}{\partial \hat{h}} \sum_{\epsilon_i} \frac{K_{\epsilon_i} \delta(\epsilon_i)}{MSE} e^{-\frac{D\delta^2(\epsilon_i)}{2MSE}} = 0$$

The SER-biased adaptive gradient may be used by the tap update/adaptation units (e.g., tap update units **510**, **512**, **514**) to determine updated DFE tap coefficients. For example, Eq. 6 may be solved for \hat{h} to continuously determine updated DFE tap coefficients. The SER-biased adaptive gradient may be used by the tap update/adaptation units in a similar manner to the way an LMS gradient may be used to update tap coefficients. In general, an LMS gradient may be a function that charts mean square error verses tap coefficients, where the LMS gradient has a single distinct minimum where tap coefficients produce the smallest mean square error. If the gradient has a negative slope, the adaptation function may increase the tap coefficients, and if the gradient has a positive slope, the adaptation function may decrease the tap coefficients. The LMS gradient may be represented by $\partial MSE / \partial \hat{h}$, which is part of the right hand expression of Eq. 6. Therefore, it can be seen in Eq. 6 that the SER-biased adaptive gradient considers the impact of the mean square error, but also considers other factors, for example, the impact of error pattern/vector distances. In other words, the Euclidian distance of error patterns is biasing the pure LMS gradient. More specifically, the term $e^{-D\delta^2(\epsilon_i)/2MSE}$ balances the gain in the summation according to the error pattern distances $\delta^2(\epsilon_i)$. The gain increases as the distance decreases to reflect that error patterns with small distance are dominant in SER performance. As MSE goes low, the right hand expression becomes dominant because low MSE is associated with high SNR, and high SNR suppresses errors caused by small error pattern distances. Respectively, the left hand term becomes dominant when the error pattern distance ($\delta^2(\epsilon_i)$) becomes small.

In some embodiments, a simplified SER-biased adaptive gradient may be used, for example, a simplified form of the Eq. 6, as shown in Eq. 7 below. The simplified SER-biased adaptive gradient may use the summation terms of Eq. 6 and may exclude the balance terms (i.e., the exponent terms). It may be the case that the balance terms can be removed because the exponent terms do not exhibit a great effect on the overall adaptive gradient, and the remaining terms (the summation terms) still consider the impact of error pattern distances. The simplified SER-biased adaptive gradient may be useful if a simpler adaptive function is desired, for example, if the design is being implemented as hardware and a smaller hardware profile is desired. Additionally, the simplified SER-biased adaptive gradient may allow for faster convergence of the DFE core.

$$\frac{\partial P_M}{\partial \hat{h}} \cong -2(\hat{h})^H \sum_{\epsilon_i} \frac{K_{\epsilon_i} A(\epsilon_i)}{\delta(\epsilon_i)} + \frac{\partial MSE}{\partial \hat{h}} \sum_{\epsilon_i} \frac{K_{\epsilon_i} \delta(\epsilon_i)}{MSE} \quad (\text{Eq. 7})$$

FIG. 6 is a block diagram depicting an example soft symbol determination circuit **600**. Soft symbol determination circuit **600** may receive as inputs FFE out **612** (e.g., similar to FFE out **312** as shown in FIG. 3) and DFE out **610** (e.g., similar to DFE out **410** as shown in FIG. 4). DFE out **610** may be generated as a result of symbols (e.g., from the sequence estimation circuit) being convolved with the DFE taps/DFE response. As explained above, DFE out **610** may comprise solely (or primarily) ISI components. FFE out **612** may be the input signal (e.g., input signal **310** to FFE **306** as shown in FIG. 3) after it has been corrected for channel distortions such

as noise. Therefore, if the FFE (e.g., **306**) was perfect, the FFE out signal **612** would be the same as the signal at the output of the total partial response (e.g., Tx partial response filter and/or Rx filter), for example, where the signal is comprised of symbols with ISI introduced by the total partial response. In other words, if the FFE was perfect, the FFE out signal **612** would be the same as the signal at the input of the Tx partial response filter plus the controlled ISI introduced by the total partial response. Therefore, if the DFE out signal **610** (solely or primarily ISI) is subtracted from the FFE out signal **612** (Tx partial response filter input+ISI), the resulting signal (soft symbols **622**) would be, in theory, the input to the Tx partial response filter (i.e., symbols). Soft symbol determination circuit **600** is configured to approximate the Tx partial response filter input based on the theory just described. The soft symbol determination circuit **600** may phase correct/rotate (e.g., using multiplier **602**) the FFE out **612** such that it is phase-corrected like the DFE out **610**. Soft symbol determination circuit **600** may receive as input a fine phase signal **616** (e.g., fine phase **316** from the sequence estimation circuit **304** as shown in FIG. 3) that may be used to perform the phase correction. The soft symbol determination circuit **600** may subtract (e.g., using subtractor **604**) DFE out from the phase-corrected FFE out. In this respect, the ISI (e.g., partial response ISI) may be canceled from the FFE out. The resulting signal (soft symbols **622**) may be estimates of the symbols that were sent from the transmitter and estimates of symbols that are output by the ESE circuit (e.g., ESE circuit **300** of FIG. 3). The soft symbols may be used as a quality indication of the symbols output by the ESE circuit. The soft symbols may be used by the DFE to generate and/or determine an error signal for use in the DFE equalizer adaptation. For example, FIG. 4 shows soft symbols **422** being sent to the error determination circuit **406**, and the error determination circuit is explained more below.

FIG. 7 is a block diagram depicting an example error determination circuit **700**. Error determination circuit **700** may receive as inputs SE symbols **714** (e.g., SE symbols **314** output by the sequence estimation circuit as shown in FIG. 3), PR error **720** (e.g., PR error **320** as shown in FIG. 3) and/or soft symbols **722** (e.g., soft symbols **422** from the soft symbol determination circuit as shown in FIG. 4). Error determination circuit **700** may generate an error output (e.g., DFE error **708**), where the error output may be based on one or more of the inputs and/or other inputs. In the various embodiments of the present disclosure, the error determination circuit may be configurable and may support, for example, one or more configurations that may exclude one or more of the components and/or inputs shown in FIG. 7 and/or one or more configurations that include more components and/or inputs than shown in FIG. 7. In one example configuration, the error determination circuit **700** may simply pass one of the input signals (e.g., PR error **720**) through (e.g., without a selector **704** as depicted in FIG. 7) to the DFE error signal **708**. In such a configuration, the SE symbols **714** and soft symbols **722** signals may be excluded as inputs, and subtractor **702** may be excluded. In one example configuration, the error determination circuit **700** may connect (e.g., without a selector **704** as depicted in FIG. 7) the symbol error **723** (as shown in FIG. 7) to the DFE error signal **708**. In such a configuration, the PR error **720** may be excluded as an input.

In the embodiment depicted in FIG. 7, DFE error output **708** is connected to the output of selector **704**. Selector **704** is configurable (e.g., based on a selection signal **730** output by a selector setting register, selector setting circuit or the like) to select between three input signals, and may pass the selected input signal through to the selector output (e.g., DFE error

708). For example, if the selection signal is equal to '0', the selector 704 may select the left-most input (e.g., PR error 720); if the selection signal is equal to '1', the selector 704 may select the center input (e.g., 0); and if the selection signal is equal to '2', the selector 704 may select the right-most input (e.g., symbol error 723). Whichever input is selected by the selector 704 may determine the error signal (e.g., DFE error 708) that is used by the DFE core (e.g., DFE core 402 of FIG. 4) to perform equalization/convergence. In this respect, different configurations of the selector 704 (i.e., different selections of input signals) may be referred to as different error modes. For example, in the first error mode (e.g., selector signal='0'), the DFE may use the PR error 720 as its error signal. In the second error mode (e.g., selector signal='1'), the DFE may use 0 (zero) as its error signal. A zero error signal may indicate that no error signal is to be used by the DFE. In this situation, DFE tap coefficients may be held at their current values (i.e., coefficient freeze). If a coefficient freeze is selected permanently (or for a significant period of time) for DFE operation going forward, the DFE tap coefficients may be set or initialized to tap coefficients corresponding to the total partial response, for example, where the incoming symbols are decimated down to the symbol rate, or to a related partial response such as a shortened version of the total partial response.

In the third error mode (e.g., selector signal='2'), the DFE may use the symbol error 723 generated by the subtractor 702 that receives as inputs SE symbols 714 and soft symbols 722, as shown in FIG. 7. The SE symbols 714 may be output from the sequence estimation circuit (e.g., 304). These SE symbols 714 may be the sequence estimation circuit's best estimation (e.g., given time constraints) of the symbols received from the transmitter and may be the same symbols that are output by the ESE circuit 300 (e.g., to a de-mapper). The United States patent application titled "Low-Complexity, Highly-Spectrally-Efficient Communications," incorporated herein by reference, as set forth above, provides example details about how SE symbols 714 (e.g., similar to signal 132 of FIG. 2) may be more or less error-prone depending on whether signal 132 is taken from a lower or higher indexed element in the symbol buffer. The soft symbols 722 may be output from the soft symbol estimation circuit, based on the DFE out, for example, as shown in FIG. 6. The soft symbols 722 may be estimated symbols that are determined by convolving the SE symbols from the sequence estimation circuit with the DFE taps and then subtracting that output (DFE out) from FFE out (shown in FIG. 6). The soft symbols signal 722 may be used as a quality indication of the symbols output by the sequence estimation circuit, and the symbol error (difference between soft symbols and symbols) may be used as an error signal to drive the DFE equalization/convergence.

In some embodiments, the Symbol error 723 may be used to control (e.g., dynamically) various components in the DFE. For example, Symbol error 723 may control the gain of the DFE (e.g., Gain Err 530 in FIG. 5). As another example, Symbol error 723 may control Selector 704. In one specific implementation, the power of Symbol error 723 may be filtered (e.g., using a FIR or IIR filter structure) to yield an average error power value. The average error power value may be compared to a reference value, and in the case that the average error power value exceeds the reference, this may indicate that the error signal has poor reliability. If the error signal has poor reliability, selector 704 may be switched to a different position (e.g., selector signal='1' to select the zero input). In this situation, the DFE may not adapt (e.g., temporarily) because of the poor reliability of the error signal. When the average error power drops below the reference value, the

selector 704 may switch to a different position (e.g., selecting a non-zero error input). In some implementations, DFE gain (e.g., Gain Err 530 in FIG. 5) may change (instead of or in conjunction with the selector changing) if the error signal has poor reliability. For example, the DFE gain value may be reduced to cause the DFE to adapt less, to protect the DFE's response from a poor-reliability error signal. When the average error power drops below the reference value, the DFE gain may increase.

In some embodiments, the selection signal 730 that determines which input of the selector should be passed through to the selector output (e.g., DFE error) may be determined dynamically (e.g., by a selector setting circuit), for example, during the operation of the system and the DFE. For example, the selection signal may change depending on channel conditions (e.g., SNR and/or multipath) or other performance indicators. As one specific example, SNR may be indicated by SNR and/or multipath estimators, and the estimator(s) output (s) may determine the selection of the sector 704. As another example, the selection signal may change depending on equalization performance (e.g., FFE and/or DFE performance), SER performance (e.g., including the performance of the sequence estimation circuit and/or the equalizers). As one specific example, the selection signal may change depending on SER patterns/error vectors. The use of SER patterns/error vectors (e.g., to drive equalizer adaptation) is discussed at various points throughout this disclosure. In some embodiments, the selection signal determination may generally change according to the following rule: if channel conditions are degrading the signal, such that channel distortion is dominant over SER, then the selector may select the PR error, and the DFE may use the PR error to converge; if SER is dominant over channel distortion, then the selector may select the symbol error, and the DFE may use the symbol error as its DFE error signal. Various other selection rules based on these and other performance metrics may be used by the error determination circuit 700.

FIG. 8 is a block diagram depicting an example equalization and sequence estimation (ESE) 800 circuit for use in a system configured for highly-spectrally-efficient communications. ESE 800 may be similar to the equalization and sequence estimation circuit 300 of FIG. 3, for example. As can be seen by comparing FIG. 3 and FIG. 8, several components and connections are similar, but the integration (e.g., certain connections) of DFE 802 into ESE 800 may be different than the integration of DFE 302 into ESE 300. For example, DFE out 810 from DFE 802 may be combined with (e.g., added to) FFE out 812. Additionally, DFE 802 may be different in some respects when compared to DFE 302. As explained above, in some embodiments of the present disclosure, the DFE (e.g., DFE 302 of FIG. 3) may perform biased equalization, meaning that the DFE may compromise between optimally equalizing to correct for signal distortions (e.g., distortions due to multipath, etc.) and achieving other optimization goals such as, for example, improving sequence estimation (e.g., by improving the minimum distance). As an alternative to such a biasing DFE (e.g., 302), a DFE (e.g., 802) may include multiple DFE cores, each of which may comprise a constrained DFE core such as, for example, the DFE core 500 of FIG. 5, an unconstrained DFE core such as the DFE core 1000 of FIG. 10, or a partial DFE core such as, for example, the partial DFE core 1100 of FIG. 11. In an example embodiment, each of the multiple DFE cores may seek to satisfy a different optimization goal.

FIG. 9 is a block diagram depicting an example DFE circuit 900 for use in a system configured for highly-spectrally-efficient communications. DFE circuit 900 may be similar to

DFE circuit **802** of FIG. **8**, for example. As can be seen in FIG. **9**, DFE circuit **900** includes DFE core **902** and DFE core **904**.

In an example embodiment, the DFE core **902** may be used to perform equalization (e.g., using a pure LMS algorithm) to compensate for channel distortions (e.g., multipath). Accordingly, tap coefficients of DFE core **902** may be allowed to adapt (e.g., using an error signal such as PR error **920**) in an unconstrained manner (e.g., no constraint function or adaptation function that is biased for SER). In such an example embodiment, the DFE core **902** may be an unconstrained DFE core such as the DFE core **1000** (described below with reference to FIG. **10**) and may receive, as inputs, PR error **920**, Fine phase **916** and SE symbols **914**. These inputs may be similar to the inputs of DFE core **402** of FIG. **4**. In an example embodiment, the DFE core **904** may be used to achieve optimal sequence estimation/symbol detection. Accordingly, tap coefficients of the DFE core **904** may be fixed and may replicate or estimate (e.g., be within a determined tolerance of) tap coefficients corresponding to a target response (e.g., h), without any biasing to compensate for channel distortions. In such an example embodiment, the DFE core **904** may be a partial DFE core such as the DFE core **1100** (described below with reference to FIG. **11**).

In an example embodiment, the DFE core **904** may perform adaptation (e.g., using an LMS type algorithm) to maximize the minimum distance (and/or minimize SER), without considering the effect of mean square error (e.g., channel distortions). In this respect, the minimum distance in the receiver (associated with \hat{h}) may improve beyond the minimum distance in the transmitter (associated with h). In this embodiment, DFE core **904** may include more circuitry beyond just registers (and associated circuitry) that hold tap coefficients. For example, DFE core **904** may include circuitry that may implement an optimization function (e.g., an optimization function that is similar to Eq. 5 explained above). For example, the optimization function may receive as input a number of error vectors, and the optimization function may consider the minimum distance (e.g., Euclidian distance) corresponding to the error vectors. The optimization function may seek to set the tap coefficients of the DFE core **904** to values that maintain, restore or improve the minimum distance. As a result, the optimization function may seek to set the tap coefficients of the DFE core **904** to values that are optimal for sequence estimation, symbol detection and minimal SER. Unlike the optimization function of Eq. 5, this optimization function may not consider error arising due to channel distortions (e.g., multipath). For example, in this embodiment, DFE core **904** may not receive PR error as an input and may not considering the effect of the mean square of PR error. In other words, the optimization function may not compromise to achieve minimum PR error. In this respect, the optimization function may be focused on improved SER, and the minimum distance (associated with taps values corresponding to \hat{h}) may improve beyond the original minimum distance (associated with tap values corresponding to h).

The use of multiple DFE cores may enable effective utilization of the non-linear model on reconstructed partial response signals (e.g., signals **205** and **209**) used for sequence estimation (e.g., performed by sequence estimation circuit **210**) while operated over the symbol candidates (during the search), as well as for equalization and carrier recovery, e.g., over signals **205** and **209** to yield signal **203** and **207** respectively. One DFE core (e.g., **904**) may hold an optimized response in presence of non-linear distortion while other DFE core (e.g., **902**) may be operated to compensate for channel response (e.g., multipath). The optimized response (e.g., stored by **904**) may be an optimal balance between AWGN,

non-linear distortion tolerance (e.g., response that is similar to the transmitter pulse shaping) and reduced complexity sequence estimation stability near threshold SNR (e.g., early taps amplitude, tail response amplitude).

FIG. **10** is a block diagram depicting an example unconstrained DFE core circuit **1000**, for example, an unconstrained DFE core that may be used as DFE **902** in the embodiment of FIG. **9**. As can be seen in FIG. **10**, unconstrained DFE core **1000** may include various components and connections that are similar to those of the constrained DFE core **500** of FIG. **5**. Unlike the constrained DFE core **500** of FIG. **5**, the unconstrained DFE core **1000** may not have a constraint function or constraints feeding into the tap update units. Unconstrained DFE core **1000** may have a similar set of delay elements and may use similar tap adaptation function (s); however, the adaptation functions (e.g., in tap update units **1010**, **1012**, **1014**, **1016**) may be unconstrained. In this respect, unconstrained DFE core **1000** may be allowed to adapt (e.g., using an error signal such as PR error) in an unconstrained manner (e.g., no constraint function, constraints or adaptation function that is biased for SER).

DFE core **1000** may generate a DFE out signal **1010** as shown in FIG. **10**. DFE out **1010** may be generated by rotating a DFE temporary out signal **1011** (output of accumulator **1018**) using multiplier **1020**. The fine phase signal **1016** may be used to perform the rotation. Similar to the constrained DFE core **500** described with regard to FIG. **5**, a phase correction (e.g., using fine phase **1016**) of DFE error **1008** may be required in unconstrained DFE core **1000** such that DFE error is phase aligned with the SE symbols **1014**, which have been generated using a phase corrected signal. Therefore, the DFE temp out signal **1011** will be phase corrected. However, the DFE out signal **1010** must be rotated (e.g., using fine phase) such that DFE out **1010** can be combined with (e.g., added) FFE out (an uncorrected signal), for example, as shown in FIG. **8**.

FIG. **11** is a block diagram depicting an example partial DFE core circuit **1100**, for example, a partial DFE core that may be used as DFE core **904** in the embodiment of FIG. **9**. Partial DFE core **1100** may include a number of taps the coefficients of which correspond to the response \hat{h} which replicates or estimates the total partial response h . \hat{h} may be an optimized observation accommodating balance between AWGN, non-linear distortion tolerance and reduced complexity sequence estimation stability at threshold SNR. In the embodiment of FIG. **11**, partial DFE core **1100** may comprise registers that hold tap coefficients (e.g., Tap Early 1, Tap Early Ne, Tap Late 1, Tap Late Ni). The register(s) of DFE core **1000** may update, for example, if the total partial response (e.g., h) changes. The tap coefficients \hat{h} of partial DFE core **1100** may be sent to (and used by) a sequence estimation circuit (e.g., sequence estimation circuit **804** of FIG. **8**), as explained further herein. In some alternate embodiments of the present disclosure, the registers of the partial DFE core **1100** may reside, for example, in the sequence estimation circuit, and it may be unnecessary for the DFE (e.g., DFE **802** of FIG. **8**) to send the tap coefficients corresponding to \hat{h} to the sequence estimation circuit (e.g., circuit **804**).

Certain embodiments of the present disclosure may be found in one or more methods for a decision feedback equalizer with multiple cores for highly-spectrally-efficient communications. With respect to the various methods described herein and depicted in associated figures, it should be understood that, in some embodiments, one or more of the steps described and/or depicted may be performed in a different order. Additionally, in some embodiments, a method may include more or less steps than are described and/or depicted.

FIG. 12 depicts a flow diagram 1200 that shows example steps in a method for a decision feedback equalizer with multiple cores for highly-spectrally-efficient communications, in accordance with one or more embodiments of the present disclosure. More specifically, FIG. 12 shows example steps that may be performed to determine and use DFE tap coefficients, for example, DFE tap coefficients Tap Early 1, Tap Early Ne, Tap Late 1, Tap Late Nl in DFE core 1000 and/or partial DFE core 1100. At step 1202, DFE core 1000 may receive an error signal (e.g., DFE error 1008). At step 1204, DFE core 1000 may determine a first plurality of DFE tap coefficients based on the error signal, as described in more detail herein. At step 1206, partial DFE core 1100 may receive externally determined coefficients (e.g., coefficients that replication or estimate coefficients from one or more partial response filters, as explained in more detail herein). At step 1208, DFE core 1100 may determine a second plurality of DFE tap coefficients based on the externally determined coefficients, as explained in more detail herein. At step 1210, DFE core 1000 may use the first plurality of DFE tap coefficients to generate a DFE out signal (e.g., DFE out 1010) that is combined with an FFE out signal (e.g., FFE out 812). At step 1212, partial DFE core 1100 may send the second plurality of DFE tap coefficients to other receive components (e.g., the sequence estimator 804). The steps of FIG. 12 may repeat and/or reoccur throughout the operation of the system. For example, the determination of the first and second sets of DFE tap coefficients may occur dynamically, such that the DFE tap coefficients are modified when relevant inputs (e.g., DFE error and/or externally determined coefficients) change.

Various embodiments of the present disclosure describe one or more systems, methods, routines and/or techniques a decision feedback equalizer with multiple cores for highly spectrally efficient communications. One or more embodiments describe an equalization circuit in a receiver that may include a decision feedback equalizer circuit. The decision feedback equalizer circuit may include a first plurality of tap coefficients that are determined based on a cost function that receives as input an error signal that is an inter-symbol-correlated (ISC) signal. The decision feedback equalizer circuit may include a second plurality of tap coefficients that are determined based on a filter with an ISC response. The cost function may determine the mean square of the error signal. The cost function may be constrained or unconstrained. The error signal may represent error caused by a channel. In some embodiments, the ISC signal is a partial response signal, and the filter with an ISC response is a partial response filter. In some embodiments, the partial response signal is generated by a sequence estimation circuit estimates, and the partial response signal estimates the output of the partial response filter. In some embodiments, the second plurality of tap coefficients are used by a sequence estimation circuit that uses the second plurality of tap coefficients to detect symbols that are encoded in an input signal to the receiver. In some embodiments, the second plurality of tap coefficients are stored in coefficient registers, and the coefficient registers are set and updated to replicate or estimate the ISC response.

In some embodiments, the decision feedback equalizer circuit may include tap update circuitry that is related to the second plurality of tap coefficients, wherein the tap update circuitry updates the second plurality of tap coefficients to minimize symbol error rate in the receiver. The tap update circuitry may receive a set of error vector(s), each error vector in the set representing the difference between estimated symbols generated in the receiver and expected symbols. The tap update circuitry may use the error vector(s) to update the second plurality of tap coefficients to minimize symbol error

rate in the receiver. In some embodiments, an output of the decision feedback equalizer circuit is mixed with an output of a feed forward equalizer. In some embodiments, the first plurality of tap coefficients compensates for channel response, and the second plurality of tap coefficients represents a filter response that is optimal in the presence of non-linear distortion.

One or more embodiments of the present disclosure describe a method performed in a receiver. The method may include generating an error signal that is based on an estimated partial response signal output by a sequence estimation circuit. The method may include determining a first plurality of tap coefficients based on the error signal, wherein the tap coefficients are determined by minimizing the mean square of the error signal. The method may include generating coefficient values based on one or more partial response pulse shaping filters, wherein the generated coefficient values replicate or estimate a total response of the one or more partial response pulse shaping filters. The method may include determining a second plurality of tap coefficients based on the generated coefficient values. The method may include sending the second plurality of tap coefficients to the sequence estimation circuit, wherein the sequence estimation circuit uses the tap coefficients to detect symbols that are encoded in an input signal to the receiver. The method may include receiving an estimated symbols signal from the sequence estimation circuit, and generating a decision feedback equalizer output signal by performing a convolution on the estimated symbols signal using the first plurality of tap coefficients. The method may include mixing the decision feedback equalizer output signal with an output of a feed forward equalizer.

In some embodiments, the method may include receiving a phase correction signal from the sequence estimation circuit, wherein the phase correction signal represents an estimation of the overall phase error in an input signal received by the receiver. The method may include phase correcting the error signal before the error signal is used to determine the first plurality of tap coefficients, wherein the phase correction signal is used to perform the phase correcting. The method may include receiving a phase correction signal from the sequence estimation circuit, wherein the phase correction signal represents an estimation of the overall phase error in an input signal received by the receiver. The method may include phase correcting the error signal before the error signal is used to determine the first plurality of tap coefficients, wherein the phase correction signal is used to perform the phase correcting. The method may include rotating the decision feedback equalizer output signal using the phase correction signal. The method may include updating the second plurality of tap coefficients to minimize symbol error rate in the receiver. The method may include updating the second plurality of tap coefficients such that a first minimum distance in the receiver improves beyond a second minimum distance in an associated transmitter. The first and second minimum distance may be related to symbols in a symbol alphabet used by the receiver and transmitter.

One or more embodiments of the present disclosure describe a decision feedback equalizer in a receiver. The decision feedback equalizer may include first equalization core that is optimized to correct an input signal that has been distorted by a channel that adds inter symbol interference. The decision feedback equalizer may include a second equalization core that is optimized to achieve minimal symbol error rate in the receiver. The second equalization core may include a set of tap coefficients that are used by a sequence estimation circuit. The second equalization core may initialize and

update the tap coefficients to replicate or estimate a total response of one or more partial response pulse shaping filters. The second equalization core may initialize the tap coefficients to replicate taps that exist in one or more partial response pulse shaping filters. The second equalization core may allow the tap coefficients to update such that a first minimum distance in the receiver improves beyond a second minimum distance in an associated transmitter. The first and second minimum distance may be related to symbols in a symbol alphabet used by the receiver and transmitter. The second equalization core may initialize the tap coefficients to replicate taps that exist in one or more partial response pulse shaping filters, and the second equalization core may allow the tap coefficients to update to minimize symbol error rate in the receiver. The first equalization core may include a set of tap coefficients that are determined based on the mean square of an error signal, the error signal being based on an equalized version of the input signal and a signal generated by the sequence estimation circuit.

The description of the different advantageous embodiments has been presented for purposes of illustration and the description and is not intended to be exhaustive or limited to the embodiments in the form disclosed. Many modifications and variations will be apparent to those of ordinary skill in the art. Further different advantageous embodiments may provide different advantages as compared to other advantageous embodiments. The embodiment or embodiments selected are chosen and described in order to best explain the principles of the embodiments of the practical application and to enable others of ordinary skill in the art to understand the disclosure for various embodiments with various modifications as are suited to the particular use contemplated.

The invention claimed is:

1. An equalization circuit in a receiver, the circuit comprising:

decision feedback equalizer circuit having a first plurality of tap coefficients that adapt during operation of the receiver to compensate for channel distortions; and a second plurality of tap coefficients that:

are initialized to replicate a response corresponding to filter of the receiver and a filter of a transmitter from which the receiver receives signals;

are occasionally or periodically calibrated, during operation of the receiver, to replicate the response corresponding to the filter of the receiver and the filter of the transmitter; and

do not adapt to compensate for channel distortions.

2. The equalization circuit of claim **1**, wherein the adaptation of the first plurality of tap coefficients is based on a cost function.

3. The equalization circuit of claim **2**, wherein the cost function is constrained.

4. The equalization circuit of claim **1**, wherein the decision feedback equalizer circuit receives as input an error signal generated based on symbol estimates determined by a sequence estimation circuit of the receiver.

5. The equalization circuit of claim **4**, wherein the error signal is generated via processing of the symbol estimates determined by the sequence estimation circuit.

6. The equalization circuit of claim **5**, wherein the error signal is a partial response signal generated via a convolution of the symbol estimates and tap coefficients based on the response corresponding to the filter of the receiver and the filter of the transmitter.

7. The equalization circuit of claim **1**, wherein the second plurality of tap coefficients are used by a sequence estimation

circuit that uses the second plurality of tap coefficients to detect symbols that are encoded in an input signal to the receiver.

8. The equalization circuit of claim **1**, wherein the second plurality of tap coefficients are stored in coefficient registers.

9. The equalization circuit of claim **1**, further comprising tap update circuitry that performs the adaptation of the first plurality of tap coefficients wherein the adaptation of the first plurality of tap coefficients is based on symbol error rate in the receiver.

10. The equalization circuit of claim **9**, wherein the tap update circuitry receives a set of error vector(s), each error vector in the set representing the difference between a vector of estimated symbols generated in the receiver and a vector of expected symbols, and wherein the tap update circuitry uses the error vector(s) to update the second plurality of tap coefficients to minimize symbol error rate in the receiver.

11. The equalization circuit of claim **1**, wherein an output of the decision feedback equalizer circuit is mixed with an output of a feed forward equalizer.

12. A method comprising:

in a receiver, processing a received signal, wherein the processing comprises:

adapting, during operation of the receiver, a first plurality of tap coefficients based on distortion in a channel over which the receiver is receiving the received signal;

initializing a second plurality of tap coefficients to replicate a response corresponding to a filter of the receiver and to a filter of a transmitter from which the receiver is receiving the received signal; and

occasionally or periodically calibrating, during operation of the receiver, the second plurality of tap coefficients to replicate the response corresponding to the filter of the receiver and the filter of the transmitter, wherein said second plurality of tap coefficients are not adapted based on the distortion in the channel over which the receiver is receiving the received signal.

13. The method of claim **12**, comprising adapting the first plurality of tap coefficients based on a cost function.

14. The method of claim **13**, wherein the cost function is constrained.

15. The method of claim **12**, wherein the processing is based on an error signal that is generated based on symbol estimates determined by a sequence estimation circuit of the receiver.

16. The method of claim **15**, wherein the error signal is generated via processing of the symbol estimates determined by the sequence estimation circuit.

17. The method of claim **16**, wherein the error signal is a partial response signal generated via a convolution of the symbol estimates and tap coefficients based on the response corresponding to the filter of the receiver and the filter of the transmitter.

18. The method of claim **12**, wherein the second plurality of tap coefficients are used by a sequence estimation circuit that uses the second plurality of tap coefficients to detect symbols that are encoded in an input signal to the receiver.

19. The method of claim **12**, wherein the second plurality of tap coefficients are stored in coefficient registers.

20. The method of claim **1**, wherein:

the receiver comprises tap update circuitry that performs the adapting of the first plurality of tap coefficients; and the adapting of the first plurality of tap coefficients is based on symbol error rate in the receiver.

21. The method of claim **20**, wherein the tap update circuitry receives a set of error vector(s), each error vector in the

set representing the difference between a vector of estimated symbols generated in the receiver and a vector of expected symbols, and wherein the tap update circuitry uses the error vector(s) to update the second plurality of tap coefficients to minimize symbol error rate in the receiver. 5

22. The equalization circuit of claim 1, comprising mixing an output of the decision feedback equalizer circuit with an output of a feed forward equalizer of the receiver.

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