



US008626089B2

(12) **United States Patent**
Singerl et al.

(10) **Patent No.:** US 8,626,089 B2
(45) **Date of Patent:** *Jan. 7, 2014

(54) **ADAPTIVE PREDISTORTER COUPLED TO A NONLINEAR ELEMENT**

(75) Inventors: **Peter Singerl**, Villach (AT); **Heinz Koepl**, Seeboden (AT)

(73) Assignee: **Infineon Technologies AG**, Neubiberg (DE)

(*) Notice: Subject to any disclaimer, the term of this patent is extended or adjusted under 35 U.S.C. 154(b) by 924 days.

This patent is subject to a terminal disclaimer.

(21) Appl. No.: **12/511,999**

(22) Filed: **Jul. 29, 2009**

(65) **Prior Publication Data**

US 2009/0291650 A1 Nov. 26, 2009

Related U.S. Application Data

(63) Continuation of application No. 11/500,107, filed on Aug. 7, 2006, now Pat. No. 7,606,539.

(51) **Int. Cl.**
H04B 1/04 (2006.01)
H03F 1/26 (2006.01)
H04K 1/02 (2006.01)

(52) **U.S. Cl.**
USPC **455/114.3**; 455/63.1; 455/67.13;
455/115.1; 330/149; 375/296; 375/297

(58) **Field of Classification Search**
USPC 455/114.3, 63.1, 67.13, 115.1,
455/117-118, 126, 114.2, 127.1, 127.2, 91;
375/296-297, 295, 284-285;
330/149-151

See application file for complete search history.

(56) **References Cited**

U.S. PATENT DOCUMENTS

5,732,333	A *	3/1998	Cox et al.	455/126
5,900,778	A *	5/1999	Stonick et al.	330/149
6,335,767	B1 *	1/2002	Twitchell et al.	348/608
6,342,810	B1 *	1/2002	Wright et al.	330/51
6,356,146	B1 *	3/2002	Wright et al.	330/2
6,377,116	B1 *	4/2002	Mattsson et al.	330/2
6,501,805	B1 *	12/2002	Twitchell	375/285
6,600,516	B1 *	7/2003	Danielsons et al.	348/608
6,775,330	B2 *	8/2004	Bach et al.	375/296
6,889,060	B2 *	5/2005	Fernando et al.	455/561
7,091,779	B2 *	8/2006	Sahlman	330/149
7,268,620	B2 *	9/2007	Nygren et al.	330/149
7,613,251	B2 *	11/2009	Shako et al.	375/296
7,688,914	B2 *	3/2010	Ohba et al.	375/296

(Continued)

OTHER PUBLICATIONS

Sampling Requirements for Volterra System Identification, Walter A. Frank, IEEE Signal Processing Letters, vol. 3, No. 9, Sep. 1996, pp. 266-268.*

(Continued)

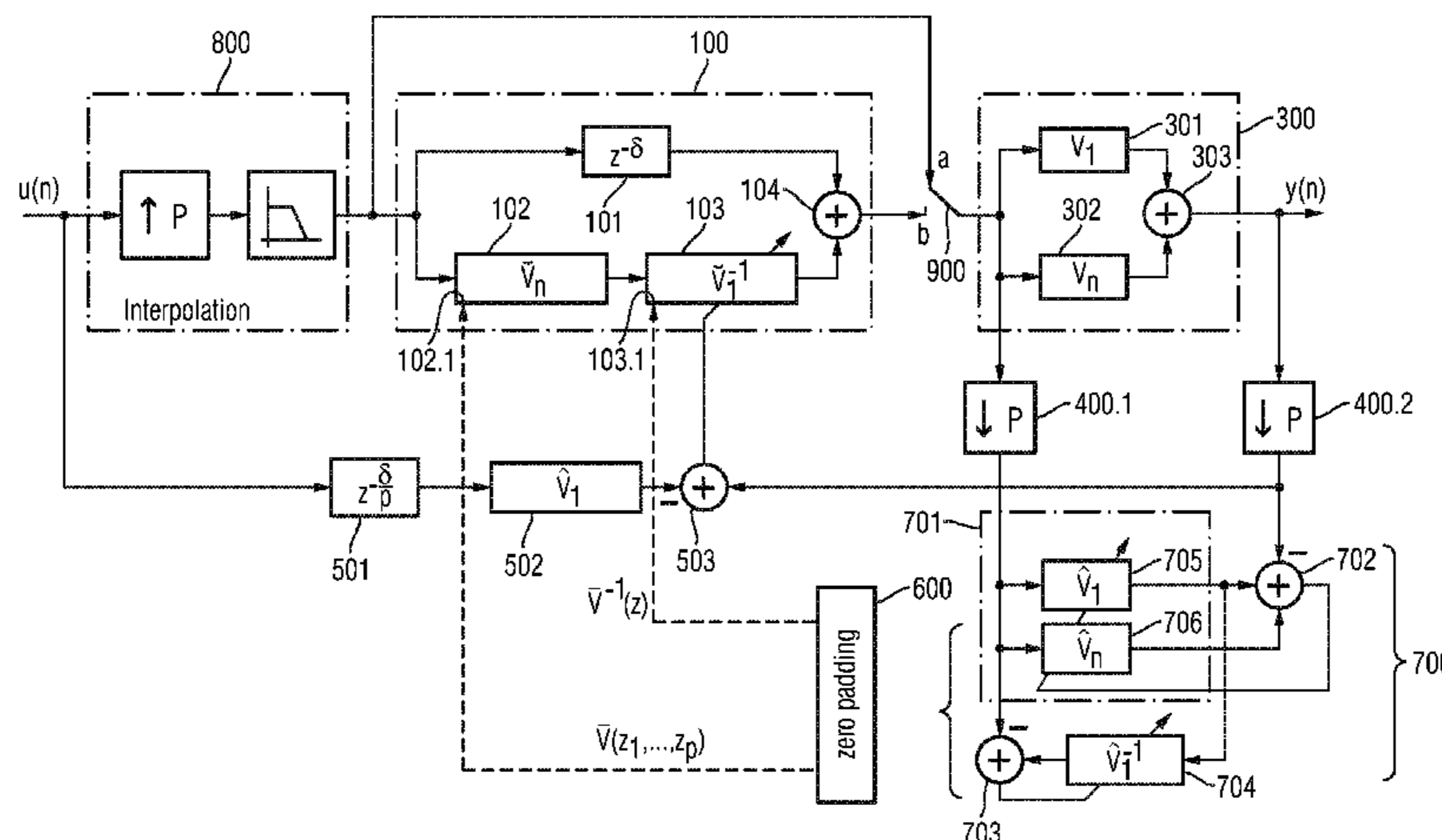
Primary Examiner — Pablo Tran

(74) Attorney, Agent, or Firm — SpryIP, LLC

(57) **ABSTRACT**

In one embodiment, a circuit comprises an adaptive predistorter and a nonlinear element coupled to an output of the predistorter. The predistorter is operative to predistort an input signal according to predistortion control parameters at a given sampling rate. The circuit further comprises an identification unit and a translation unit. The identification unit identifies nonlinearity parameters representing the nonlinearity of the nonlinear element by sampling the output of the nonlinear element at an identification sampling rate lower than twice a bandwidth at the output of the nonlinear element. The translation unit translates the identified nonlinearity parameters into the predistortion control parameters. Other systems and methods are disclosed.

15 Claims, 3 Drawing Sheets



(56)

References Cited

2011/0110473 A1* 5/2011 Keehr et al. 375/350

U.S. PATENT DOCUMENTS

7,795,858 B2* 9/2010 Tuffillaro et al. 324/76.21
7,813,496 B2* 10/2010 Kubin et al. 379/406.02
7,885,797 B2* 2/2011 Koppl et al. 703/13
7,894,788 B2* 2/2011 Keehr et al. 455/296
8,294,605 B1* 10/2012 Pagnanelli 341/144
8,295,790 B2* 10/2012 Koren et al. 455/114.3
2003/0223508 A1* 12/2003 Ding et al. 375/296
2005/0212596 A1* 9/2005 Batruni 330/149
2005/0219089 A1* 10/2005 Batruni 341/118
2008/0133982 A1* 6/2008 Rawlins et al. 714/699
2011/0069749 A1* 3/2011 Forrester et al. 375/232

OTHER PUBLICATIONS

Prosecution History from U.S. Appl. No. 11/500,107, filed Aug. 7, 2006, to include: references of Jun. 17, 2009 List of references cited by examiner; references of Jun. 17, 2009 List of References cited by applicant and considered by examiner; references of Jun. 17, 2009 List of References cited by applicant and considered by examiner; and references of Dec. 18, 2006 Information Disclosure Statement (IDS) Form (SB08).

* cited by examiner

FIG 1

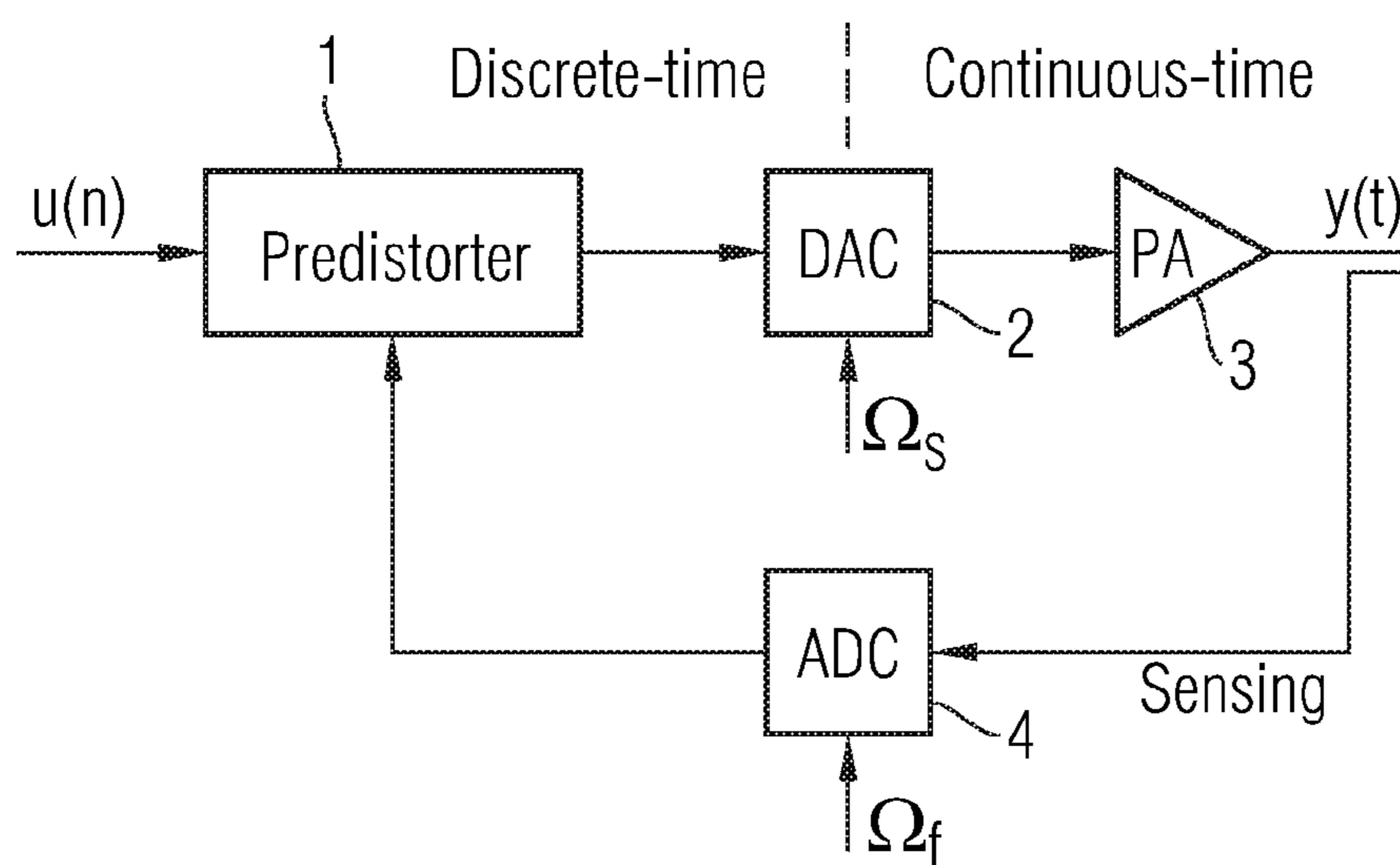


FIG 2

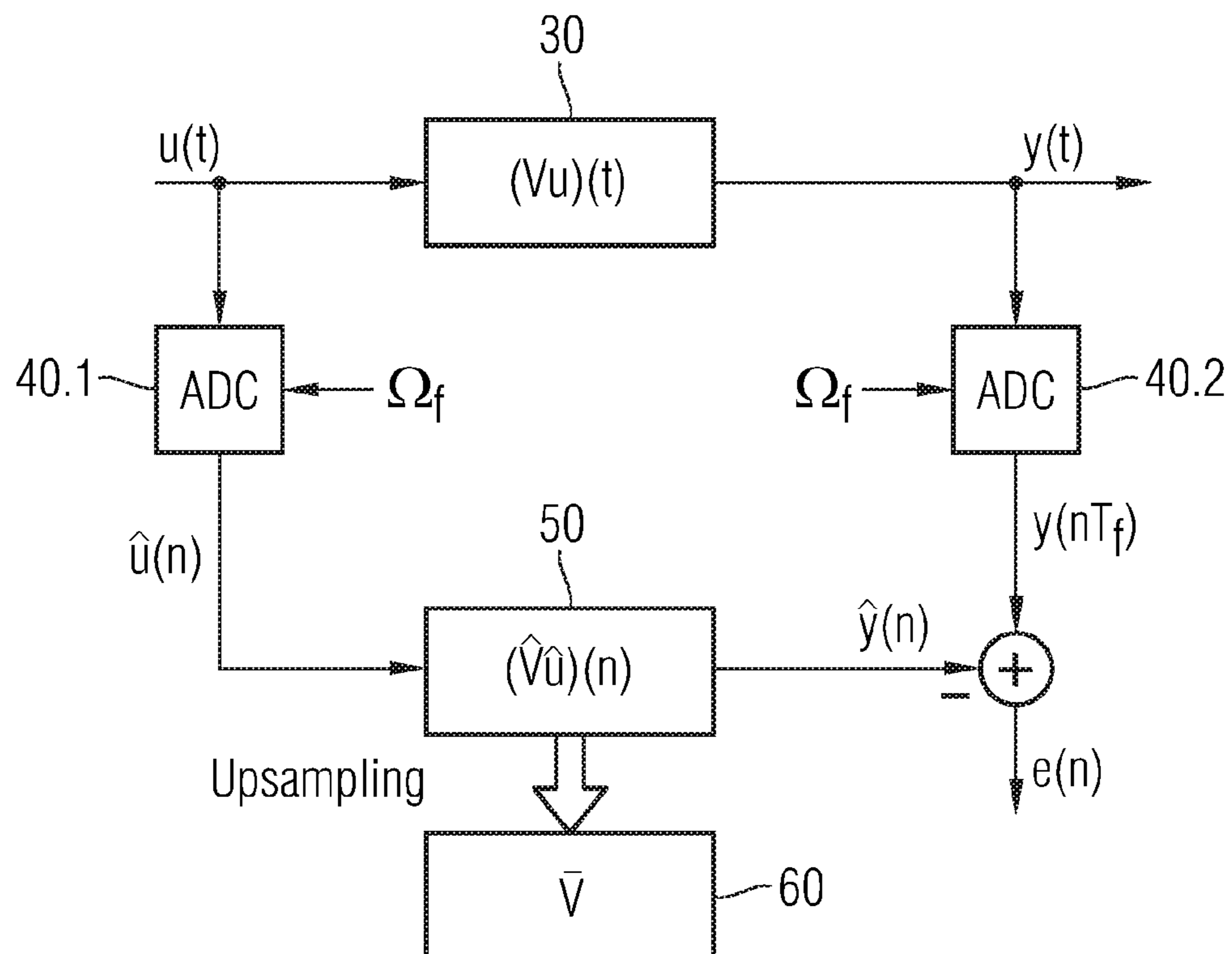


FIG 3

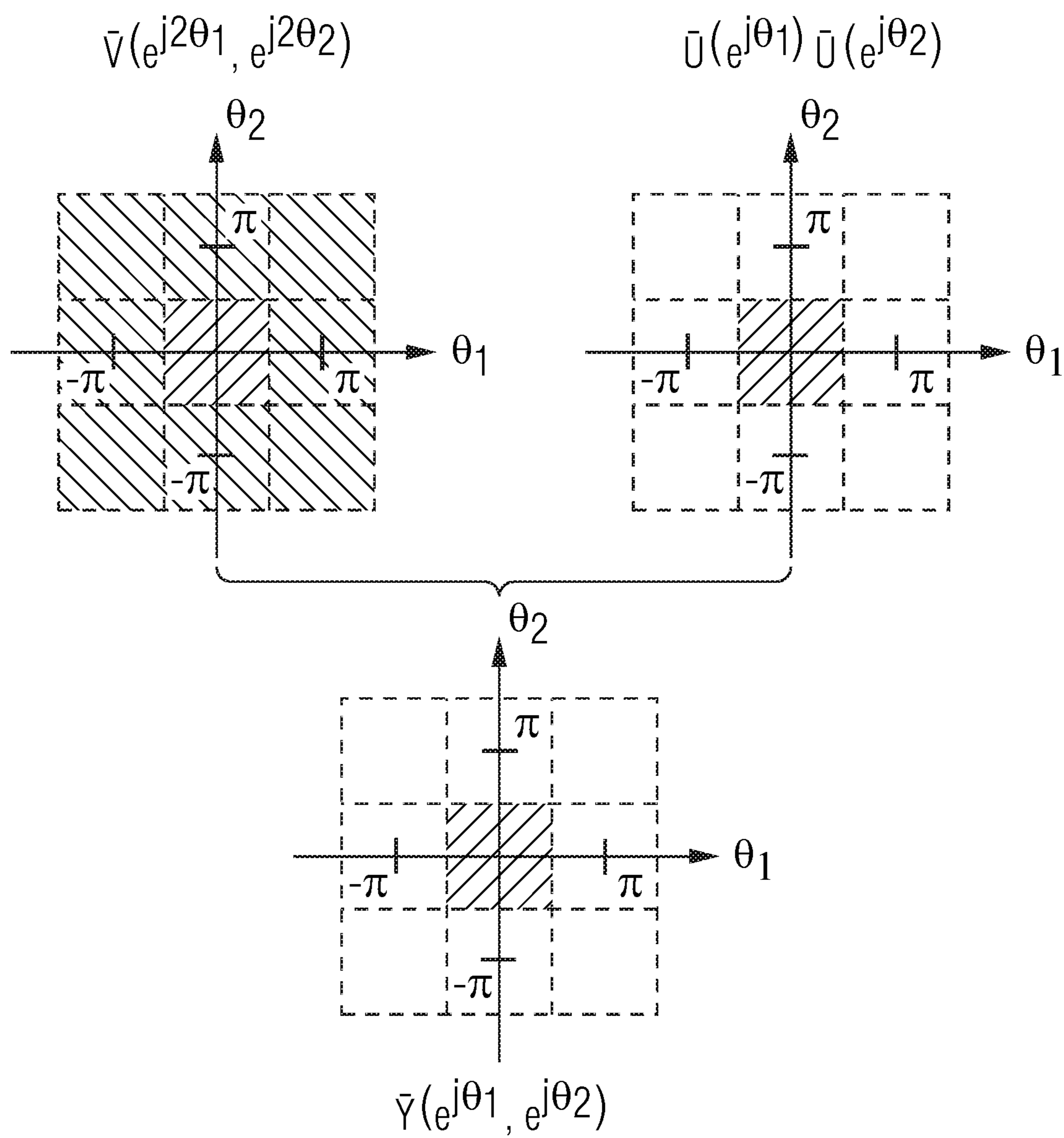
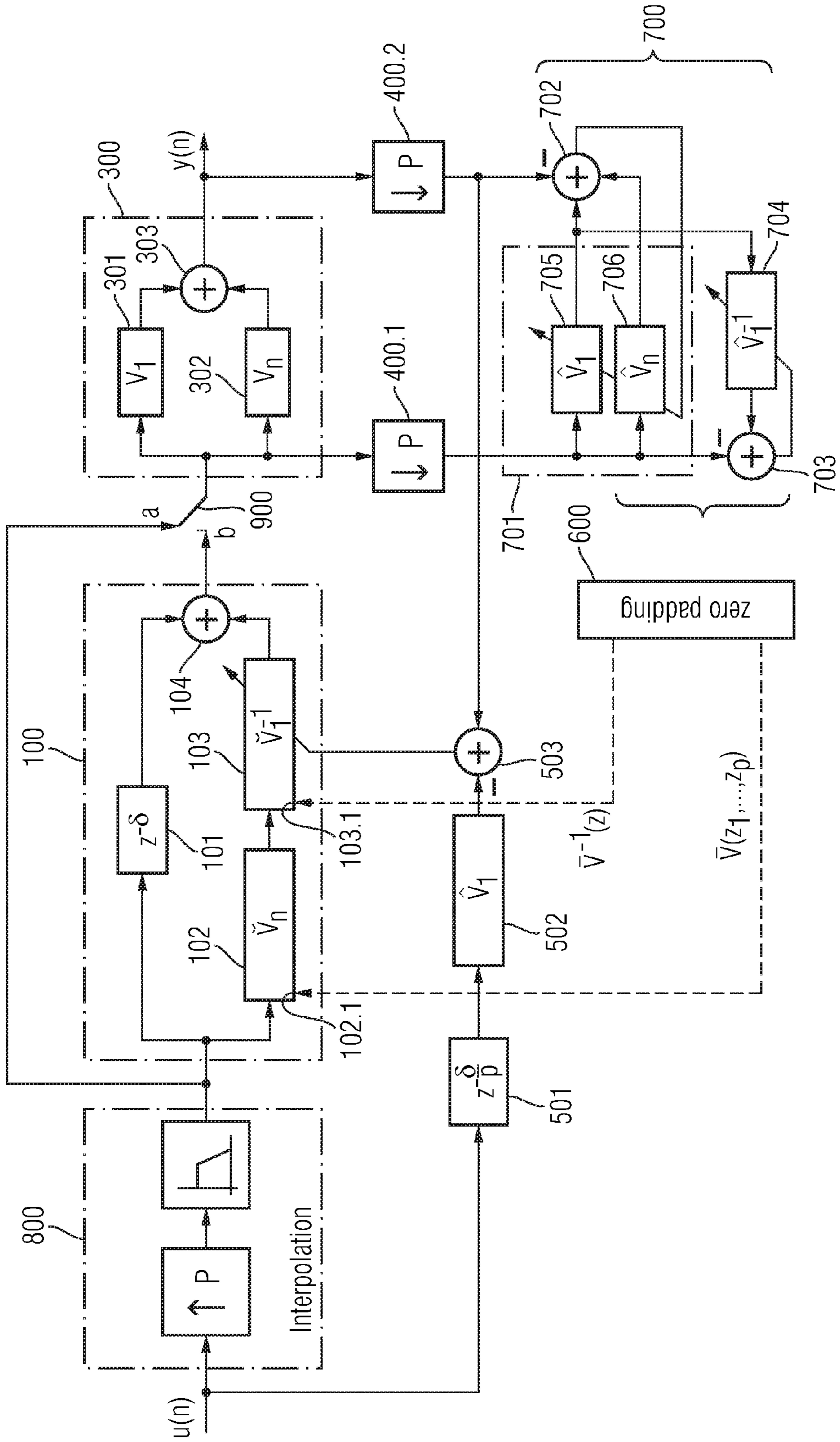


FIG 4



1

ADAPTIVE PREDISTORTER COUPLED TO A NONLINEAR ELEMENT

RELATED APPLICATIONS

This application is a Continuation of co-pending application Ser. No. 11/500,107. The priority of the application Ser. No. 11/500,107 is hereby claimed, and the entirety of the application is hereby incorporated herein by reference.

BACKGROUND

In many technical fields, linearization of a nonlinear element is used to compensate for unwanted effects caused by the nonlinear behaviour of the nonlinear element. One possibility to linearize a nonlinear element is to predistort the signal input into the nonlinear element to ensure that the output signal of the nonlinear element is, in the ideal case, linearly related to the input signal of the predistorter.

In general, such predistorter is a highly complex nonlinear system which incorporates memory. An important step in predistortion is the identification of the nonlinear element based on input-output measurements. Once the nonlinear element has been identified, it may prove possible to calculate appropriate predistorter parameters for linearization of the system containing the predistorter and the nonlinear element.

The technique of predistortion is widely used in modern wireline and wireless communication systems. Typically, communication systems employ a power amplifier with high output power requirements. Such power amplifiers are often driven in the nonlinear region to obtain the highest possible efficiency. This leads in general to spectral regrowth and intermodulation distortion in the signal band. Basically, there are two approaches to minimize these unwanted effects. The first one is the employment of an oversized power transistor which is purely driven in the linear range even for maximum output power requirements. This approach is highly cost intensive during production and also during operation because of the high price of such transistor and its low signal to DC efficiency. The second approach, signal predistortion, allows to create low price devices fulfilling a given spectral mask for the transmission signal even though the power amplifier is driven in the nonlinear region.

Identification of the nonlinear power amplifier or, more general, the nonlinear element is usually accomplished at a sufficiently high sampling rate to cover the information in the out-off-band region caused by the nonlinearity. Usually, the sampling frequency for identification of the nonlinear element is at least twice the bandwidth of the output signal, i.e. 2P-times higher than the signal bandwidth at the input of the nonlinear element, where the factor P denotes the highest odd-order nonlinearity of the nonlinear element. Because the nonlinear behaviour of the nonlinear element is generally unknown before identification, high sampling rates of a few hundreds of Msamples/s are used in communication applications.

According to a generalized sampling theorem, it is possible to uniquely identify a nonlinear element on a sampling frequency which is twice the input signal bandwidth or, in other words, which can satisfy the Nyquist theorem for the input signal of the nonlinear element.

SUMMARY OF THE INVENTION

The following presents a simplified summary of the invention in order to provide a basic understanding of some aspects of the invention. This summary is not an extensive overview

2

of the invention, and is neither intended to identify key or critical elements of the invention nor to delineate the scope of the invention. Rather, the purpose of the summary is to present some concepts of the invention in a simplified form as a prelude to the more detailed description that is presented later.

In one embodiment, the invention relates to a circuit that includes an adaptive predistorter to predistort an input signal according to a predistortion control parameter. A nonlinear element is coupled to an output of the predistorter, the output having a bandwidth. The circuit also includes an identification unit to identify a parameter representing the nonlinearity of the nonlinear element and to sample the output of the nonlinear element at an identification sampling rate lower than twice the bandwidth at the output of the nonlinear element. In addition, the circuit includes a translation unit to translate the parameter into the predistortion control parameter.

The following description and annexed drawings set forth in detail certain illustrative aspects and implementations of the invention. These are indicative of but a few of the various ways in which the principles of the invention may be employed.

BRIEF DESCRIPTION OF THE DRAWINGS

Aspects of the invention are made more evident in the following detailed description of some embodiments when read in conjunction with the attached drawing figures, wherein:

FIG. 1 is one embodiment of a block diagram of a communication transmitter with digital predistortion,

FIG. 2 is one embodiment of a diagram illustrating a system identification setup at a low-rate sampling frequency Ω_f and the generation of a high-rate operator \bar{V} for predistortion and modeling purposes,

FIG. 3 is one embodiment of a diagram illustrating two-dimensional zero-padding and spectral masking by a high rate input signal for a 2nd-order Volterra kernel, and

FIG. 4 is a block diagram showing another embodiment of the invention.

DETAILED DESCRIPTION

In the following description and claims, the term Volterra kernel is used. This term has a well defined meaning in nonlinear system theory. The concept of describing nonlinear systems in terms of Volterra kernels will not be explained in detail herein. For an intuitive understanding of the concept of Volterra kernels, it is to be noted that Volterra kernels are used to describe a nonlinear system in a similar way as the impulse response is used to describe a linear system. In linear system theory, the output signal of a linear system is the convolution of the input signal with the impulse response. Analogously, the output signal of a nonlinear system is the multi-dimensional convolution of the input signal with a series expansion of Volterra kernels. In fact, the first order Volterra kernel is identical to the impulse response of a linear system. As the concept of describing a system by an impulse response is limited to linear systems, the Volterra kernel representation may be intuitively understood as a generalization of the impulse response concept to nonlinear systems.

FIG. 1 depicts a simplified block diagram of a communication transmitter with digital predistortion. A digital input signal $u(n)$ is input into an adaptive predistorter 1. The output of the predistorter 1 is fed into a digital-to-analog converter (DAC) 2 converting the input discrete time signal into an

3

output continuous-time signal. This continuous-time signal is input into a power amplifier (PA) **3** to generate a high power analog output signal $y(t)$.

The predistorter **1** acts on the digital input signal $u(n)$ by a nonlinear operator which is in one embodiment the inverse of the nonlinearity of the power amplifier **3** to obtain an overall linear response. Because the nonlinear behavior of the power amplifier **3** is in general unknown, a feedback path is applied to identify the power amplifier **3** and to compute the inverse for the predistorter **1**. The feedback path comprises an analog-to-digital converter (ADC) **4** for converting an output-sensed signal back into the discrete-time domain.

To compensate the out-of-band spectral components caused by the nonlinearity, the predistorter **1** can be operated at a sufficiently high sampling frequency Ω_s , which is at least $2P$ -times higher than the transmission signal bandwidth B at the input of the power amplifier **3**, where the factor P denotes the highest order of significant nonlinearity in the power amplifier **3**. Therefore, the DAC **2** operates at the high sampling rate Ω_s . On the other hand, the identification of a Volterra system—in FIG. **1** the nonlinear power amplifier **3**—can be accomplished at a sampling rate that is just twice the bandwidth B of the input signal of the power amplifier **3**. Therefore, according to the embodiment of the invention shown in FIG. **1**, the sampling rate of the ADC **4** may be reduced to $\Omega_f = \Omega_s/P$. Thus, the Volterra kernels of the power amplifier **3** are identified at this lower sampling rate.

After Volterra kernel identification at low sampling rate Ω_f , the identified Volterra kernels describing the nonlinear behaviour of the power amplifier **3** are used to find the correct setting of the predistorter **1**. This step involves Volterra kernel upsampling.

Volterra kernel identification and Volterra kernel upsampling are illustrated in FIGS. **2** and **3** and described below. The embodiments of the invention as well as the following description are not restricted to power amplifiers **3** as nonlinear elements. It can be assumed that the nonlinear element can be modeled by an additive decomposition of a dominant linear part and a perturbative nonlinear part. As the entire class of such weakly nonlinear systems may be modeled in terms of its Volterra series, a Volterra representation of the nonlinear element is used herein. However, all other approximate models for weakly nonlinear systems could also be used, including but not limited to: the Wiener model, Hammerstein model, and the “linear-nonlinear-linear” (LNL) structure.

The following notation and definitions are introduced. A continuous-time weakly nonlinear system **30**, often representing nonlinear power amplifiers, will be referred to as the system $V: u \rightarrow y$, with $u, y \in L_2(\mathbb{R}_+)$, where $L_2(\mathbb{R}_+)$ denotes the space of square integrable signals and \mathbb{R}_+ is the set of positive reals. Its associated set of Laplace-domain Volterra kernels is $\{V(s_1, \dots, s_p)\}$ with $p=1, \dots, P$, where the order of homogeneity of a kernel is not given explicitly but can be read off from the number of its arguments.

A discrete-time Volterra model **50** operating at the Nyquist rate Ω_f of the input signal will be referred to as the low-rate model $\hat{V}: \hat{u} \rightarrow \hat{y}$ with $\hat{u}, \hat{y} \in I_2(Z_+)$, where $I_2(Z_+)$ denotes the space of square summable infinite sequences and Z_+ is the set of positive integers. Its associated Z -transform kernels are $\{\hat{V}(z_1, \dots, z_p)\}$ with $p=1, \dots, P$. The ADCs **40.1** and **40.2** provide the input for the low-rate model \hat{V} and correspond to ADC **4** in FIG. **1**.

The upsampled version of the Volterra model **50** is discrete time upsampled Volterra model **60** operating at an integer multiple of Ω_f , especially at the Nyquist rate of the output signal Ω_s . It will be termed the upsampled model $\bar{V}: \bar{u} \rightarrow \bar{y}$

4

with $\bar{u}, \bar{y} \in I_2(Z_+)$. Its associated Z -transform kernels are $\{\bar{V}(z_1, \dots, z_p)\}$ with $p=1, \dots, P$.

With a slight deviation from standard notation, but for the sake of conciseness and without loss of generality in case of band-limited input signals, it is associated with the system V its exact equivalent discrete-time system $I_2(Z_+) \rightarrow I_2(Z_+)$ at Ω_s with kernels $\{V(z_1, \dots, z_p)\}$. The following can be used in a frequency domain analysis, i.e., $s_k = j\Omega k$ and $z_k = e^{j\theta k}$ with $k=1, \dots, p$ and j being the imaginary unit.

Referring to kernel identification, one can find a low-rate model \hat{V} reproducing exactly the sampled output $y(nT_f)$ of the system V with $T_f = 2\pi/\Omega_f$. This is depicted in FIG. **2**, where minimizing $e(n) = y(nT_f) - \hat{y}(n)$ is a possibility to find that low-rate model \hat{V} .

Upsampling of the Volterra kernels $\{\bar{V}(z_1, \dots, z_p)\}$ is performed by zero-padding without a following interpolation. The multidimensional zero-padding of Volterra kernels corresponds to a frequency scaling and to a periodic extension of $\{\hat{V}(z_1, \dots, z_p)\}$. The kernels of the upsampled model are obtained by $\bar{V}(z_1, \dots, z_p) = \hat{V}(z_1^P, \dots, z_p^P)$. Thus, to upsample a Volterra model, one solely can replace each delay element z^{-1} in the Volterra kernels by z^{-P} .

Note that the periodic extension implies that the kernels of the obtained upsampled model coincide with those of the system V over a limited frequency region, i.e., $\bar{V}(e^{j\theta_1}, \dots, e^{j\theta_p}) = V(e^{j\theta_1}, \dots, e^{j\theta_p})$ for $\theta_k \in [-\pi/P, \pi/P)$, with $\theta_k = 2\pi\Omega k/\Omega_s$ and $k=1, \dots, p$. But at the high rate, the interpolated input signal $\bar{U}(e^{j\theta})$ is nonzero for $\theta \in [-\pi/P, \pi/P)$, such that the system responses of \bar{V} and V are identical. This process is illustrated for the response of a homogeneous 2nd-order Volterra system ($P=2$) in FIG. **3**. \bar{Y} is the z -transform of the upsampled model output signal $\bar{y}(t)$ and \bar{U} is the z -transform of the upsampled model input signal $u(t)$. The shaded areas in FIG. **3** represent the nonzero values of the frequency-domain kernels \bar{V} and the frequency domain signals \bar{Y} and \bar{U} of the upsampled model, respectively.

FIG. **4** illustrates another embodiment of the invention. The circuit depicted in FIG. **4** comprises an interpolator **800** which upsamples and interpolates the input signal $u(n)$. The P -times upsampled and interpolated input signal $u(n)$ is input into a predistorter **100**. The predistorter **100** has a simple predistortion architecture comprising the functional elements **101**, **102**, **103** and an adder **104**. More specifically, the predistorter **100** comprises a parallel structure of a delay element **101** and a cascade of a purely nonlinear element **102** and a purely linear element **103**. The nonlinear and linear characteristics of the elements **102** and **103** can be controlled by parameters received at the control inputs **102.1** and **103.1** of the elements **102**, **103**, respectively. The adder **104** sums the output of the delay element **101** and the output of the purely linear element **103**.

The operation of the functional elements **101**, **102**, **103** can be expressed by operators. The delay element **101** applies the delay operation $z^{-\delta}$ with $\delta = nP$, $n \in Z_+$, the nonlinear element **102** applies the nonlinear part \check{V}_n of an operator \check{V} and the linear element **103** applies the inverse of the linear part \check{V}_1 of the operator \check{V} . Thus, the overall function of the predistorter **100** can be expressed by the operator P with

$$P = z^{-\delta} + \check{V}_1^{-1} \circ \check{V}_n, \quad (1)$$

where “ \circ ” denotes the composition of operators. The delay of δ samples is used to compensate the latency of the cascade $\check{V}_1^{-1} \circ \check{V}_n$. It is shown later under section “Mathematical Description of Predistorter Architecture” that this specific predistorter architecture is capable of linearizing the nonlin

5

ear element **300** when selecting a proper operator \check{V} and that the kernels of this proper operator \check{V} are given by

$$\check{V}_1(z) = -V_1(z)z^\delta \text{ for } p=1 \text{ and} \quad (2)$$

$$\check{V}_p(z_1, \dots, z_p) = V_p(z_1, \dots, z_p) \text{ for } p \geq 2. \quad (3)$$

Returning to FIG. 4, the output of the predistorter **100** is coupled to the input of a nonlinear element **300** via a switch **900**, if the switch **900** is in position b. If the switch **900** is in position a, the predistorter **100** is bypassed.

The nonlinear element **300** is modeled by a purely linear unit **301** represented by the linear part V_1 of the operator V , a nonlinear unit **302** represented by the nonlinear part V_n of the operator V and an adder **303** adding the output of the linear and nonlinear units **301**, **302**. The adder **303** outputs the output signal $y(n)$.

The input of the nonlinear element **300** is down-sampled in a down-sampling stage **400.1** by the factor P and the output of the nonlinear element **300** is down-sampled in a down-sampling stage **400.2** by the factor P . It is also possible that a smaller down-sampling factor than P is used. In any case, the sampling rate at the output of the down-sampling stages **400.1**, **400.2** is lower than twice the bandwidth at the output of the nonlinear element **300**.

The down-sampled signals are input into an identification unit such as a low-rate kernel identification stage **700**. The low-rate kernel identification stage **700** can identify a nonlinearity parameter representing the nonlinearity of the nonlinear element **300** and sample the output of the nonlinear element at an identification sampling rate that is lower than twice the bandwidth at the output of the nonlinear element. The low-rate kernel identification stage **700** comprises a low-rate nonlinear element modeling unit **701**. Further, it may comprise some circuitry (adder **702**, adder **703** and unit **704**) adapted to minimize a cost function over the nonlinearity parameter for the purpose of identifying the nonlinear element **300**.

The low-rate nonlinear element modeling unit **701** comprises a linear stage **705** operating according to the linear part \hat{V}_1 of the low-rate model operator \hat{V} and a nonlinear stage **706** operating according to the nonlinear part \hat{V}_n of the low-rate model operator \hat{V} . The operation characteristics of both stages **705**, **706** may be changed by adjusting parameters controlling the function of the respective stages **705**, **706**. As there are many different possibilities to parameterize the low-rate operator parts \hat{V}_1 , \hat{V}_n , no specific parameterization is outlined here.

During the initial identification of the nonlinear element **300**, the predistorter **100** is bypassed by setting the switch **900** in position a. Identification of the nonlinear element **300** means determination of the low-rate model operator \hat{V} , i.e. to find the set of parameters for the stages **705**, **706** which set the characteristic of the low-rate nonlinear element modeling unit **701** to the low-rate model operator \hat{V} reproducing exactly the sampled output $y(nTf)$ of the nonlinear element operator V with $Tf=2\pi/\Omega f$. This may be accomplished by minimizing the cost function

$$E = \sum_{m=m_1}^{m_2} |e[m]|^2 \rightarrow \min, \quad e[m] = y[m] - \hat{y}[m], \quad (4)$$

where $y[m]$ is the decimated output signal of the nonlinear element **300** at the output of the down-sampling stage **400.2** and $\hat{y}[m]$ is the sum of the output signals of the linear and nonlinear stages **705**, **706**. In other words, the input param-

6

eters of the stages **705**, **706** are varied until the output of the adder **702** reaches a minimum. Although one method has been described, it will be appreciated that other methods and especially other cost functions may be applied to identify the nonlinear element **300** in the low-rate nonlinear element modeling unit **701**.

In order to obtain the parameters representing the inverse \hat{V}_1^{-1} of the linear part \hat{V}_1 of the operator \hat{V} , input parameters of the inverse linear stage **704** are adjusted until the output of adder **703** reaches a minimum. Also here, other methods for obtaining the parameters representing the inverse of the linear part of the operator \hat{V} are possible.

In one embodiment, a translation unit can translate a parameter representing the nonlinearity of the nonlinear element into a predistortion control parameter. The translation unit can comprise a transformation unit that is adapted to calculate parameters which are an upsampled representation of the identified nonlinearity parameter.

The parameters calculated by the transformation unit can be calculated by applying a multi-dimensional zero padding in the time domain. In one embodiment, the translation unit includes an optimizing unit to calculate the predistortion control parameter by minimizing a second cost function over the parameters calculated by the transformation unit.

The low-rate identified operator \hat{V} is used to calculate the input parameters for the nonlinear stage **102** and the inverse linear stage **103** of the predistorter **100**. Calculation of these parameters is done by obtaining the kernels of the upsampled operator \bar{V} . As already described in connection with FIG. 2, the low-rate identified operator \hat{V} can be recalculated to the high-rate operator \bar{V} by a multi-dimensional zero padding. As zero padding in the time domain results in a simple frequency scaling in the frequency domain, this operation can be accomplished with almost no additional costs—each unit-sample delay is replaced by a P -sample delay according to

$$\bar{V}(z_1, \dots, z_p) = \hat{V}(z_1^P, \dots, z_p^P). \quad (5)$$

Then, the upsampled operator \bar{V} is used in the predistorter **100** with

$$\check{V}(z_1, \dots, z_p) = \bar{V}(z_1, \dots, z_p). \quad (6)$$

It is to be noted that, although the upsampled operator \bar{V} is able to reproduce the exact output of the system (i.e. of the operator V), the upsampled operator \bar{V} generally does not perform in the same way when applied to the predistorter **100**.

To see this, note that the kernels of the unsampled model (i.e. operator \bar{V}) and the nonlinear system (i.e. operator V) coincide over the bandwidth of the input signal $\bar{U}(e^{j\theta})$. The structure of the predistorter $P \equiv z^{-\delta} + \hat{V}_1^{-1} \circ \hat{V}_n$ reveals that, owing to the spectral regrowth of \hat{V}_n , the linear operator \hat{V}_1^{-1} acts on a signal with a bandwidth that is in general P -times the input signal bandwidth. Thus, the output signal of the upsampled version of \hat{V}_1^{-1} differs from the correct version given in (2) and (3) by an amount that depends on the flatness of the linear frequency response of the system. The more flat the response is, the less is the effect of the periodic extension in the upsampled version outside the input signal bandwidth.

To correct for the periodic extensions of $\check{V}(z)$, one can apply a low-rate adaptation, where the initialization is taken to be the upsampled response $\bar{V}(z)$. This optional low-rate adaptation is implemented by the delay stage **501**, the linear stage **502** operating according to the linear part \hat{V}_1 of the operator \hat{V} and the adder **503**. The linear stage **502** may be identical to the linear stage **705** and uses the parameters obtained in the optimization process described above for low-rate identification of the linear element **300**. The adaptation is performed by varying the characteristics of the inverse linear

stage **103** (for instance by varying the parameters provides at the control input **103.1**) until the output of the adder **503** settles at a minimum. Different to the optimization process used for low-rate identification of the linear element **300**, this optimization is to be done with the switch **900** being in position b.

It is to be noted that the calculation of the predistorter parameters as described above by upsampling the nonlinear kernels of the low-rate identified nonlinear element **300** and copying these high-rate kernels into the predistorter **100** is generally not feasible for predistorter architectures.

With the applied predistortion architecture $P = z^{-\delta} + \check{V}_1^{-1} \circ \check{V}_n$, as the operator \check{V}_n with kernels $\{\check{V}(z_1, \dots, z_p)\}$, $p=2, \dots, P$, operates on the input signal bandwidth, its response is identical to the response of \check{V}_n with $\{V(z_1, \dots, z_p)\}$ and thus requires no adaptation. This feature allows for efficient low-rate adaptive predistortion, where the few parameters of $\check{V}(z)$ need to be adapted, while the mass of parameters from the higher-order frequency responses requires no adaptation. This is in contrast to the application of the upsampled kernels $\{\check{V}(z_1, \dots, z_p)\}$ to the so-called pth-order inverse predistorter architecture, which, for $p=3$, can be written, for instance, by the following expression

$$\check{P} = z^{-\delta} \circ \{1 - V_1^{-1} \circ [V_2 \circ V_1^{-1} \circ V_2 + V_2 \circ (1 - V_1^{-1} \circ V_2) + V_3]\}. \quad (7)$$

In the case of upsampled kernels, every composition of operators where a nonlinear operator with $k>1$ is preceding one or more operators, an adaptation of all consecutive operators following V_k is necessary to yield equivalent performance to a 3rd-order inverse utilizing the correct high-rate kernels of the system V .

The block diagram shown in FIG. 4 may be implemented in various circuits including purely digital systems or mixed signal systems. In digital systems, no analog-to-digital conversion is performed and therefore, the down-sampling stages **400.1** and **400.2** are decimation stages. In mixed signal systems as, for instance, shown in FIG. 1, the down-sampling stage **400.2** is implemented by an analog-to-digital converter corresponding to the ADC **4** shown in FIG. 2. In this case, the DAC **2** (cf. FIG. 1) would be implemented between the switch **900** and the input of the nonlinear element **300**. Thus, the down-sampling stage **400.1** would still be in the digital domain and does not need to be replaced by another ADC.

Further, it is to be noted that for purely digital systems as well as for mixed signal systems, the nonlinear element **300** could be implemented by virtually any nonlinear circuitry and is not restricted to amplifiers. In addition, although described and illustrated as a hardware structure, the functionality and features of the present system can also be performed by appropriate software routines or a combination of hardware and software. Thus, the present invention should not be limited to any particular implementation.

As a practical implementation, the nonlinear element **300** may be a power amplifier for a VDSL (Very High Data Rate Digital Subscriber Line) analog front-end transmitter device that complies with the distortion ratios of the VDSL standard Band Plan **998**. To be able to apply predistortion over a realistic distortion range, such circuit was miss-tuned to yield a harmonic distortion ratio of 40 dB. The operator V has been obtained by fitting a simple 5th-order Volterra model to the I/O measurement of a transistor-level circuit simulation. The standard 4-band VDSL system can utilize a bandwidth of up to 12 MHz, where each band is DMT (Discrete Multi-Tone Modulation) modulated. The frequency allocation for this simulation according to the standard Band Plan **998** comprises two downstream bands with bandwidth of each 2.59

MHz, located at center frequencies 2.225 MHz and 6.885 MHz. According to the above, the sampling rate of the digital predistortion for this system can be at least $\Omega_S = 2\pi \times 81.8$ MHz. Conventionally, this directly translates to the sampling rate of the ADC **4** in the feedback path used for the parameterization of the predistorter. The proposed approach can use an ADC sampling rate of $\Omega_f = 2\pi \times 16.36$ MHz resulting in a cost efficient implementation of such circuitry.

Mathematical Description of Predistorter Architecture

First, it is shown that the predistorter architecture depicted in FIG. 4 and defined in equation (1) is capable of linearizing a nonlinear element **300** modeled by the operator V .

The composition of the predistorter **100** and the nonlinear element is given by the expression

$$L = (V_1 + V_n) \circ (z^{-\delta} + \check{V}_1^{-1} \circ \check{V}_n). \quad (8)$$

This expression yields a linearization if $\|V_n\| \ll \|V_1\|$ with $\check{V}_1 = -z^{-\delta} \circ V_1$ and $\check{V}_n = V_n$, where the operator norm corresponds to the signal norm with $\|V_1\| = \sup_{|u|<1} \|V_1 u\|$. Qualitatively this becomes clear by decomposing (8) as

$$L = z^{-\delta} \circ V_1 + V_1 \circ \check{V}_1^{-1} \circ \check{V}_n + V_n \circ (z^{-\delta} + \check{V}_1^{-1} \circ \check{V}_n), \quad (9)$$

where it is used that the delay operator commutes with all time-invariant operators. The above assumption on weak nonlinearity implies $\|V_n \circ \check{V}_1^{-1} \circ V_n\| \ll \|V_n\|$, such that the last term in (9) may be approximated by $V_n \circ (z^{-\delta} + \check{V}_1^{-1} \circ \check{V}_n) \approx V_n \circ z^{-\delta}$. Thus, the overall system response is rendered as $L \approx z^{-\delta} \circ V_1$, i.e. is linear.

Second, a quantitative analysis of the linearization performance of the predistorter **100** is provided by employing the Volterra series representation of the involved operators.

The set of Volterra kernels of \check{V} is denoted by $\{\check{V}(z_1, \dots, z_p)\}$ with $p=1, \dots, P$. The set of kernels $\{L(z_1, \dots, z_p)\}$ for $p=0, 1, 2, \dots, P_2$ of L is expressed in terms of the kernels of the operators according to the decomposition in (9). For clarity of presentation, we indicate the order of homogeneity of a Volterra kernel by an additional subscript in the following derivation. Allowing for non-minimum phase systems with $\check{V}_1(z) \check{V}_1^{-1}(z) = Z^{-\delta}$, the application of the Z-domain cascade rule for discrete-time Volterra series to (9) yields

$$L_p(z_1, \dots, z_p) = \quad (10)$$

$$\sum_{n=1}^p \sum_{m_1=1} \dots \sum_{m_n=1} \left[V_n(z_1 \dots z_{\omega(1)}, \dots, z_{\alpha(n)} \dots z_{\omega(n)}) \times \prod_{k=1}^n \check{V}_1^{-1}(z_{\alpha(k)} \dots z_{\omega(k)}) \check{V}_{m_k}(z_{\alpha(k)}, \dots, z_{\omega(k)}) \right]$$

with the lower and upper index bound functions

$$\alpha(k) = \alpha(m_1, \dots, m_{k-1}) = 1 + \sum_{j=1}^{k-1} m_j$$

with $\alpha(1)=1$ and

$$\omega(k) = \omega(m_1, \dots, m_k) = \sum_{j=1}^k m_j,$$

respectively.

Separating terms in (10) for $n=1$ and $n=p$ gives

$$L_p(z_1, \dots, z_p) = V_1(z_1 \dots z_p) \check{V}_1^{-1}(z_1 \dots z_p) \times \check{V}_p(z_1, \dots, z_p) + V_p(z_1, \dots, z_p) (z_1 \dots z_p)^{-\delta} + \sum_{n=2}^{p-1} \sum_{\substack{m_1=1 \\ \omega(n)=p}} \dots \sum_{m_n=1} \left[V_n(z_1 \dots z_{\omega(1)}, \dots, z_{\alpha(n)} \dots z_{\omega(n)}) \times \prod_{k=1}^n \check{V}_1^{-1}(z_{\alpha(k)} \dots z_{\omega(k)}) \check{V}_{m_k}(z_{\alpha(k)}, \dots, z_{\omega(k)}) \right] \quad (11)$$

where the first two summands in (11) equalize if the linear frequency domain kernel is $\check{V}_1(z) = -V_1(z)z^\delta$ as already noted in (2) and the kernels for $p \geq 2$ are $\check{V}_p(z_1, \dots, z_p) + V_p(z_1, \dots, z_p)$ as already noted in (3).

In conclusion, the system with an adjusted predistorter **300** results in the following Volterra kernels

$$L_1(z_1) = V_1(z)z^{-\delta} \quad (12)$$

$$L_2(z_1, z_2) = 0$$

$$L_3(z_1, z_2, z_3) = - \left[\begin{array}{c} V_2(z_1 z_2, z_3) V_1^{-1}(z_1 z_2) V_2(z_1, z_2) + \\ V_2(z_1, z_2 z_3) V_1^{-1}(z_2 z_3) V_2(z_2, z_3) \end{array} \right] (z_1 z_2 z_3)^{-\delta}$$

$$\vdots$$

$$L_p(z_1, \dots, z_p) = - \sum_{n=2}^{p-1} \sum_{\substack{m_1=1 \\ \omega(n)=p}} \dots \sum_{m_n=1} \left[V_n(z_1 \dots z_{\omega(1)}, \dots, z_{\alpha(n)} \dots z_{\omega(n)}) \prod_{k=1}^n (z_{\alpha(k)} \dots z_{\omega(k)})^{-\delta} \times \check{V}_1^{-1}(z_{\alpha(k)} \dots z_{\omega(k)}) V_{m_k}(z_{\alpha(k)}, \dots, z_{\omega(k)}) \right]$$

Although “perfect” equalization is possible for the 2nd-order Volterra kernel $L_2(z_1, z_2)$, the resulting distortion of the predistorted non-linear element output signal is much lower compared to the non-compensated non-linear element output signal if $\|V_n\| \ll \|V_1\|$ because all the kernels $L_p(z_1, \dots, z_p)$ for $p \geq 3$ in (12) incorporate the multiplicative inverse of the linear kernel $V_1(z)$.

Although the invention has been illustrated and described with respect to one or more implementations, alterations and/or modifications may be made to the illustrated examples without departing from the spirit and scope of the appended claims. In particular regard to the various functions performed by the above described components or structures (blocks, units, engines, assemblies, devices, circuits, systems, etc.), the terms (including a reference to a “means”) used to describe such components are intended to correspond, unless otherwise indicated, to any component or structure which performs the specified function of the described component (e.g., that is functionally equivalent), even though not structurally equivalent to the disclosed structure which performs the function in the herein illustrated exemplary implementations of the invention. In addition, while a particular feature of the invention may have been disclosed with respect to only one of several implementations, such feature may be

combined with one or more other features of the other implementations as may be desired and advantageous for any given or particular application. Furthermore, to the extent that the terms “including”, “includes”, “having”, “has”, “with”, or variants thereof are used in either the detailed description and the claims, such terms are intended to be inclusive in a manner similar to the term “comprising”.

What is claimed is:

1. A circuit comprising:

a nonlinear element having an output, and an identification unit coupled to the nonlinear element, the identification unit to identify a nonlinearity parameter associated with the nonlinear element and to sample the output of the nonlinear element at a sampling rate lower than twice a bandwidth at the output of the nonlinear element, the identification unit adapted to calculate the nonlinearity parameter by minimizing a first cost function over the nonlinearity parameter.

2. The circuit according to claim 1, further comprising: an adaptive predistorter having a parallel structure of a delay stage and a cascade of an adaptive nonlinear stage and an adaptive linear stage.

3. The circuit according to claim 2, wherein: the delay stage causes a P-sample delay, where P is the ratio of the sampling rate of the adaptive predistorter to the sampling rate.

4. The circuit according to claim 1, further comprising: a translation unit adapted to calculate parameters which are an upsampled representation of the identified nonlinearity parameter and to calculate the predistortion control parameter.

5. The circuit according to claim 4, wherein: the parameters calculated by the transformation unit are calculated by applying a multi-dimensional zero padding in the time domain.

6. The circuit according to claim 4, wherein: the translation unit comprises an optimizing unit to calculate the predistortion control parameter by minimizing a second cost function over the parameters calculated by the transformation unit.

7. The circuit according to claim 2, comprising: a digital-to-analog converter coupled to the predistorter.

8. The circuit according to claim 1, comprising: an analog-to-digital converter coupled to the identification unit.

9. The circuit according to claim 8, wherein: the analog-to-digital converter is adapted to operate at the sampling rate.

10. A method, comprising:

identifying nonlinearity parameters representing nonlinearity of a nonlinear element by sampling an output of the nonlinear element at a sampling rate lower than twice a bandwidth of a signal, the identifying of the nonlinearity parameters including calculating by minimizing a first cost function over the nonlinearity parameters, and translating the identified parameters into the predistortion control parameters.

11. The method according to claim 10, wherein: the translating includes calculating transformed parameters which are an upsampled representation of the identified nonlinearity parameters.

12. The method according to claim 11, further comprising: calculating the predistortion control parameters by minimizing a second cost function over transformed parameters calculated in the translating act.

- 13.** A method, comprising:
identifying parameters representing Volterra kernels of a
nonlinear element by sampling an output of the nonlin-
ear element at a sampling rate lower than twice a band-
width at the output of the nonlinear element, 5
calculating the identified parameters representing Volterra
kernels of the nonlinear element by minimizing a first
cost function over these parameters, and
translating the identified parameters into predistortion con-
trol parameters. 10
- 14.** The method according to claim **13**, further comprising:
controlling a predistorter using the predistortion control
parameters.
- 15.** The method according to claim **13**, wherein:
the translating act comprises calculating transformed 15
parameters representing Volterra kernels which are
upsampled Volterra kernels of the nonlinear element.

* * * * *