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Chui

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(54) **SYSTEM AND METHOD FOR PERFORMING WAVELET AND INVERSE WAVELET TRANSFORMATIONS OF DIGITAL DATA USING SEMI-ORTHOGONAL WAVELETS**

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Related U.S. Application Data

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(51) **Int. Cl.**⁷ **G06K 9/36**

(52) **U.S. Cl.** **382/248; 382/233; 375/240.19**

(58) **Field of Search** **382/232, 233, 382/234, 248, 249; 375/240.18, 240.19**

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Primary Examiner—Jon Chang

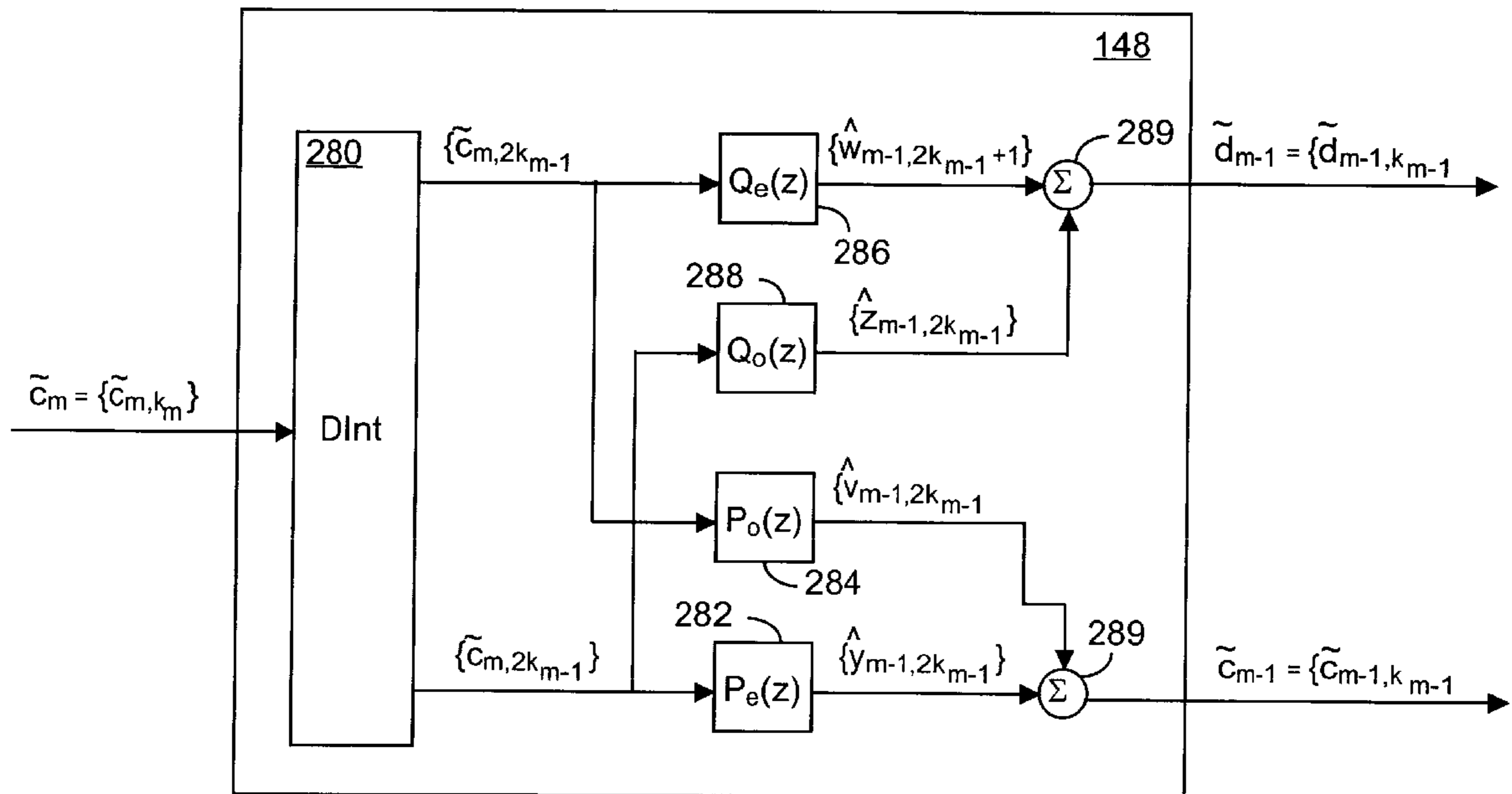
Assistant Examiner—Jingge Wu

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(57) **ABSTRACT**

A wavelet transform system and an inverse wavelet transform system are disclosed that respectively implement a wavelet transform and an inverse wavelet transform. Semi-orthogonal standard wavelets are used as the basic wavelets in the wavelet transform and the inverse wavelet transform. As a result, two finite sequences of decomposition coefficients are used for decomposition in the wavelet transform. Furthermore, two finite sequences of reconstruction coefficients that are derived from the two finite sequences of decomposition coefficients are used for reconstruction in the inverse wavelet transform. The finite sequences of decomposition and reconstruction coefficients are not infinite sequences of coefficients that have been truncated. Furthermore, in one embodiment, downsampling is not used in the wavelet transform and upsampling is not used in the inverse wavelet transform.

12 Claims, 19 Drawing Sheets



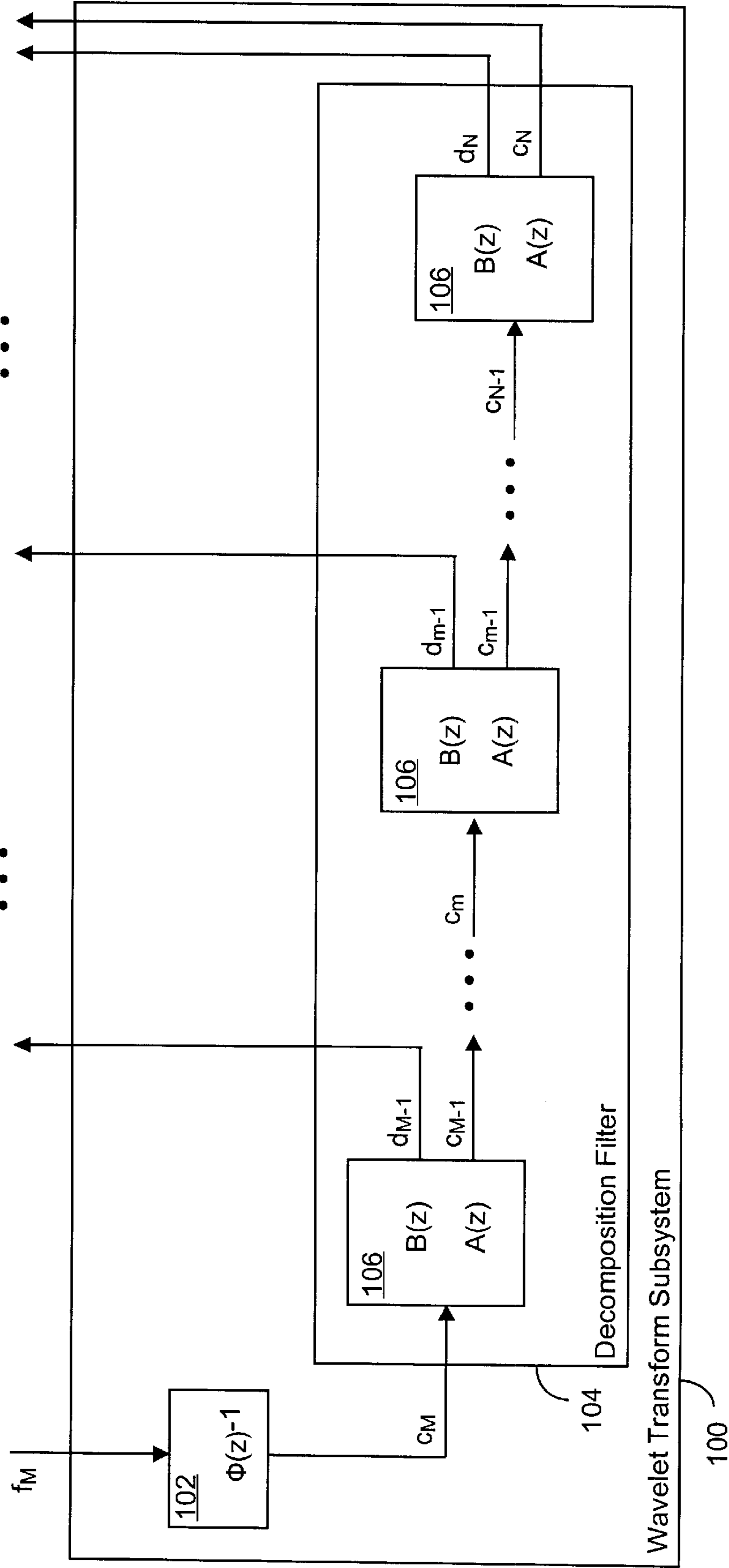


Fig. 1

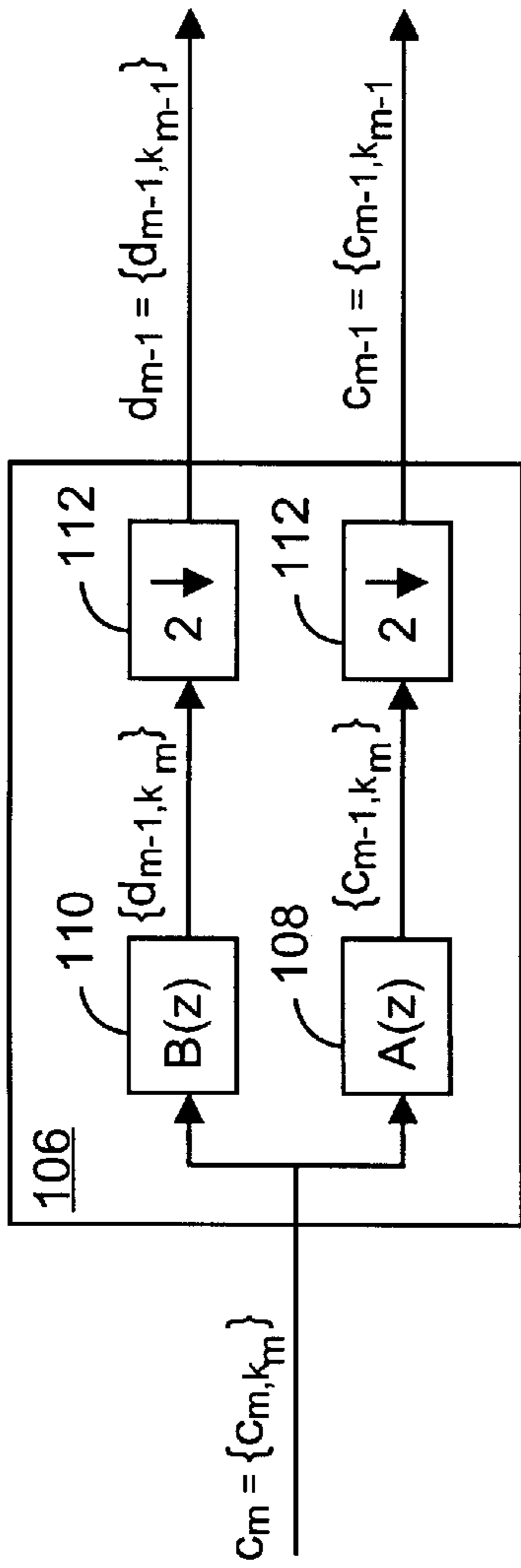


Fig. 2

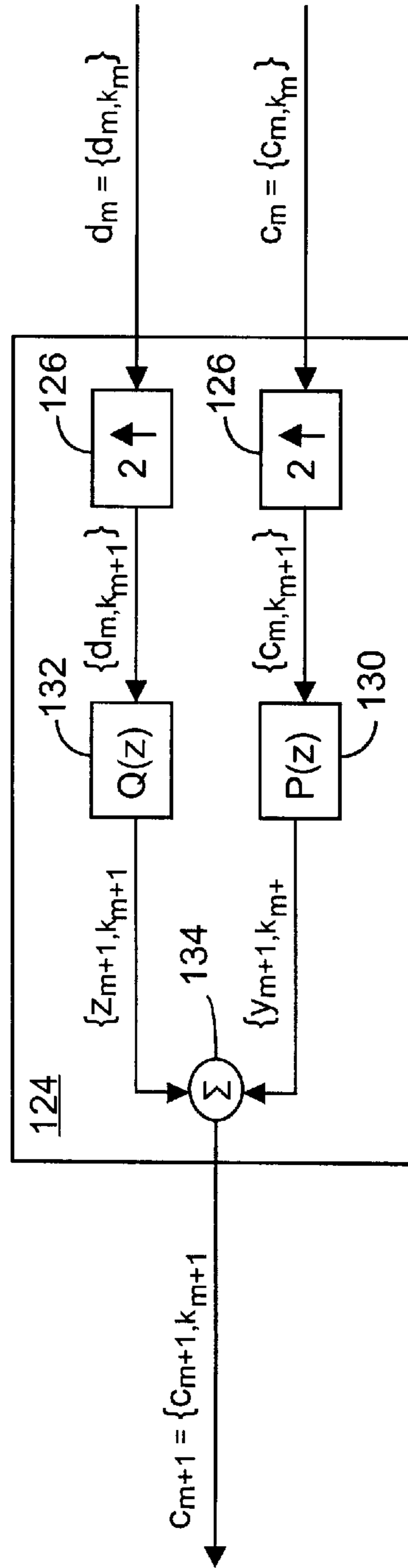


Fig. 4

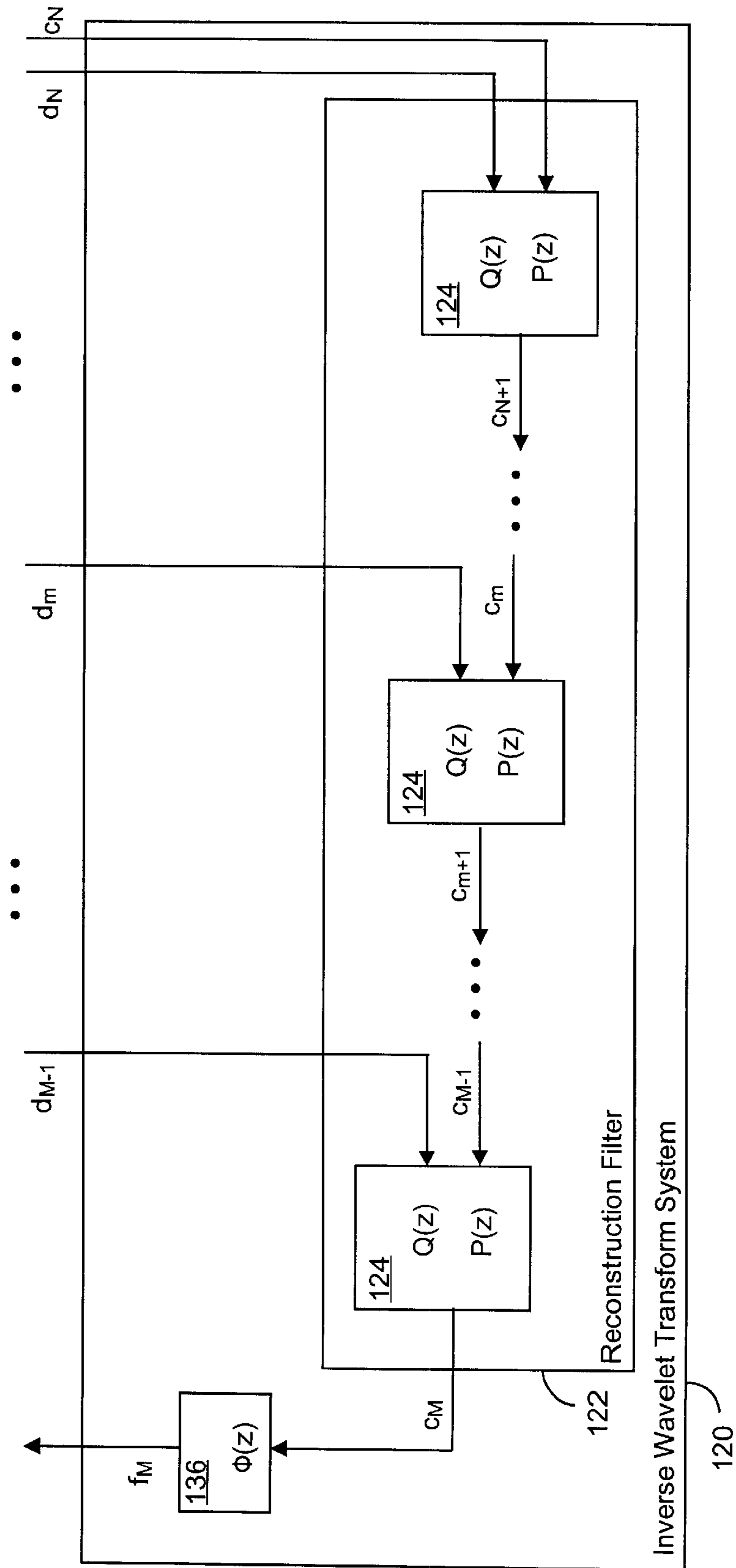


Fig. 3

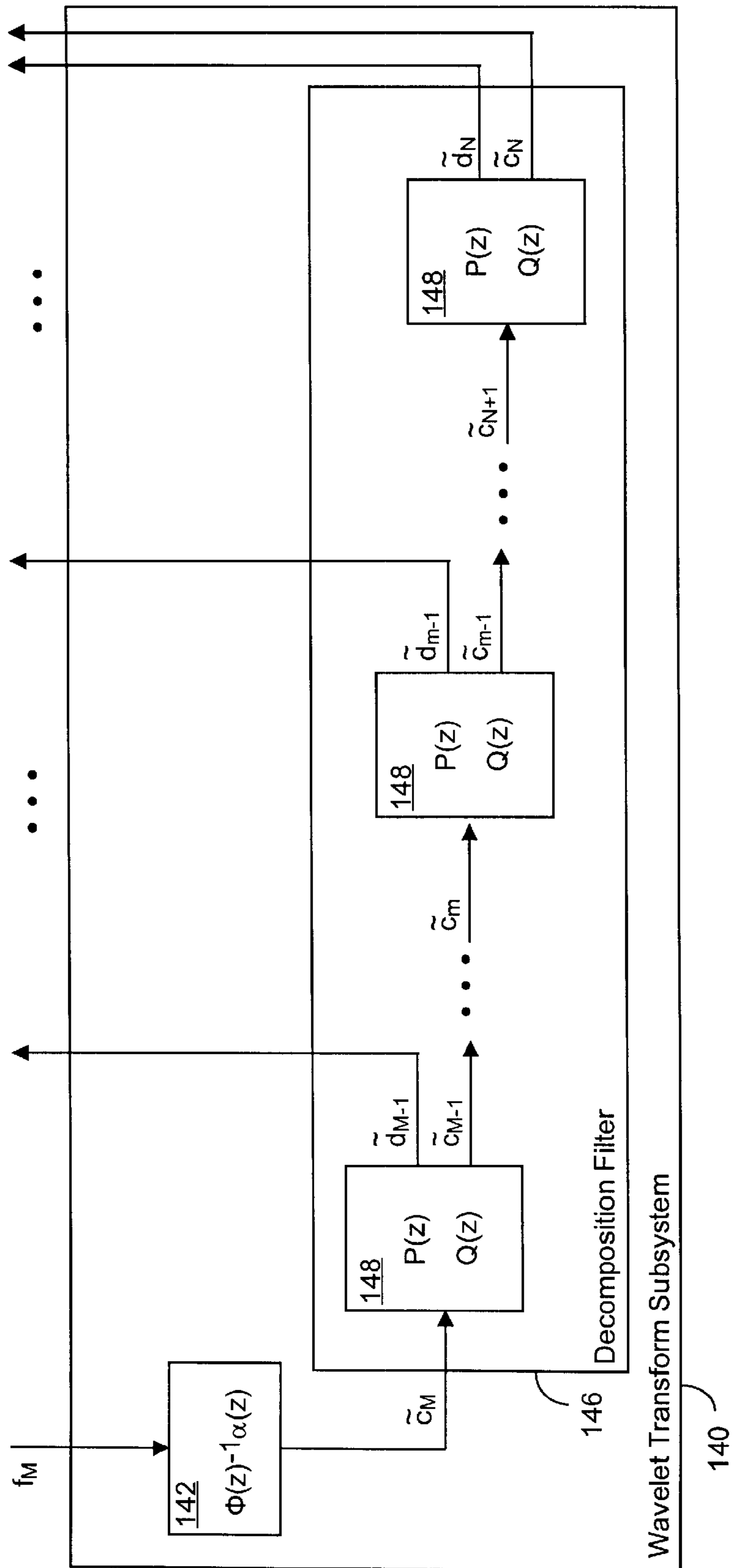


Fig. 5

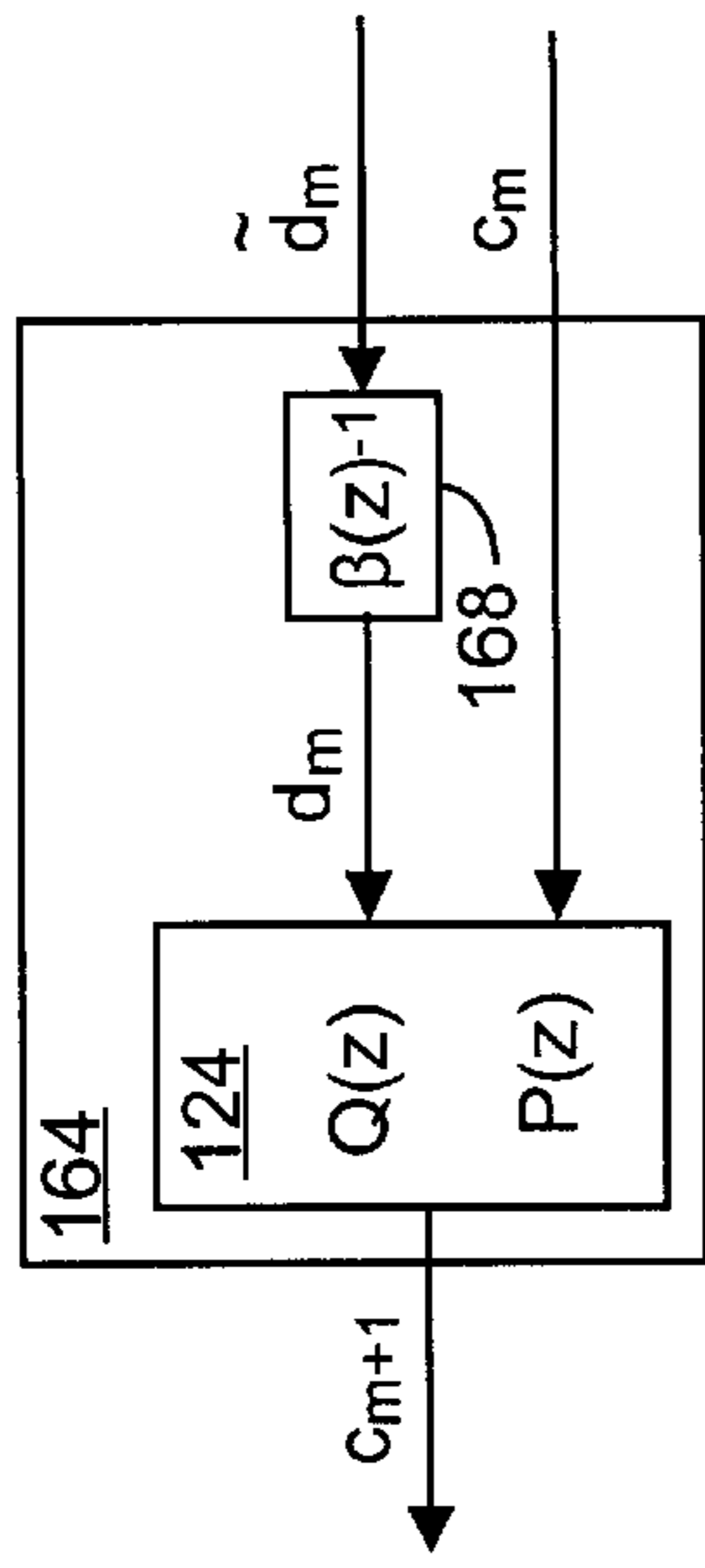


Fig. 10

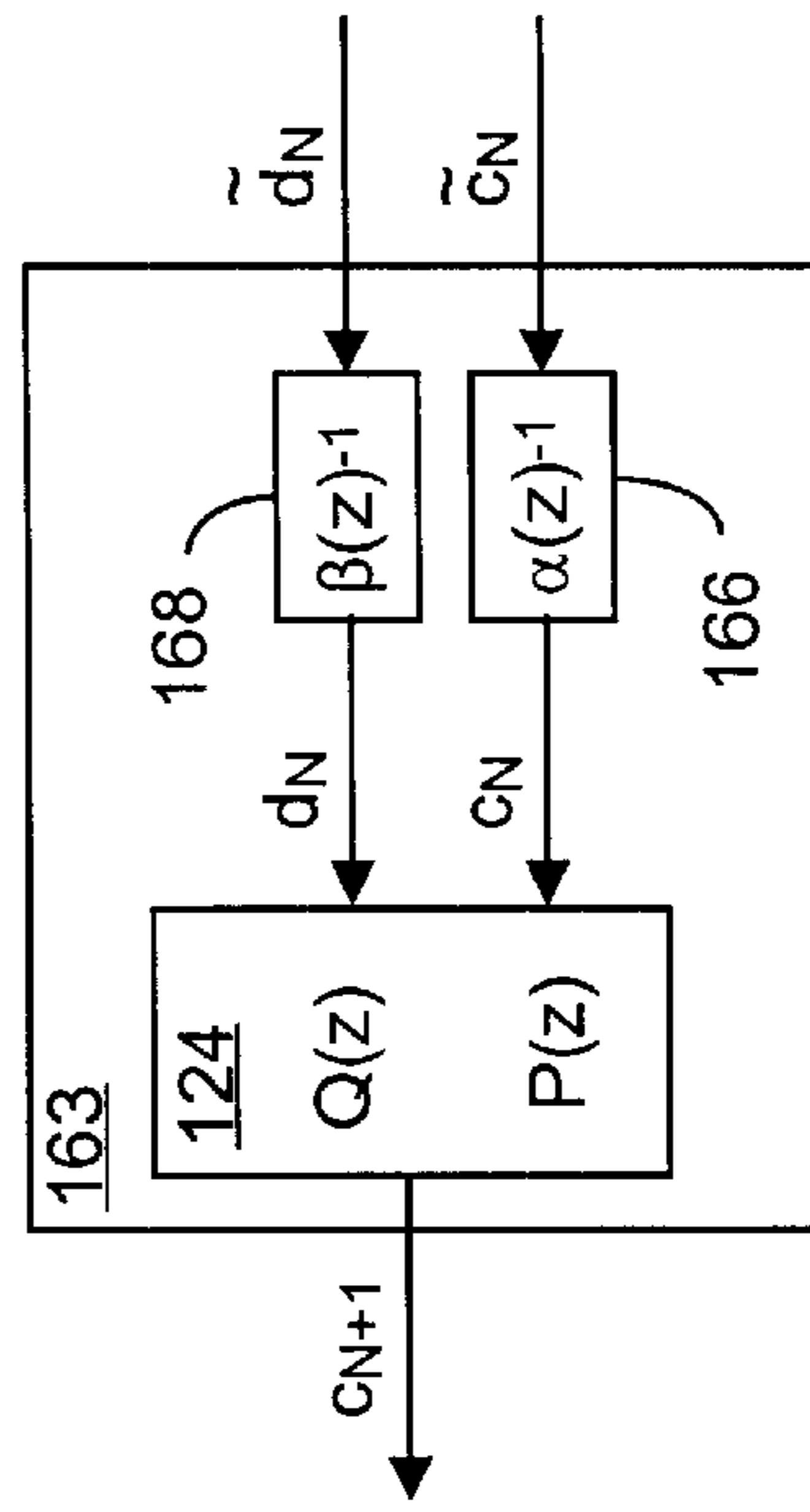


Fig. 9

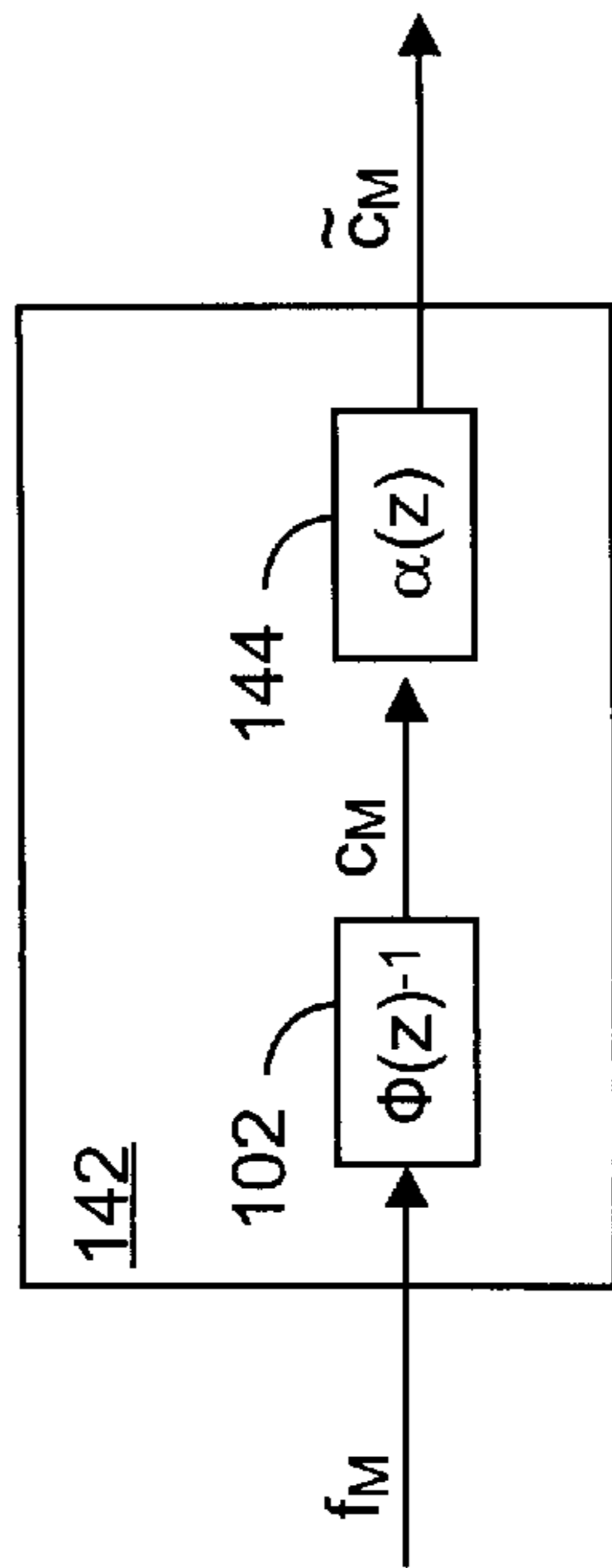


Fig. 6

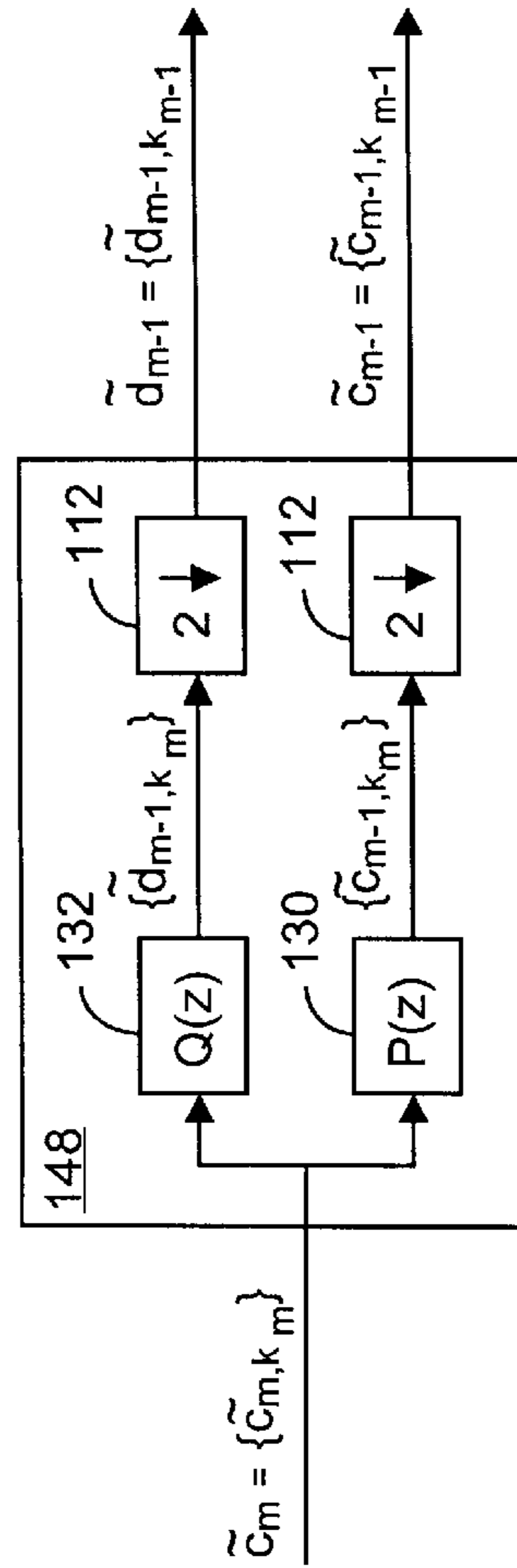


Fig. 7

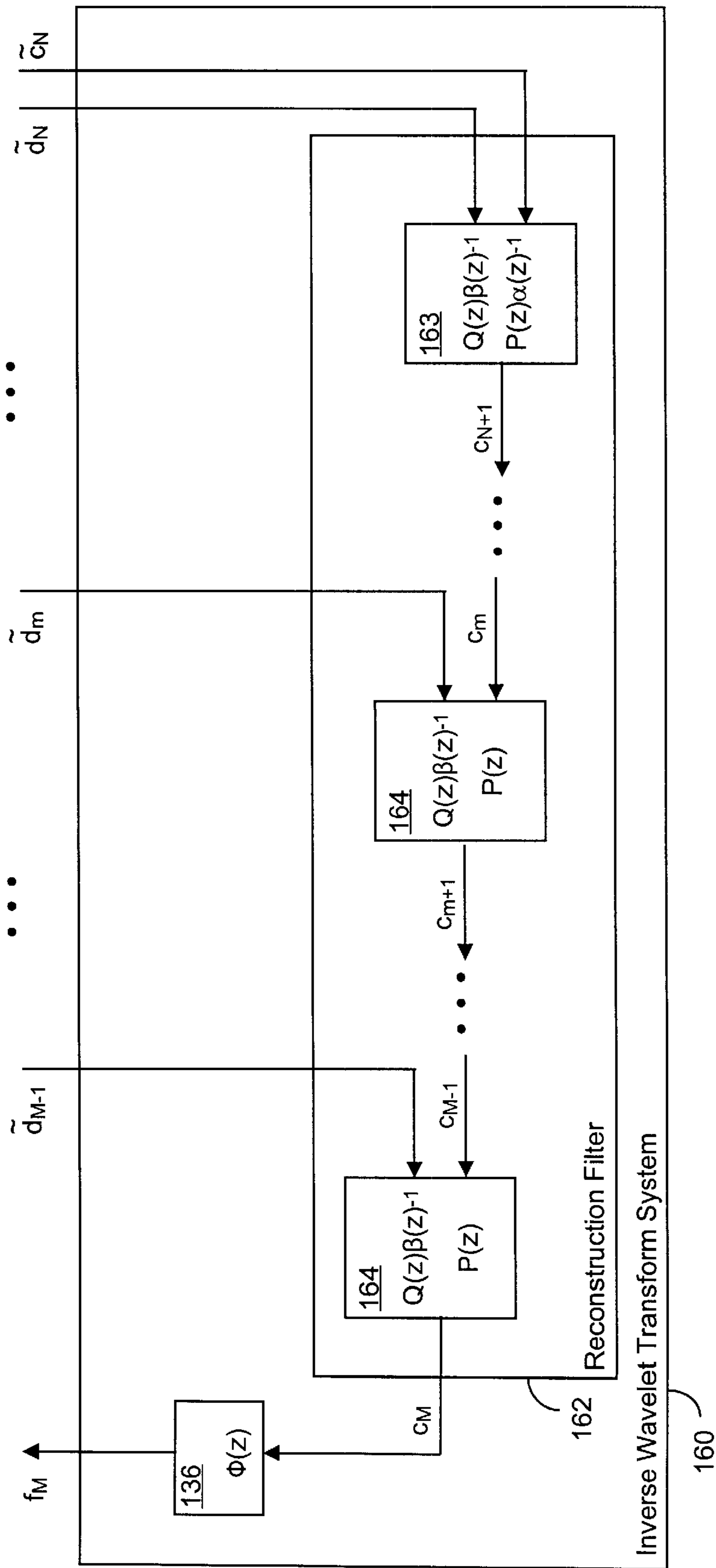


Fig. 8

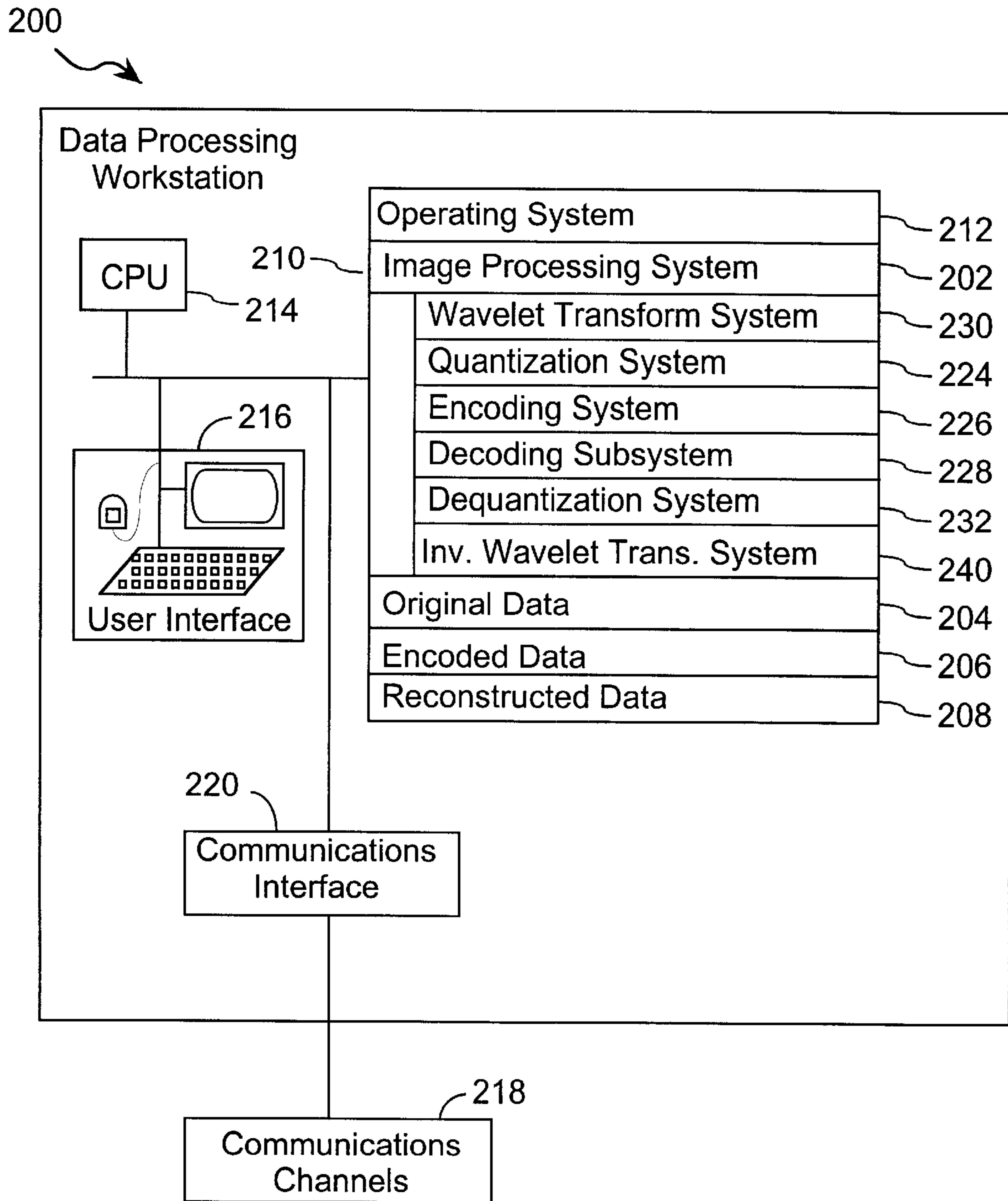


Fig. 11

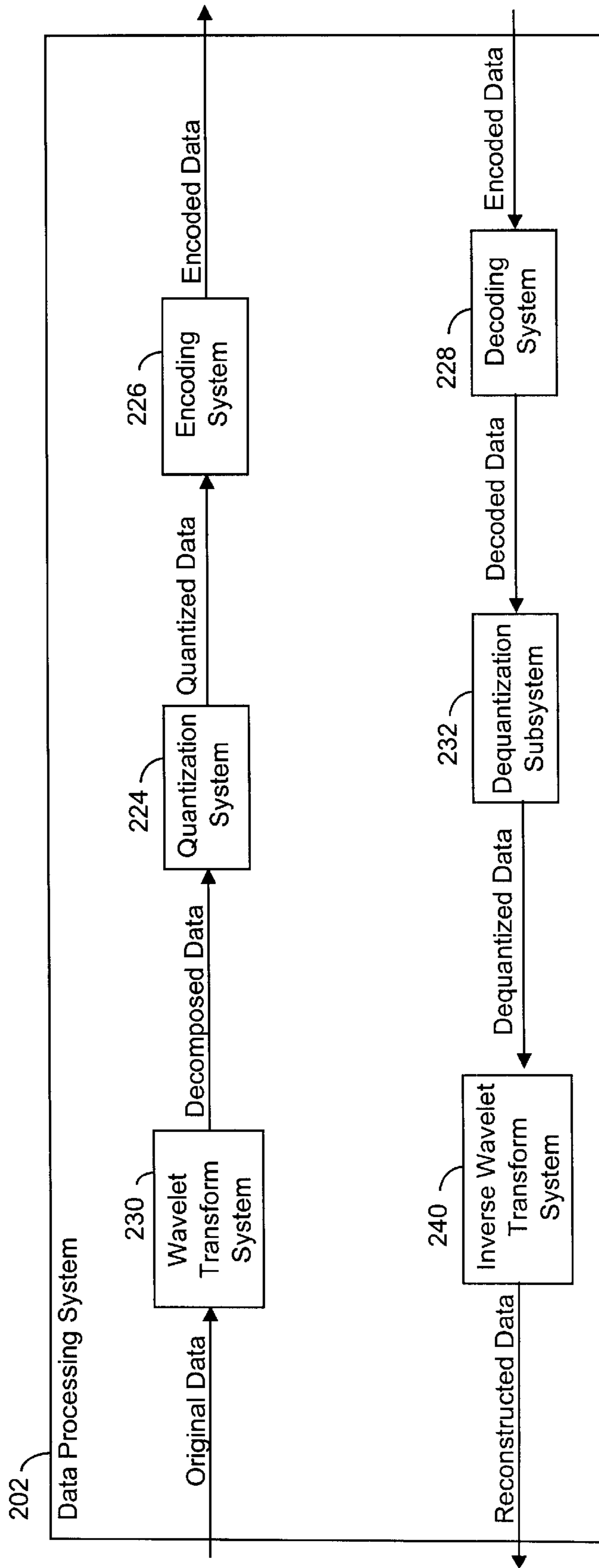


Fig. 12

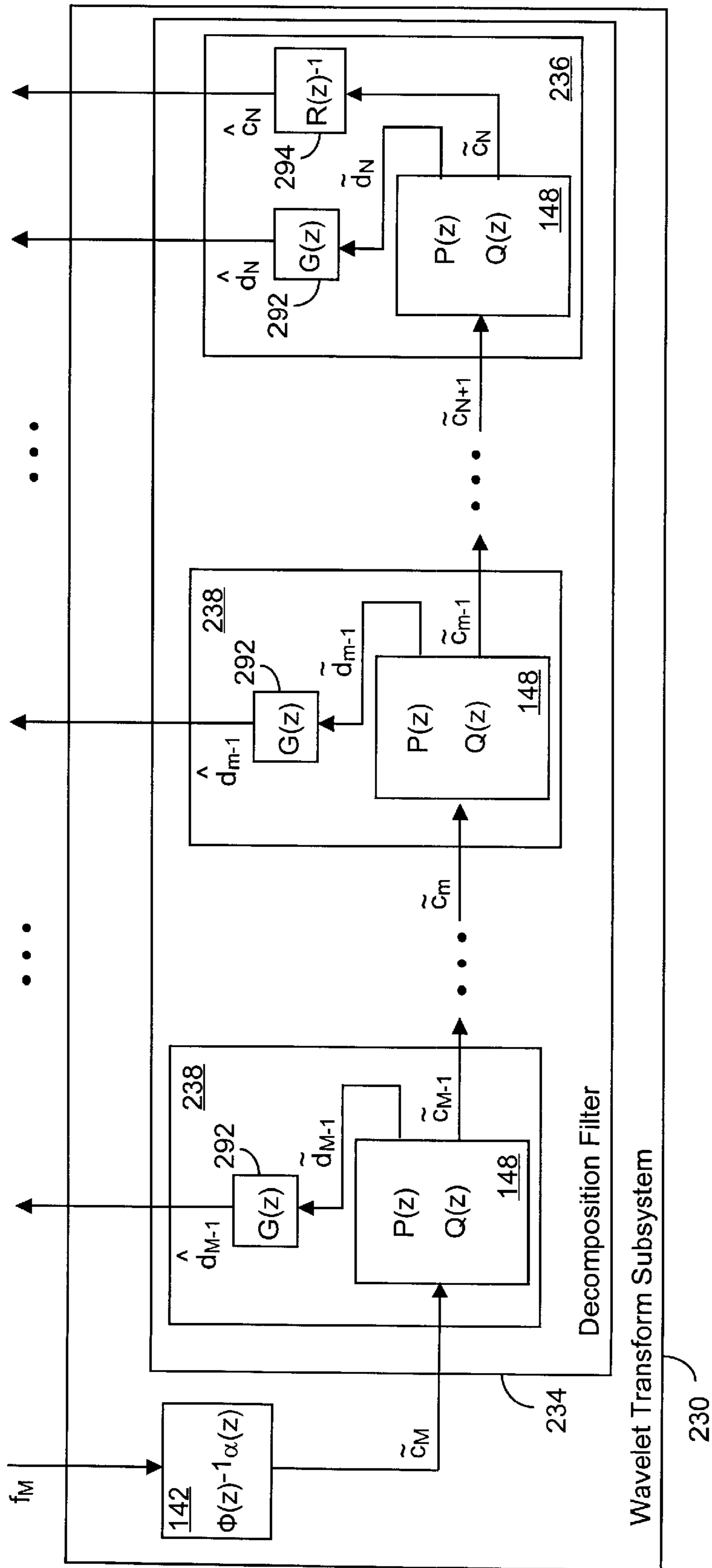


Fig. 13

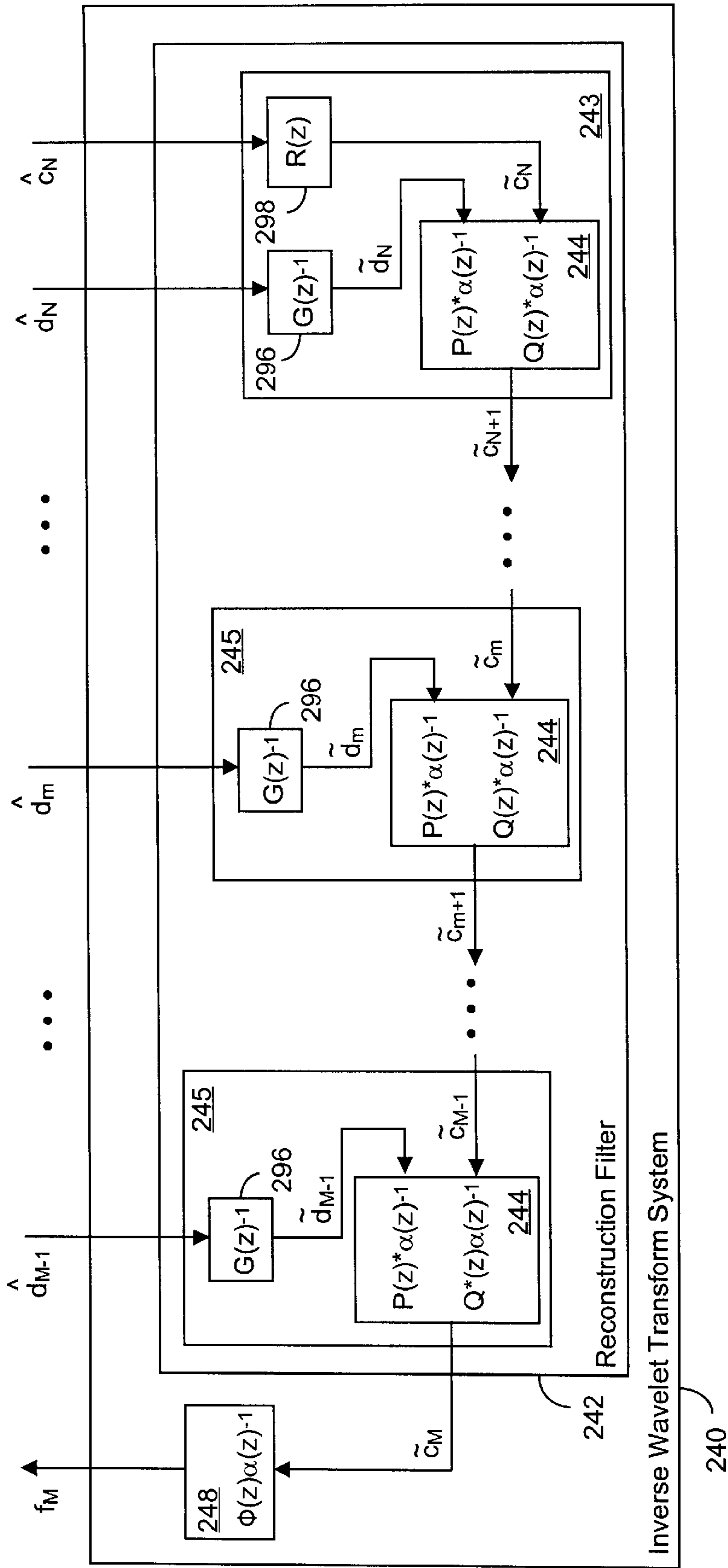


Fig. 14

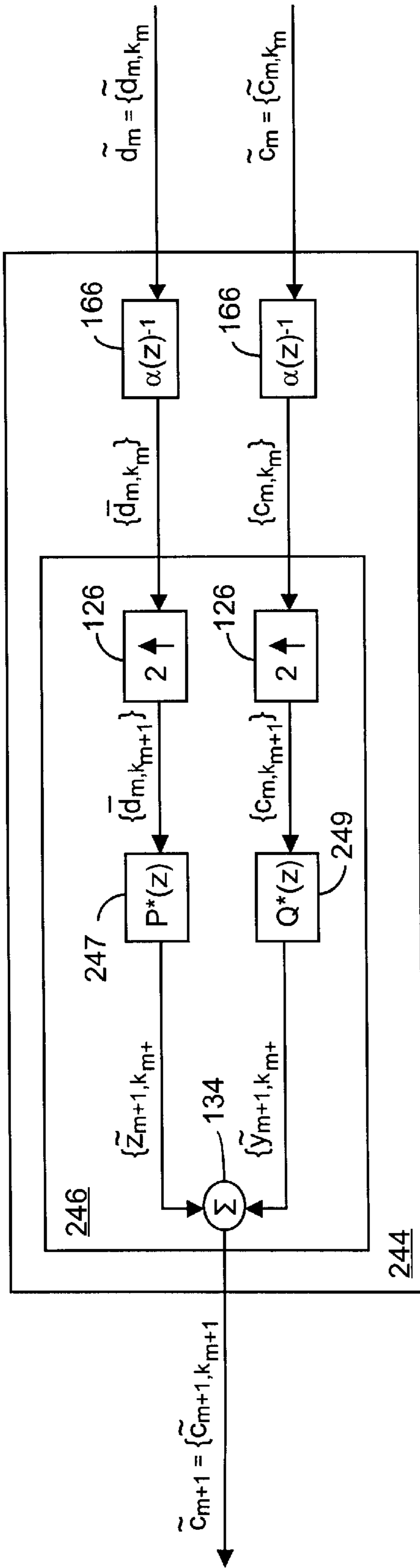


Fig. 15

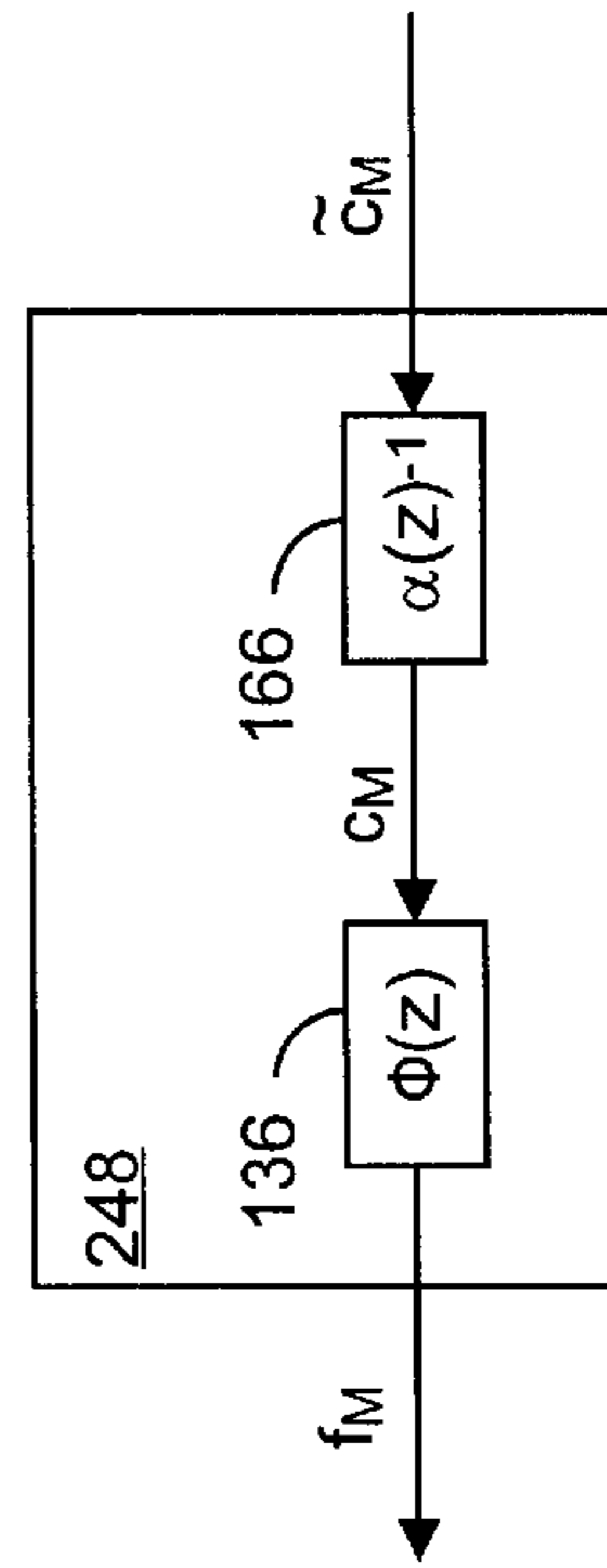


Fig. 16

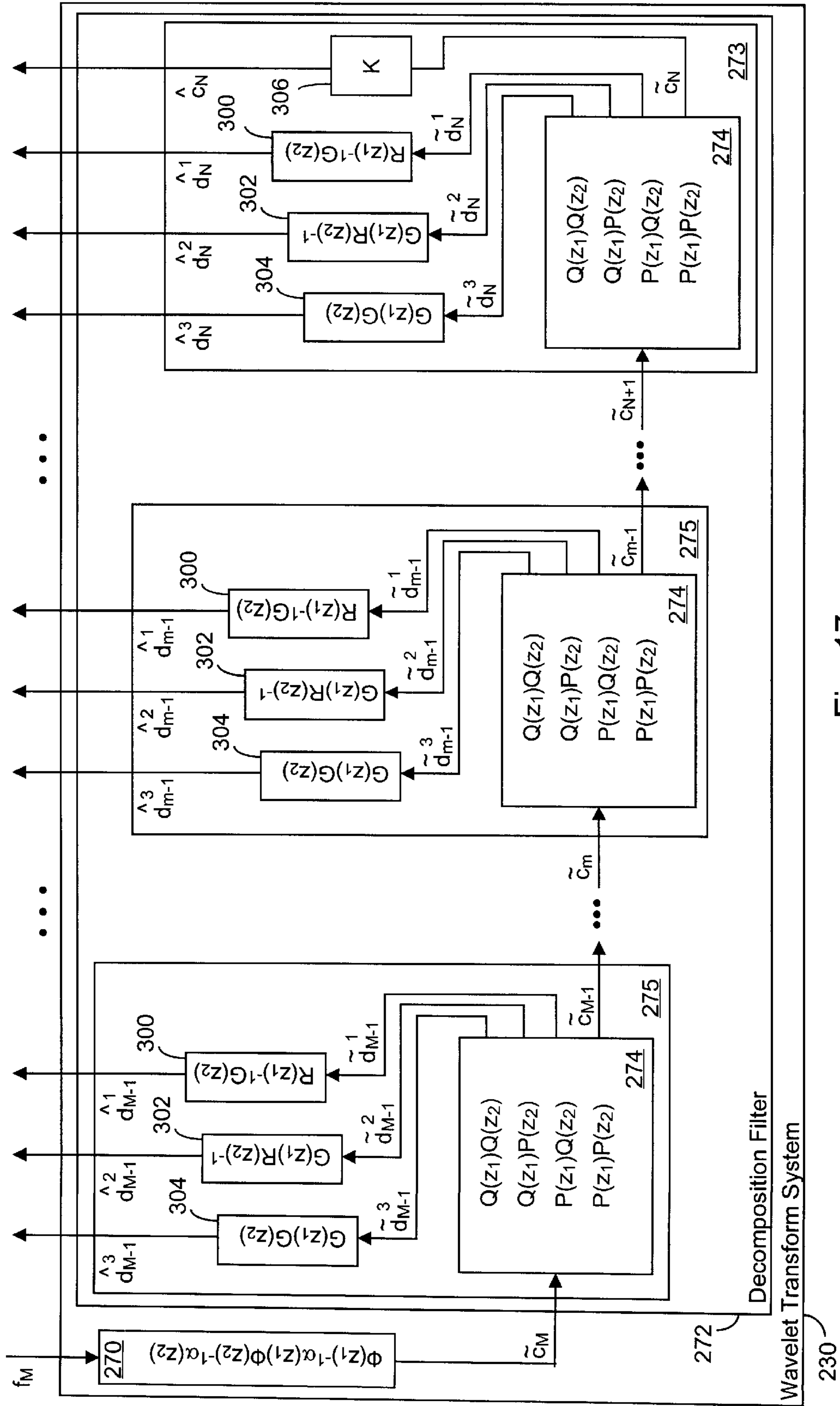


Fig. 17

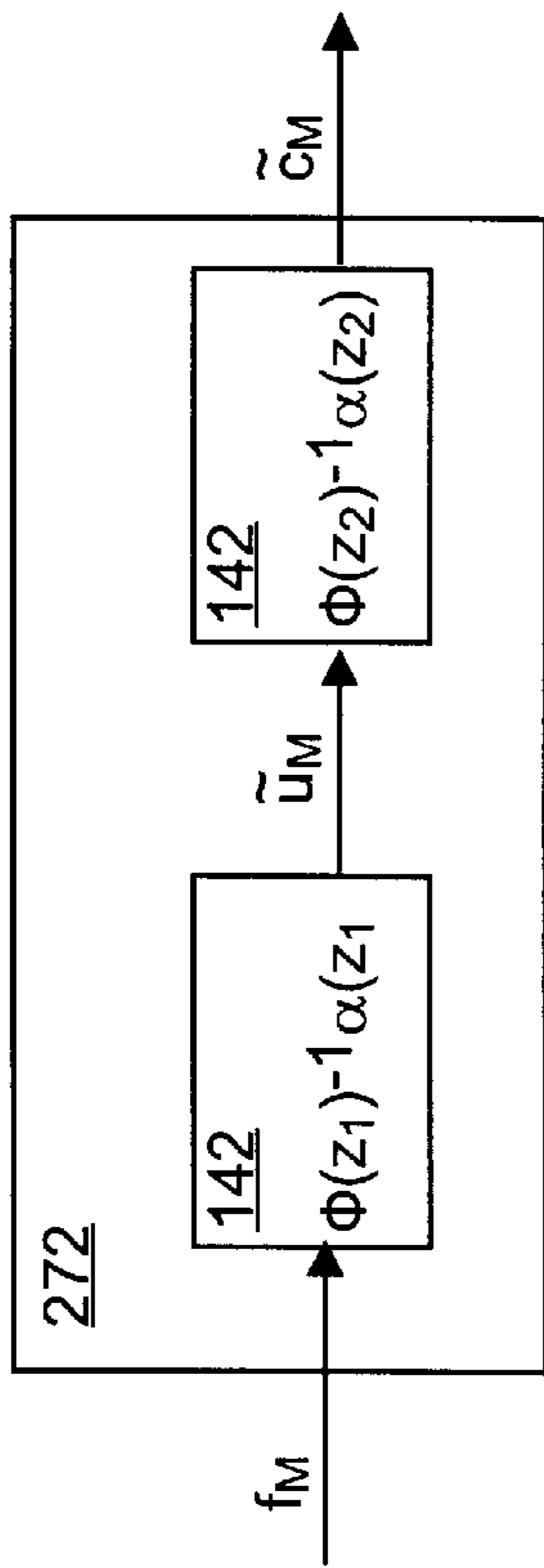


Fig. 18

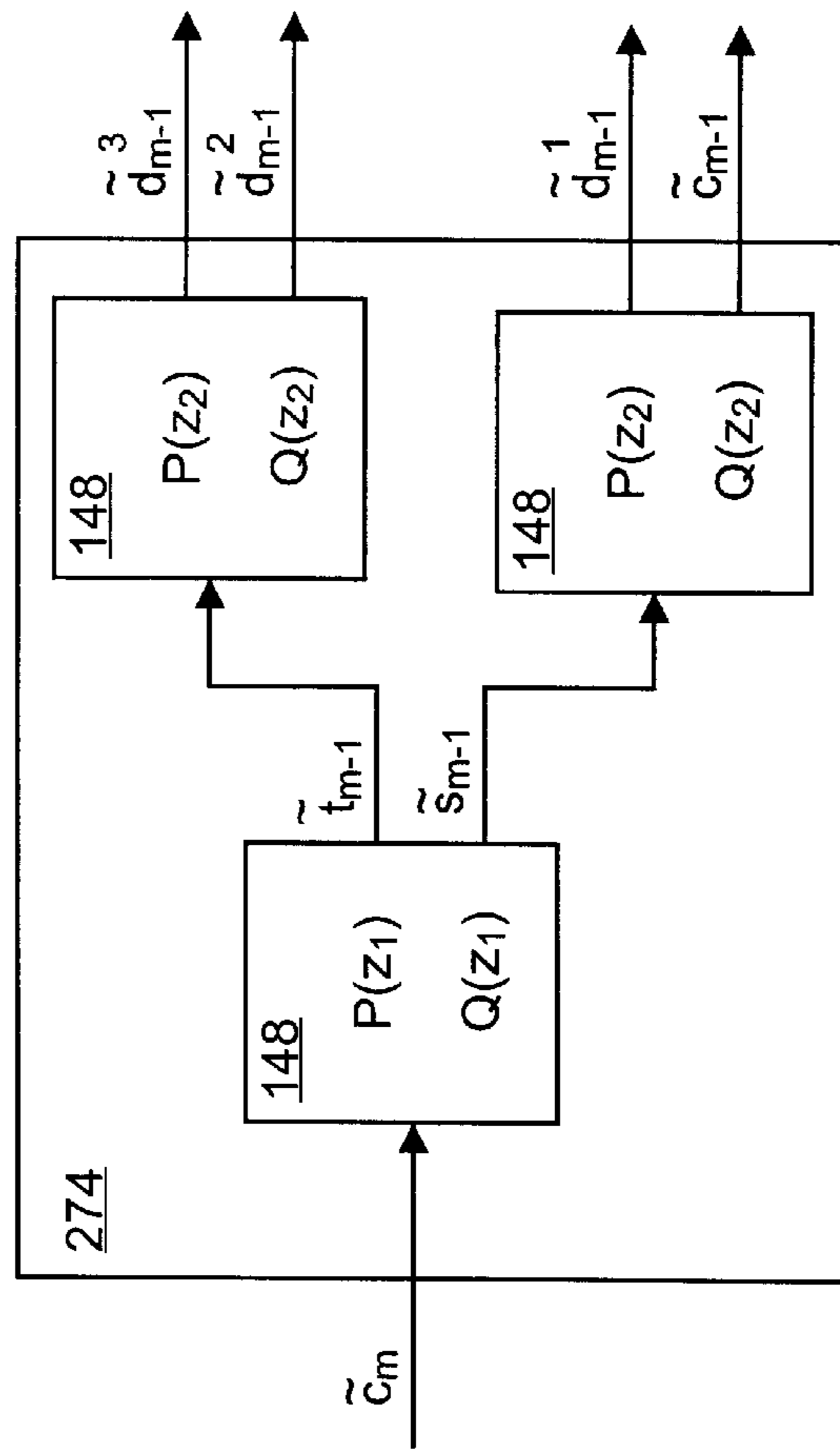


Fig. 19

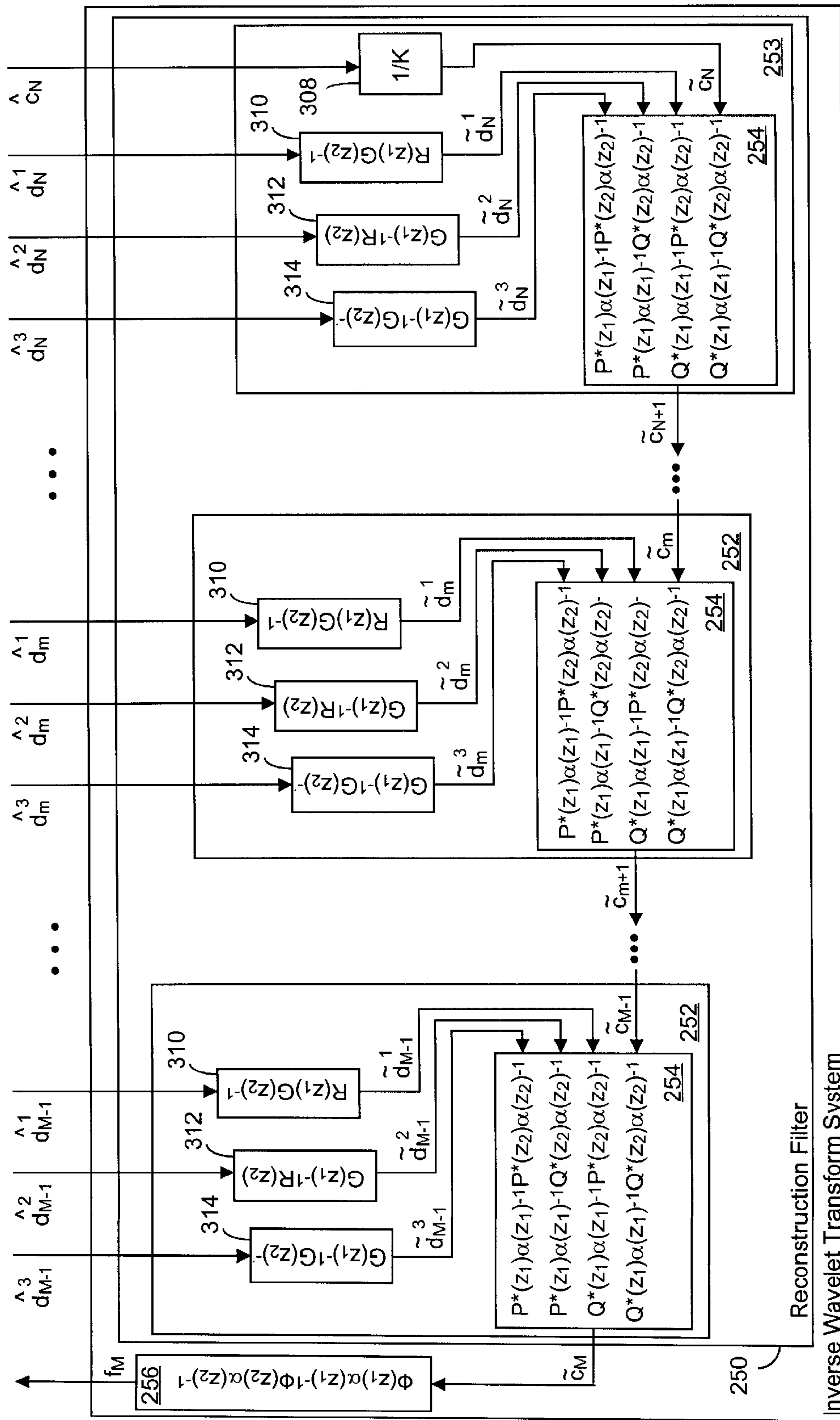


Fig. 20

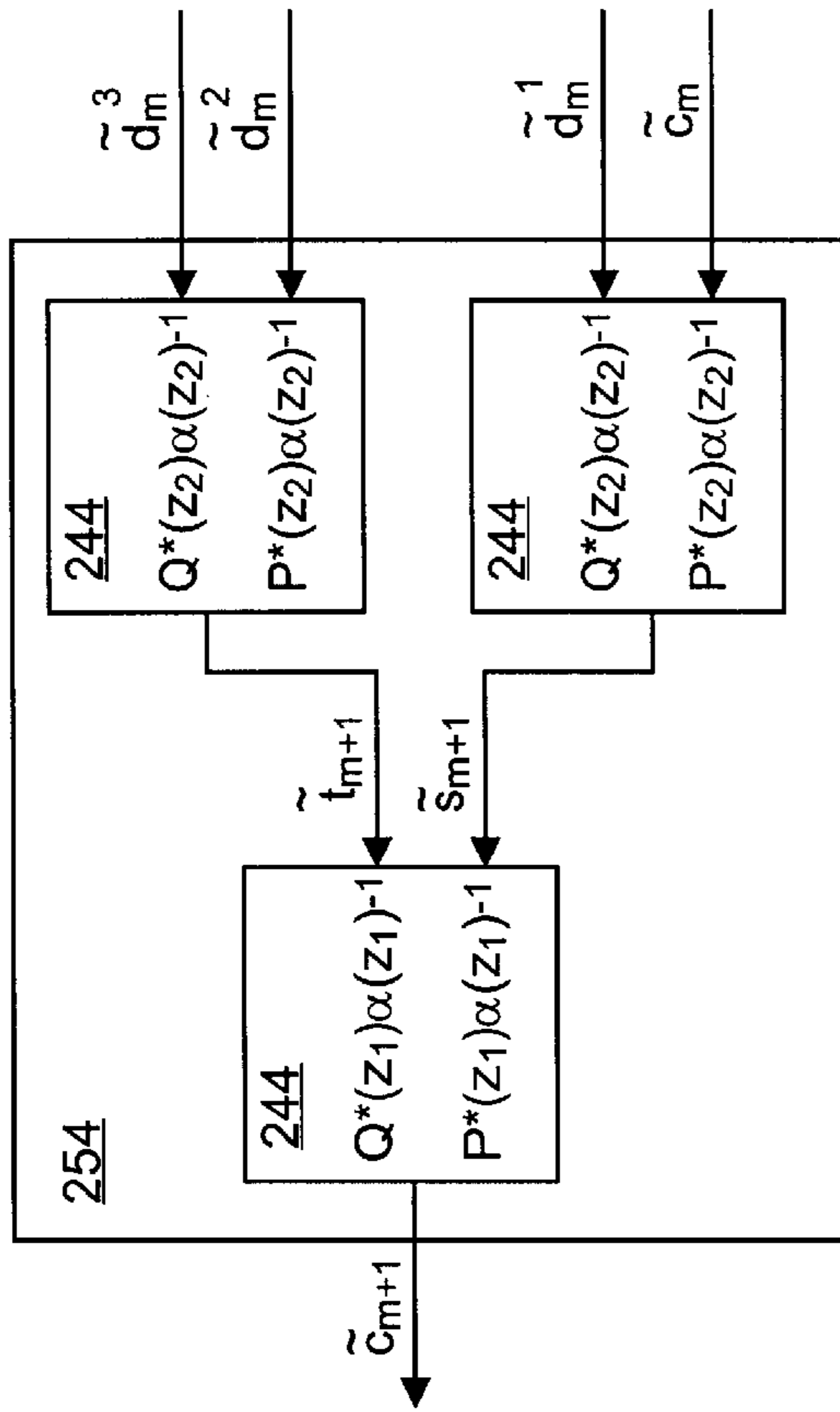


Fig. 21

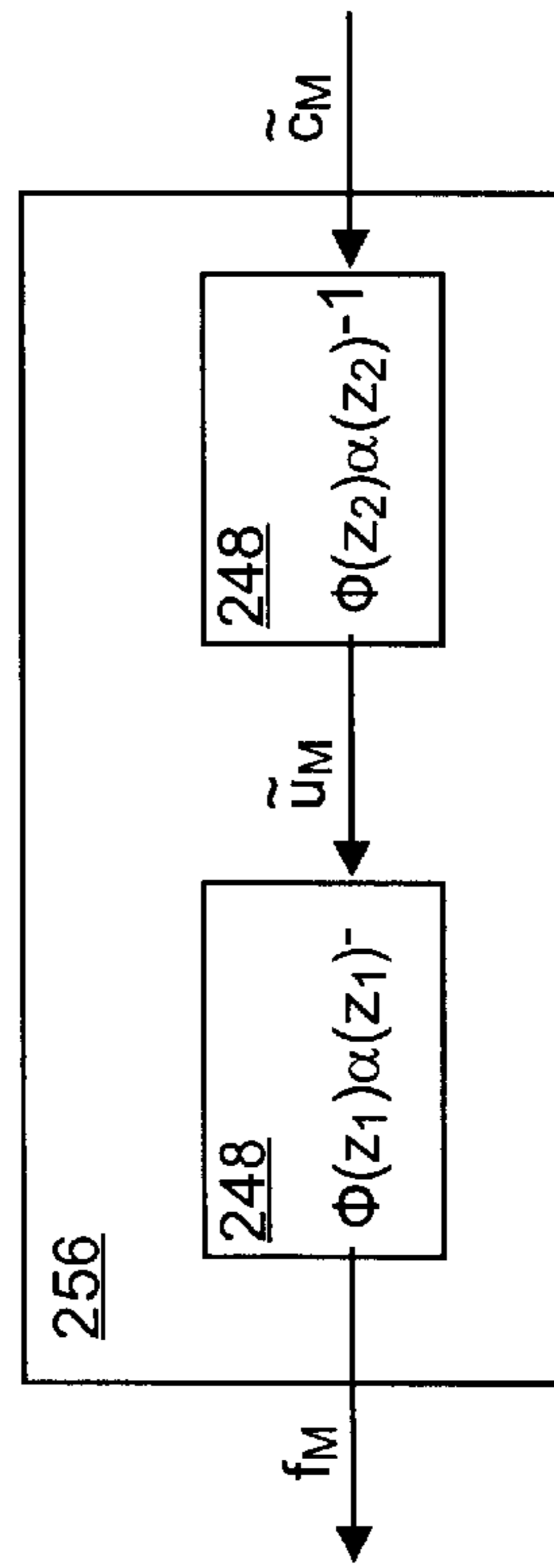


Fig. 22

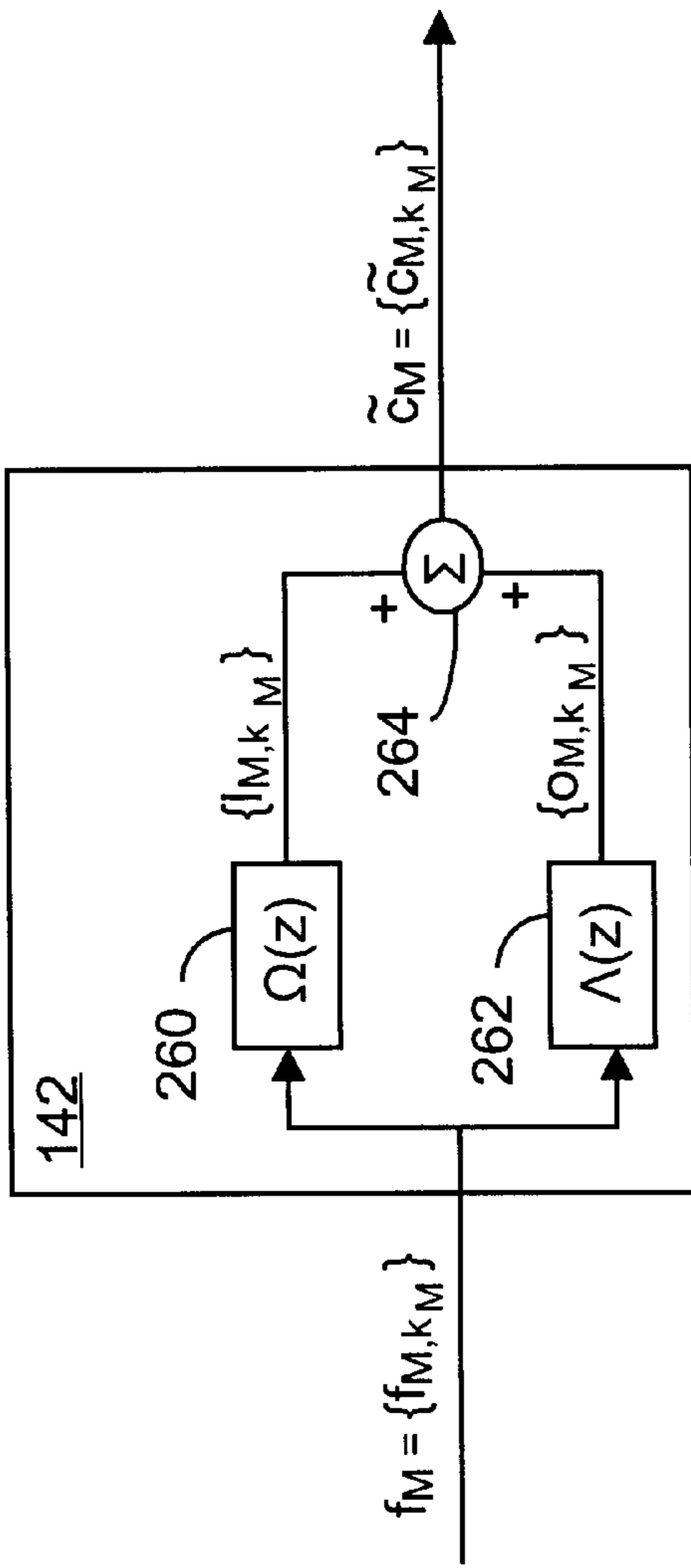


Fig. 23

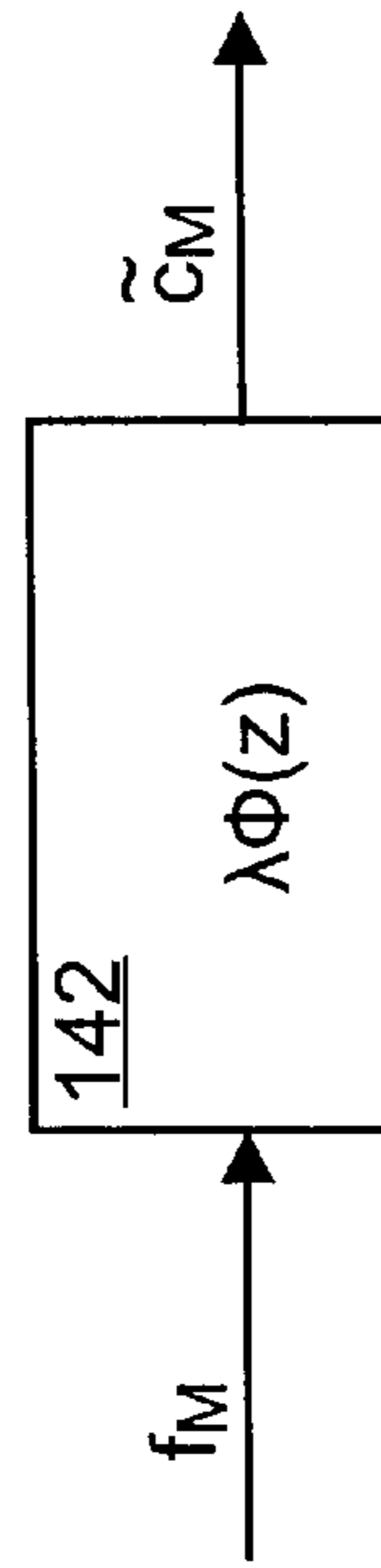


Fig. 24

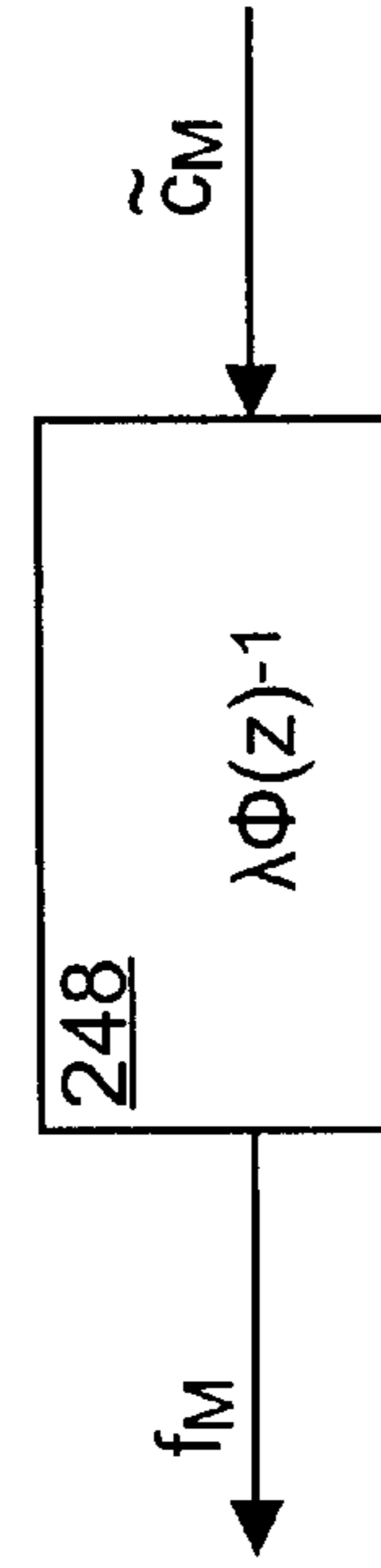


Fig. 25

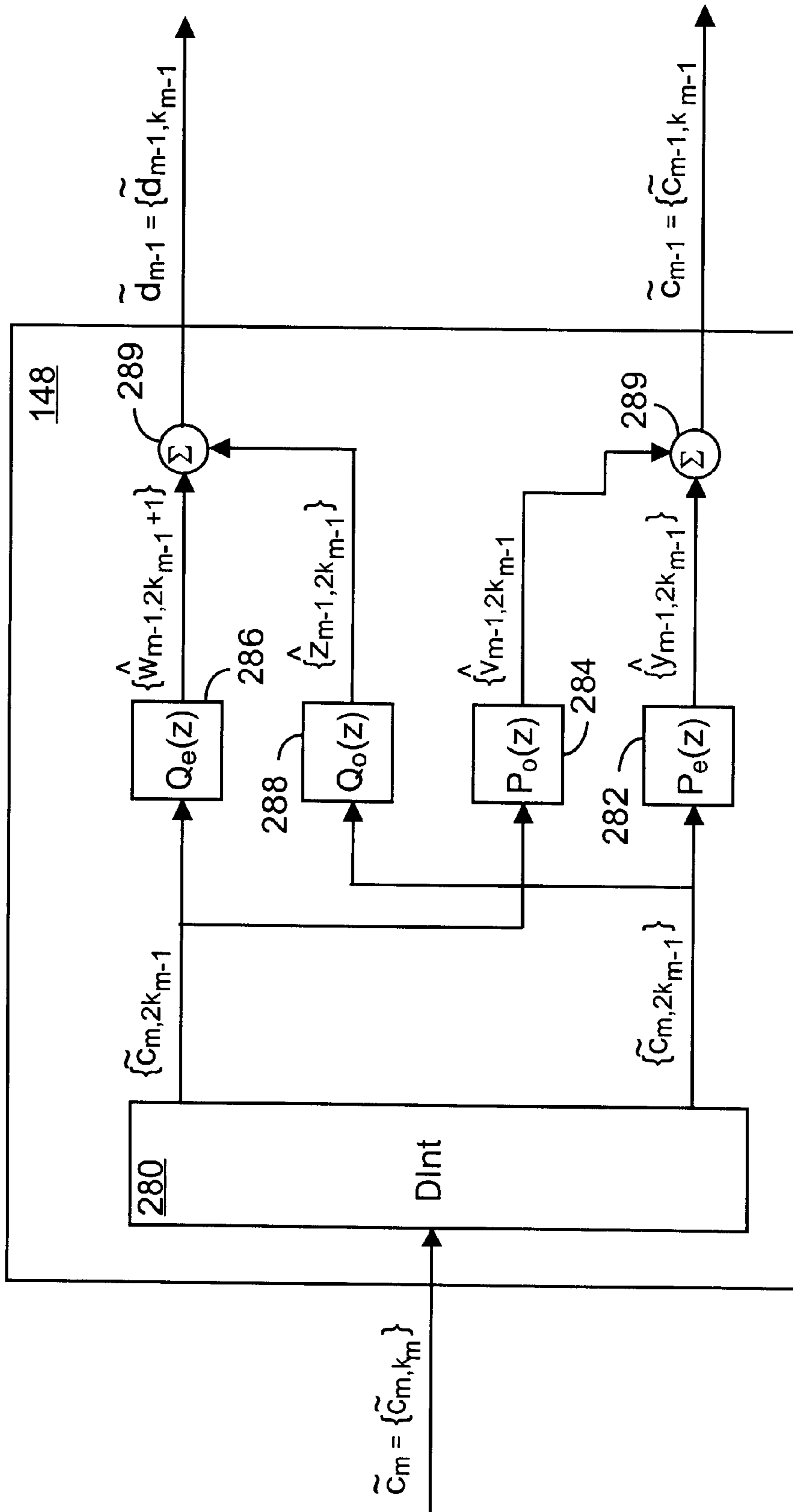


Fig. 26

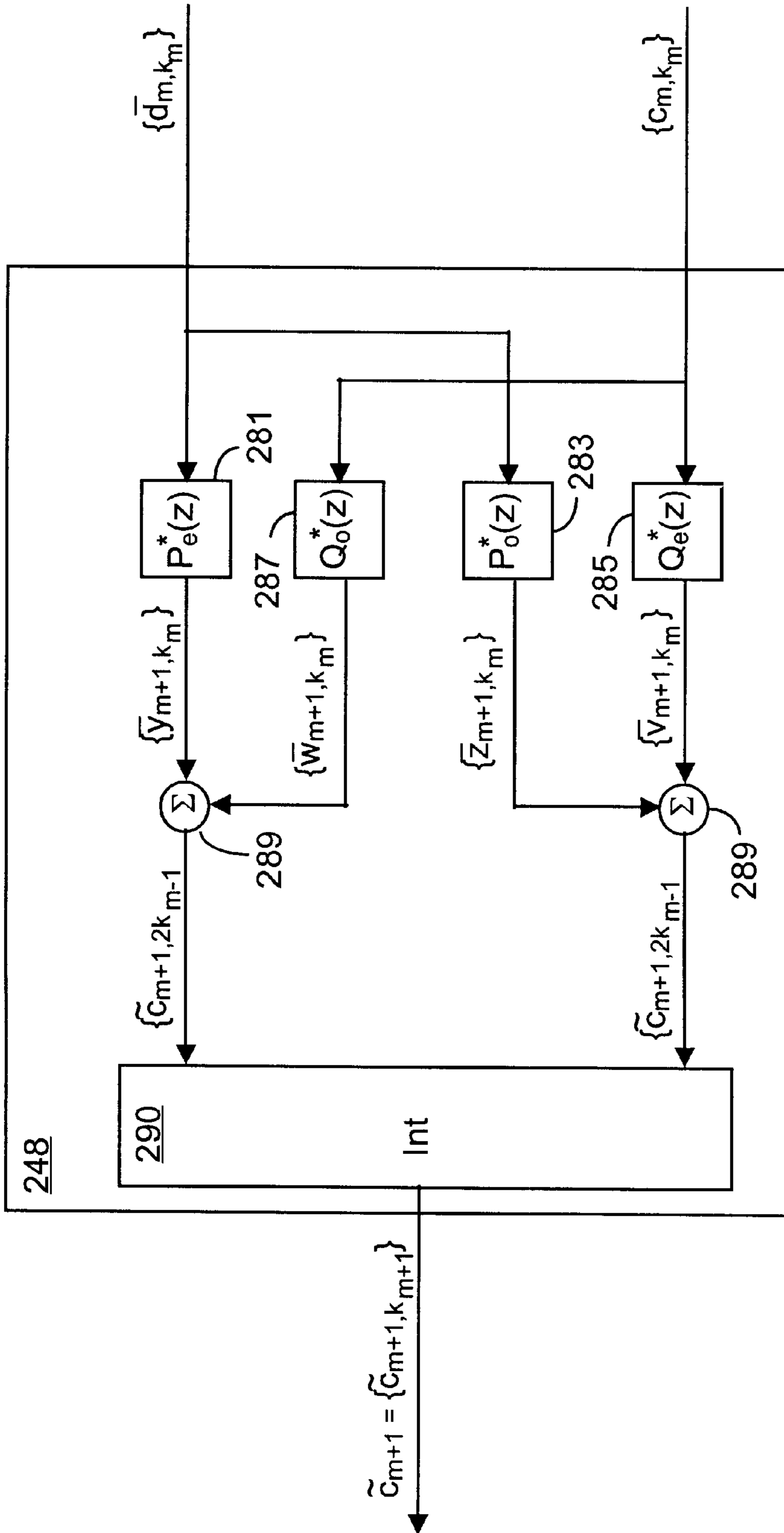


Fig. 27

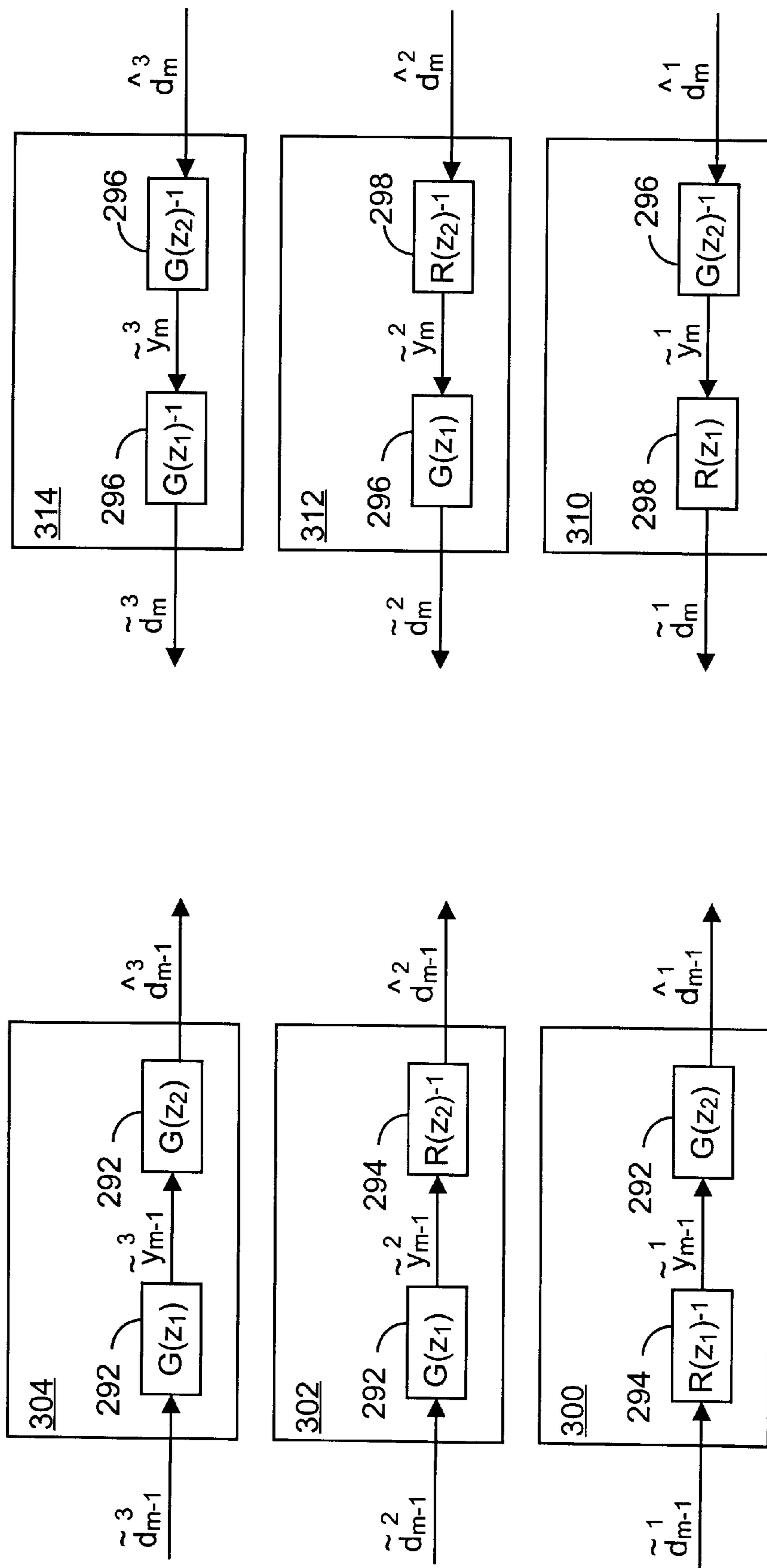


Fig. 28

Fig. 29

SYSTEM AND METHOD FOR PERFORMING WAVELET AND INVERSE WAVELET TRANSFORMATIONS OF DIGITAL DATA USING SEMI-ORTHOGONAL WAVELETS

This is a continuation-in-part of U.S. patent application Ser. No. 08/921,141, filed Aug. 29, 1997.

The present invention relates generally to systems and methods for processing digital data. In particular, it pertains to a system and method for performing wavelet and inverse wavelet transformations of digital data using semi-orthogonal wavelets.

BACKGROUND OF THE INVENTION

The use of IWTs (integral wavelet transforms) and inverse IWTs is well established in MRA (multi-resolution analysis) processing of 1-D (one dimensional) digital data, such as audio signals, and/or 2-D (two dimensional) digital data, such as image data. A special feature of IWTs and inverse IWTs is that they provide narrow windowing of short duration high frequency data while also providing wide windowing of long duration low frequency data. This is generally described in Chui, C. K., "An Introduction to Wavelets", Academic Press, Boston, Mass., 1992, which is hereby incorporated by reference. The following discussion provides examples of how IWTs and inverse IWTs have been implemented in the past.

Wavelet Transform System Using Dual Wavelets $\{\psi_{m,k_m}(x)\}$ as Basic Wavelets

FIG. 1 shows a 1-D wavelet transform system **100**, which is an improved version of the wavelet transform system shown in "An Introduction to Wavelets." This system **100** incorporates some aspects of the present invention, but first we will first explain the conventional aspects of this system. The wavelet transform system **100** implements a 1-D IWT that, for each resolution level m at which a decomposition is made, uses dual wavelets $\{\psi_{m,k_m}(x)\}$ to corresponding standard wavelets $\{\psi_{m,k_m}(x)\}$ as the basic wavelets in the 1-D IWT and uses dual scaling functions $\{\phi_{m,k_m}(x)\}$ to corresponding standard scaling functions $\{\phi_{m,k_m}(x)\}$ as the basic scaling functions in the 1-D IWT. Each standard wavelet $\psi_{m,k_m}(x)$ and standard scaling function $\phi_{m,k_m}(x)$ is given by:

$$\begin{aligned}\psi_{m,k_m}(x) &= 2^{m/2} \psi(2^m x - k_m) \\ \phi_{m,k_m}(x) &= 2^{m/2} \phi(2^m x - k_m)\end{aligned}\quad (1)$$

where k_m is a corresponding index for the resolution level m and the normalization factor $2^{m/2}$ will be suppressed hereafter for computational efficiency.

In performing the 1-D IWT, the wavelet transform system **100** decomposes a 1-D set of original data samples f_M at an original resolution level $m=M$ into a 1-D set of standard scaling function coefficients c_N in an L (low) frequency band at the resolution level $m=N$ and 1-D sets of standard wavelet coefficients d_{M-1} to d_N in H (high) frequency bands at respectively the resolution levels $m=M-1$ to N .

The set of original data samples $f_M = \{f_{M,n}\} = f_M(2^{-M}n)$ is given by a 1-D function $f_M(x)$, where $x=2^{-M}n$. The 1-D function $f_M(x)$ approximates another 1-D function $f(x)$ at the original resolution level M . The set of original data samples f_M extends in one spatial dimension, namely the x direction, and is first pre-processed by a pre-decomposition filter **102** of the 1-D wavelet transform system **100**. The pre-decomposition filter has a transfer function $\phi(z)^{-1}$ for mapping (i.e., converts) the 1-D set of original data samples f_M

into a 1-D set of standard scaling function coefficients c_M in an L frequency band at the original resolution level M .

The transfer function $\phi(z)^{-1}$ is obtained from the following relationship at the resolution level m between a 1-D function $f_m(x)$, the standard scaling functions $\{\phi_{m,k_m}(x)\}$ and the set of standard scaling function coefficients $c_m = \{c_{m,k_m}\}$:

$$f_m(x) = \sum_{k_m} c_{m,k_m} \phi_{m,k_m}(x) \quad (2)$$

where the 1-D function $f_m(x)$ approximates the function $f(x)$ at the resolution level m . The transfer function $\phi(z)^{-1}$ is the inverse of a transfer function $\phi(z)$. The transfer function $\phi(z)$ is a polynomial that has the sequence of mapping coefficients $\{\phi_n\} = \{\phi_{0,0}(n)\}$ as its coefficients while the transfer function $\phi(z)^{-1}$ is a rational function that has a corresponding sequence of mapping coefficients $\{\Theta_n\}$ as its poles. Thus, the pre-decomposition filter **102** comprises a 1-D IIR (infinite impulse response) filter that applies the sequence of mapping coefficients $\{\Theta_n\}$ to the set of original data samples $f_M = \{f_{M,n}\}$ to generate the set of standard scaling function coefficients $c_M = \{c_{M,k_M}\}$.

Then, the decomposition filter **104** of the 1-D wavelet transform system **100** decomposes the set of standard scaling function coefficients c_M into the sets of standard scaling function and wavelet coefficients c_N and d_{M-1} to d_N . To do this, in the present invention the decomposition filter **104** has a corresponding decomposition filter stage **106** for each resolution level $m=M$ to $N+1$ at which a decomposition is made. The decomposition filter stage **106** for each resolution level m decomposes a 1-D set of standard scaling function coefficients c_m in an L frequency band at the higher resolution level m into a 1-D set of standard scaling function coefficients c_{m-1} in an L frequency band and a 1-D set of wavelet coefficients d_{m-1} in an H frequency band at the next lower resolution level $m-1$.

This is done by the decomposition filter stage **106** according to the function $f_m(x)$ given in Eq. (2). Here, for each resolution level m , the function $f_m(x)$ further provides the following relationship between the set of standard scaling function coefficients c_m at each resolution level m and the sets of standard scaling function and wavelet coefficients $c_{m-1} = \{c_{m-1,k_{m-1}}\}$ and $d_{m-1} = \{d_{m-1,k_{m-1}}\}$ at the next lower resolution level $m-1$:

$$\sum_{k_m} c_{m,k_m} \phi_{m,k_m}(x) = \sum_{k_{m-1}} d_{m-1,k_{m-1}} \psi_{m-1,k_{m-1}}(x) + \sum_{k_{m-1}} c_{m-1,k_{m-1}} \phi_{m-1,k_{m-1}}(x). \quad (3)$$

In Eq. (3), the set of standard scaling function coefficients $\{c_{m-1,k_{m-1}}\}$ has the indexes $\{k_{m-1}\} = \{(k_m-1)/2\}$ for odd indexes $\{k_m\}$ and the set of standard wavelet coefficients $\{d_{m-1,k_{m-1}}\}$ has the indexes $\{k_{m-1}\} = \{k_m/2\}$ for even indexes $\{k_m\}$.

Furthermore, there exists two 1-D sequence of decomposition coefficients $\{a_n\}$ and $\{b_n\}$ such that each standard scaling function $\phi_{m,k_m}(x)$ at a higher resolution level m is related to and can be decomposed into the standard wavelets and scaling functions $\{\psi_{m,k_m}(x)\}$ and $\{\phi_{m,k_m}(x)\}$ at the next lower resolution level $m-1$. This decomposition relation is given as follows:

$$\phi_{m,k_m}(x) = \sum_{k_{m-1}} a_{k_m-2k_{m-1}} \phi_{m-1,k_{m-1}}(x) + b_{k_m-2k_{m-1}} \psi_{m-1,k_{m-1}}(x). \quad (4)$$

In view of Eqs. (3) and (4), the sets of standard scaling function and wavelet coefficients $\{c_{m-1,k_{m-1}}\}$ and $\{d_{m-1,k_{m-1}}\}$ at the resolution level $m-1$ are obtained according to the decomposition sequences:

$$c_{m-1,k_{m-1}} = \sum_{k_m} a_{k_m-2k_{m-1}} c_{m,k_m} \quad (5)$$

$$d_{m-1,k_{m-1}} = \sum_{k_m} b_{k_m-2k_{m-1}} c_{m,k_m}.$$

It must be noted here that the dual wavelets $\{\psi_{m,k_m}(x)\}$ and the dual scaling functions $\{\phi_{m,k_m}(x)\}$ are used respectively as the basic wavelets and scaling functions in the 1-D IWT since the 1-D IWT is defined by:

$$c_{m,k_m} = \int_{-\infty}^{\infty} f(x) \tilde{\phi}_{m,k_m}(x) dx \quad (6)$$

$$d_{m,k_m} = \int_{-\infty}^{\infty} f(x) \tilde{\psi}_{m,k_m}(x) dx.$$

Thus, the transfer functions $A(z)$ and $B(z)$ of the decomposition filter stage **106** are polynomials that respectively have the sets of decomposition coefficients $\{a_n\}$ and $\{b_n\}$ as their coefficients.

As is well known, when the standard wavelets $\{\psi_{m,k_m}(x)\}$ are s.o. wavelets, the dual wavelets $\{\psi_{m,k_m}(x)\}$ are also s.o. wavelets. In this case, the sequences of decomposition coefficients $\{a_n\}$ and $\{b_n\}$ are infinite. This is described in detail for a subset of s.o. wavelets known as spline wavelets in "An introduction to Wavelets" and in Chui, C. K. et al., "Spline-Wavelet Signal Analyzers and Method for Processing Signals", U.S. Pat. No. 5,262,958, issued Nov. 16, 1993, which is hereby incorporated by reference.

Referring to FIG. 2, when the standard wavelets $\{\psi_{m,k_m}(x)\}$ are s.o. wavelets, the sequences of coefficients may be truncated $\{a_n\}$ and $\{b_n\}$ in order to use FIR (finite impulse response) filters **108** and **110** in the decomposition filter stage **106** for each resolution level m , as described in U.S. Pat. No. 5,262,958. The filters **108** and **110** respectively perform the transfer functions $A(z)$ and $B(z)$ on the set of standard scaling function coefficients c_m at the resolution level m . This is done by applying the sequences of decomposition coefficients $\{a_n\}$ and $\{b_n\}$ to the set of standard scaling function coefficients $c_m = \{c_{m,k_m}\}$ to respectively generate sets of intermediate coefficients $\{c_{m-1,k_{m-1}}\}$ and $\{d_{m-1,k_{m-1}}\}$. The sets of intermediate coefficients $\{c_{m-1,k_{m-1}}\}$ and $\{d_{m-1,k_{m-1}}\}$ are then downsampled by the downsamplers **112** of the decomposition filter stage **106** to respectively generate the sets of standard scaling function and wavelet coefficients $c_{m-1} = \{c_{m-1,k_{m-1}}\}$ and $d_{m-1} = \{d_{m-1,k_{m-1}}\}$ at the resolution level $m-1$.

Inverse Wavelet Transform System Using Standard Wavelets $\{\psi_{m,k_m}(x)\}$ as Basic Wavelets

Conversely, FIG. 3 shows an inverse wavelet transform system **120** corresponding to the system shown in FIG. 1. The wavelet transform system **120** implements a corresponding 1-D inverse IWT to the 1-D IWT just described. Thus, the standard scaling functions and wavelets $\{\phi_{m,k_m}(x)\}$

and $\{\psi_{m,k_m}(x)\}$ are used as the basic scaling functions and wavelets in the 1-D inverse IWT to reconstruct the 1-D sets of standard scaling function and wavelet coefficients c_N and d_{M-1} to d_N into a 1-D set of reconstructed data samples f_M .

In order to do so, the reconstruction filter **122** of the inverse wavelet transform system **120** first reconstructs the 1-D set of standard scaling function and wavelet coefficients c_N and d_{M-1} to d_N into the 1-D set of standard scaling function coefficients c_M . In particular, for each resolution level $m=N$ to $M-1$ at which a reconstruction is made, the reconstruction filter **122** has a corresponding reconstruction filter stage **124** that reconstructs the sets of standard scaling function and wavelet coefficients c_m and d_m in the L and H frequency bands at the lower resolution level m into the set of standard scaling function coefficients c_{m+1} in the L frequency band at the next higher resolution level $m+1$.

This is done by the reconstruction filter stage **124** for each resolution level m with two 1-D sequences of reconstruction coefficients $\{p_n\}$ and $\{q_n\}$. Here, in one two-scale relation, each standard scaling function $\phi_{m,k_m}(x)$ at a lower resolution level m is related to and can be reconstructed into the standard scaling functions $\{\phi_{m+1,k_{m+1}}(x)\}$ at the next higher resolution level $m+1$ with the sequence of reconstruction coefficients $\{p_n\}$. Similarly, in another two-scale relation, each standard wavelet $\psi_{m,k_m}(x)$ at a lower resolution level m is related to and can be reconstructed into the standard scaling functions $\{\phi_{m+1,k_{m+1}}(x)\}$ at the next higher resolution level $m+1$ with the sequence of reconstruction coefficients $\{q_n\}$. These two-scale relations are given by:

$$\phi_{m,k_m}(x) = \sum_{k_{m+1}} p_{k_{m+1}} \phi_{m+1,2k_m+k_{m+1}}(x) \quad (7)$$

$$\psi_{m,k_m}(x) = \sum_{k_{m+1}} q_{k_{m+1}} \phi_{m+1,2k_m+k_{m+1}}(x).$$

From Eqs. (3) and (7), each scaling function coefficient $c_{m+1,k_{m+1}}$ is obtained according to the reconstruction sequence:

$$c_{m+1,k_{m+1}} = \sum_{k_m} (p_{k_{m+1}-2k_m} c_{m,k_m} + q_{k_{m+1}-2k_m} d_{m,k_m}). \quad (8)$$

By combining Eqs. (3) and (8), it can be seen that the standard scaling functions and wavelets $\{\phi_{m,k_m}(x)\}$ and $\{\psi_{m,k_m}(x)\}$ are used as the basic scaling functions and wavelets in the 1-D inverse IWT.

As described in "An Introduction to Wavelets" and U.S. Pat. No. 5,262,958, the sequences of reconstruction coefficients $\{p_n\}$ and $\{q_n\}$ are finite when the standard wavelets $\{\psi_{m,k_m}(x)\}$ are the basic wavelets in the 1-D inverse IWT and are spline wavelets. Similar to the corresponding 1-D IWT, this is also true for the more general case where the standard wavelets $\{\psi_{m,k_m}(x)\}$ are s.o. wavelets.

Therefore, in the case where semi-orthogonal standard wavelets $\{\psi_{m,k_m}(x)\}$ are used in the 1-D inverse IWT, each reconstruction filter stage **124** has transfer functions $P(z)$ and $Q(z)$. These transfer functions $P(z)$ and $Q(z)$ are polynomials that respectively have the sets of decomposition coefficients $\{p_n\}$ and $\{q_n\}$ as their coefficients.

Referring to FIG. 4, in order to perform the transfer functions $P(z)$ and $Q(z)$, the reconstruction filter stage **124** at each resolution level m has upsamplers **126**. The upsamplers **126** respectively upsample the sets of scaling function and wavelet coefficients $c_m = \{c_{m,k_m}\}$ and $d_m = \{d_{m,k_m}\}$ at the resolution level m to respectively generate the sets of intermediate coefficients $\{c_{m,k_{m+1}}\}$ and $\{d_{m,k_{m+1}}\}$.

The reconstruction filter stage **124** at each resolution level m also includes FIR filters **130** and **132**. The filters **130** and **132** respectively perform the transfer functions $P(z)$ and $Q(z)$ by applying the sequences of decomposition coefficients $\{p_n\}$ and $\{q_n\}$ respectively to the sets of intermediate coefficients $\{c_{m,k_{m+1}}\}$ and $\{d_{m,k_{m+1}}\}$ to respectively generate the sets of intermediate coefficients $\{y_{m+1,k_{m+1}}\}$ and $\{z_{m+1,k_{m+1}}\}$. The sets of intermediate coefficients $\{y_{m+1,k_{m+1}}\}$ and $\{z_{m+1,k_{m+1}}\}$ are then component-wise summed (i.e., each intermediate coefficient $y_{m+1,k_{m+1}}$ is summed with the corresponding intermediate coefficient $z_{m+1,k_{m+1}}$) by the summing stage **134** to generate the set of standard scaling function coefficients $c_{m+1}=\{c_{m+1,k_{m+1}}\}$ at the resolution level $m+1$.

Referring back to FIG. **3**, the post-reconstruction filter **136** of the inverse wavelet transform system **120** then maps the set of standard scaling function coefficients c_M into the set of reconstructed data samples f_M in accordance with Eq. (2). The post-reconstruction filter **136** comprises an FIR filter and has a transfer function $\phi(z)$ that is the inverse of the transfer function $\phi(z)^{-1}$ of the pre-decomposition filter **102**. Thus, the sequence of coefficients $\{\phi_n\}$ are applied to the set of standard scaling function coefficients c_M to generate the set of reconstructed data samples f_M .

Wavelet Transform System Using S.O. Standard Wavelets $\{\psi_{m,k_m}(x)\}$ as Basic Wavelets

FIG. **5** shows a 1-D wavelet transform system **140** in which standard scaling function $\{\phi_{m,k_m}(x)\}$ and s.o. standard wavelets $\{\psi_{m,k_m}(x)\}$ are used as the basic scaling functions and wavelets in the 1-D IWT. As will be explained shortly, this is done in order to take advantage of the fact that the finite sequences of coefficients $\{p_n\}$ and $\{q_n\}$ are used as the sequences of decomposition coefficients in this 1-D IWT instead of the infinite sequences of coefficients $\{a_n\}$ and $\{b_n\}$. This concept is described in Chui, C. K. et al., U.S. Pat. No. 5,600,373, entitled "Method and Apparatus for Video Image Compression and Decompression Using Boundary-Spline-Wavelets", issued Feb. 4, 1997, which is hereby incorporated by reference, for the specific case where the standard wavelets $\{\psi_{m,k_m}(x)\}$ are boundary spline wavelets in a 2-D IWT which uses banded matrices for the sequences of coefficients $\{p_n\}$ and $\{q_n\}$. However, in accordance with the present invention, it may be extended to the more general case where the standard wavelets $\{\psi_{m,k_m}(x)\}$ are semi-orthogonal wavelets. For simplicity, this more general case will be described hereafter for the 1-D IWT.

Like the 1-D wavelet transform system **100** described earlier, the 1-D wavelet transform system **140** includes a pre-decomposition filter **142**. However, the pre-decomposition filter **142** has a transfer function $\phi(z)^{-1}\alpha(z)$ for mapping the 1-D set of original data samples f_M into a 1-D set of dual scaling function coefficients \tilde{c}_M in an L frequency band at the resolution level M .

Referring to FIG. **6**, in order to do this, the pre-decomposition filter **142** of the present invention includes as a mapping filter stage **102** the same pre-decomposition filter **102** as that shown in FIG. **1**. Thus, with the transfer function $\phi(z)^{-1}$, this mapping filter stage **102** maps the set of original data samples f_M into the set of standard scaling function coefficients c_M .

The pre-decomposition filter **142** of the present invention also has a mapping filter stage **144** that maps the set of standard scaling function coefficients c_M to the set of dual scaling function coefficients \tilde{c}_M . This generates a change of bases so that the standard scaling functions $\{\phi_{m,k_m}(x)\}$ and the semi-orthogonal standard wavelets $\{\psi_{m,k_m}(x)\}$ are used

as the basic scaling functions and wavelets in the 1-D IWT instead of the dual scaling functions and wavelets $\{\phi_{m,k_m}(x)\}$ and $\{\psi_{m,k_m}(x)\}$.

In order to do this, it is important to note that the 1-D function $f_m(x)$ given in Eq. (2) is also related to the dual scaling functions $\{\phi_{m,k_m}(x)\}$ and a 1-D set of dual scaling function coefficients $\tilde{c}_m=\{\tilde{c}_{m,k_m}\}$ at the resolution level m according to the following relationship:

$$f_m(x) = \sum_{k_m} \tilde{c}_{m,k_m} \tilde{\phi}_{m,k_m}(x). \quad (9)$$

Moreover, each dual scaling function coefficient \tilde{c}_{m,k_m} and the set of standard scaling function coefficients $\{c_{m,k_m}\}$ are related by a sequence of mapping coefficients $\{\alpha_n\}$ as follows:

$$\tilde{c}_{m,k_m} = \sum_n \alpha_n c_{m,k_m-n} \quad (10)$$

where each mapping coefficient α_n is given by the following change of bases formula:

$$\alpha_n = \int_{-\infty}^{\infty} \phi_{m,0}(x) \phi_{m,n}(x) dx. \quad (11)$$

The transfer function $\alpha(z)$ for the mapping filter stage **144** is a polynomial that has the sequence of mapping coefficients $\{\alpha_n\}$ as its coefficients. Thus, for $m=M$ in Eqs. (9) to (11), the mapping filter stage **144** is an FIR filter and performs the transfer function $\alpha(z)$ by applying the sequence of mapping coefficients $\{\alpha_n\}$ to the set of standard scaling function coefficients $c_M=\{c_{M,k_M}\}$ to generate the set of dual scaling function coefficients $\tilde{c}_M=\{\tilde{c}_{M,k_M}\}$.

Then, referring back to FIG. **5**, the decomposition filter **146** of the 1-D wavelet transform system **140** decomposes the 1-D set of dual scaling function coefficients \tilde{c}_M in the L frequency band at the original resolution level M into a 1-D set of dual scaling function coefficients \tilde{c}_N in an L frequency band at the resolution level N and 1-D sets of dual wavelet coefficients \tilde{d}_{M-1} to \tilde{d}_N in H frequency bands at respectively the resolution levels $M-1$ to N . To do so, the decomposition filter **146** has a corresponding decomposition filter stage **148** for each resolution level $m=M$ to $N+1$ at which a decomposition is made. The decomposition filter stage **148** for each resolution level m decomposes a 1-D set of dual scaling function coefficients \tilde{c}_m in an L frequency band at the higher resolution level m into a 1-D set of dual scaling function coefficients \tilde{c}_{m-1} in an L frequency band and a 1-D set of dual wavelet coefficients \tilde{d}_{m-1} in an H frequency band at the next lower resolution level $m-1$.

This is done in view of the fact that, when a change of bases is made and the standard scaling functions $\{\phi_{m,k_m}(x)\}$ and the semi-orthogonal standard wavelets $\{\psi_{m,k_m}(x)\}$ are used as the basic scaling functions and wavelets in the 1-D IWT, the relationships in Eqs. (1) to (8) are switched (i.e., interchanged) with those for when the dual scaling functions and wavelets $\{\phi_{m,k_m}(x)\}$ and $\{\psi_{m,k_m}(x)\}$ are the basic wavelets in the 1-D IWT. More specifically, at each resolution level M , the standard scaling functions $\{\phi_{m,k_m}(x)\}$ and the semi-orthogonal standard wavelets $\{\psi_{m,k_m}(x)\}$ are respectively switched with the corresponding dual scaling functions and wavelets $\{\phi_{m,k_m}(x)\}$ and $\{\psi_{m,k_m}(x)\}$, the sets of standard scaling function and wavelet coefficients c_m, c_{m-1} ,

and d_{m-1} are respectively switched with the corresponding sets of dual scaling function and wavelet coefficients \tilde{c}_m , \tilde{c}_{m-1} , and \tilde{d}_{m-1} , and the sequences of coefficients $\{a_n\}$ and $\{b_n\}$ are respectively switched with the sequences of coefficients $\{p_n\}$ and $\{q_n\}$. This is further described in "An Introduction to Wavelets".

Thus, the decomposition filter stage **148** at each resolution level m has the transfer functions $P(z)$ and $Q(z)$. The decomposition filter stage **148** performs the transfer functions $P(z)$ and $Q(z)$ on the set of dual scaling function coefficients \tilde{c}_m to generate the sets of dual wavelet and scaling functions coefficients \tilde{c}_{m-1} and \tilde{d}_{m-1} .

More specifically, as shown in FIG. 7, in accordance with the present invention the decomposition filter stage **148** at each resolution level m has the same filters **130** and **132** as those shown in FIG. 4 for each reconstruction filter stage **124** of the inverse wavelet transform system **120** of FIG. 3. In this case, the filter stages **130** and **132** respectively perform the transfer functions $P(z)$ and $Q(z)$ (which are generally shorter, and thus more computationally efficient, than $A(z)$ and $B(z)$ of the system shown in FIG. 1) by respectively applying the sequences of decomposition coefficients $\{p_n\}$ and $\{q_n\}$ to the set of dual scaling function coefficients $\tilde{c}_m = \{\tilde{c}_{m,k_m}\}$ to respectively generate the sets of dual wavelet and scaling functions coefficients $\{\tilde{d}_{m-1,k_m}\}$ and $\{\tilde{c}_{m-1,k_m}\}$. The sets of dual wavelet and scaling functions coefficients $\{\tilde{d}_{m-1,k_m}\}$ and $\{\tilde{c}_{m-1,k_m}\}$ are then respectively downsampled by the downsamplers **112** of the decomposition filter stage **146** to respectively generate the sets of standard wavelet and scaling functions coefficients $\tilde{d}_{m-1} = \{\tilde{d}_{m-1,k_{m-1}}\}$ and $\tilde{c}_{m-1} = \{\tilde{c}_{m-1,k_{m-1}}\}$. The downsamplers **112** are the same as those shown in FIG. 2 for each decomposition filter stage **106** of the wavelet transform system **100** of FIG. 1.

Inverse Wavelet Transform System Using Standard Wavelets $\{\psi_{m,k_m}(x)\}$ as Basic Wavelets

FIG. 8 shows an inverse wavelet transform system **160** that implements a corresponding 1-D inverse IWT to the 1-D IWT implemented by the wavelet transform system **140** of FIG. 5. Like the 1-D inverse IWT performed by the inverse wavelet transform system **120** of FIG. 3, the 1-D inverse IWT implemented by the inverse wavelet transform system **160** uses the standard scaling functions $\{\phi_{m,k_m}(x)\}$ and the semi-orthogonal standard wavelets $\{\psi_{m,k_m}(x)\}$ as the basic scaling functions and wavelets in the 1-D inverse IWT.

In order to implement the 1-D inverse IWT, the inverse wavelet transform system **160** includes a reconstruction filter **162**. The reconstruction filter **162** reconstructs the sets of dual scaling function and wavelet coefficients \tilde{c}_N and \tilde{d}_{M-1} to \tilde{d}_N into the set of standard scaling function coefficients c_M . In order to do so, the reconstruction filter **162** includes a corresponding reconstruction filter stage **163** for the first resolution level $m=N$ at which a reconstruction is made and a corresponding reconstruction filter stage **164** for every other resolution level $m=N+1$ to $M-1$ at which a reconstruction is made. The reconstruction filter stage **163** has the transfer functions $Q(z)\beta(z)^{-1}$ and $P(z)\alpha(z)^{-1}$ while each reconstruction filter stage **164** has the transfer functions $Q(z)\beta(z)^{-1}$ and $P(z)$.

Referring to FIG. 9, the reconstruction filter stage **163** includes a mapping filter substage **166** which has the transfer function $\alpha(z)^{-1}$. The mapping filter substage **166** uses the transfer function $\alpha(z)^{-1}$ to map the set of dual scaling function coefficients \tilde{c}_N to the set of standard scaling function coefficients c_N at the resolution level N . Furthermore,

referring to FIGS. 9 and 10, each reconstruction filter **163** and **164** includes a mapping filter substage **168** that has a transfer function $\beta(z)^{-1}$ for mapping the set of dual wavelet coefficients \tilde{d}_m to the set of standard wavelet coefficients d_m at the corresponding resolution level m . In this way, a change of bases is made so that the standard scaling functions $\{\phi_{m,k_m}(x)\}$ and the semi-orthogonal standard wavelets $\{\psi_{m,k_m}(x)\}$ are used as the basic scaling functions and wavelets in the 1-D inverse IWT rather than the dual scaling functions and wavelets $\{\phi_{m,k_m}(x)\}$ and $\{\psi_{m,k_m}(x)\}$.

Referring back to FIG. 9, the transfer function $\alpha(z)^{-1}$ of the mapping filter substage **166** is the inverse of the transfer function $\alpha(z)$ described earlier. Thus, the transfer function $\alpha(z)^{-1}$ is a rational function which has a sequence of mapping coefficients $\{\Delta_n\}$ as its poles. Thus, the mapping filter substage **166** is an IIR filter that applies the sequence of mapping coefficients $\{\Delta_n\}$ to the set of dual scaling function coefficients \tilde{c}_N to generate the set of standard scaling function coefficients c_N .

Moreover, turning again to FIGS. 9 and 10, the transfer function $\beta(z)^{-1}$ for each mapping filter substage **168** is determined in view of the following relationship at the corresponding resolution level m :

$$\sum_{k_m} d_{m,k_m} \psi_{m,k_m}(x) = \sum_{k_m} \tilde{d}_{m,k_m} \tilde{\psi}_{m,k_m}(x). \quad (12)$$

Furthermore, at this resolution level $m-1$, there exists a sequence of mapping coefficients $\{\beta_n\}$ such that:

$$\tilde{d}_{m,k_m}(x) = \sum_n \beta_n d_{m,k_m-n}(x) \quad (13)$$

where each mapping coefficient β_n is given by:

$$\beta_n = \int_{-\infty}^{\infty} \psi_{m,0}(x) \psi_{m,n}(x) dx. \quad (14)$$

The transfer function $\beta(z)^{-1}$ is the inverse of a transfer function $\beta(z)$. The transfer function $\beta(z)$ is a polynomial that has the sequence of mapping coefficients $\{\beta_n\}$ as its coefficients. Thus, the transfer function $\beta(z)^{-1}$ is a rational function which has a corresponding sequence of mapping coefficients $\{\delta_n\}$ as its poles. As result, the mapping filter substage **168** is an IIR filter and applies the sequence of mapping coefficients $\{\delta_n\}$ to the set of dual wavelet coefficients \tilde{d}_m to generate the set of standard wavelet coefficients d_m .

Referring to FIGS. 9 and 10, each reconstruction filter stage **163** and **164** includes a reconstruction filter stage **124** for the corresponding resolution level m . In accordance with the present invention, this reconstruction filter stage **124** is the same as the reconstruction filter stage **124** shown in FIG. 4 for the inverse wavelet transform system **120** of FIG. 3. Thus, this reconstruction filter stage **124** has the transfer functions $P(z)$ and $Q(z)$ for reconstructing the sets of standard scaling function and wavelet coefficients c_m and d_m at the lower resolution level m into the set of standard scaling function coefficients c_{m+1} , at the next higher resolution level $m+1$.

Referring back to FIG. 8, the inverse wavelet transform system **160** further includes the same post-reconstruction filter **136** as that shown in FIG. 4 for the inverse wavelet transform system **120** of FIG. 3. Thus, as described earlier, the post-reconstruction filter **136** maps the set of standard

scaling function coefficients c_M into the set of reconstructed data samples f_M .

As those skilled in the art will recognize, the concepts just described may be extended to a 2-D IWT and a corresponding 2-D inverse IWT. This extension is straight forward and therefore will not be described at this point. However, it must be noted here that the sequence of mapping coefficients $\{\delta_n\}$ is large. Thus, unfortunately, the transfer function $\beta(z)^{-1}$ is computationally complex and difficult to implement. For example, in the 2-D inverse IWT described in U.S. Pat. No. 5,600,373, the sequence of mapping coefficients $\{\delta_n\}$ are provided as a banded matrix with many non-zero bands. This results in the 2-D inverse IWT being slow and inefficient when used to decompress 2-D image data.

SUMMARY OF THE INVENTION

In summary, the present invention comprises a wavelet transform system that implements a wavelet transform. Semi-orthogonal standard wavelets are used as the basic wavelets in the wavelet transform and related standard scaling functions are used as the basic scaling functions in the inverse wavelet transform. The standard scaling functions at a lower resolution level m are related to the standard scaling functions at a next higher resolution level $m+1$ by a first finite sequence of coefficients in a first two-scale relation. Similarly, the semi-orthogonal standard wavelets at the lower resolution level m are related to the standard scaling functions at the next higher resolution level $m+1$ by a second finite sequence of coefficients in a second two-scale relation.

The wavelet transform system includes a pre-decomposition filter. The pre-decomposition filter maps a set of original data samples into a set of dual scaling function coefficients at a resolution level $m=M$. The set of original data samples are given by a first function $f_m(x)$ that approximates a second function $f(x)$ at the resolution level $m=M$.

The wavelet transform system further includes a decomposition filter that decomposes the set of dual scaling function coefficients at the resolution level M into a set of dual scaling function coefficients at a resolution level $m=N$ and sets of dual wavelet coefficients at respective resolution levels $m=M-1$ to N . The decomposition filter includes a corresponding decomposition filter stage for each resolution level $m=M$ to $N+1$ at which a decomposition is made.

The decomposition filter stage at each resolution level m decomposes a set of dual scaling function coefficients at the higher resolution level m into a set of dual scaling function coefficients and a set of dual wavelet coefficients at the next lower resolution level $m-1$. In doing so, the first finite sequence of coefficients is applied to the set of standard scaling function coefficients at the higher resolution level m to generate the set of dual scaling function coefficients at the resolution level $m-1$. Similarly, the second finite sequence of coefficients is applied to the set of standard scaling function coefficients at the higher resolution level m to generate the set of dual wavelet coefficients at the resolution level $m-1$.

The present invention also comprises an inverse wavelet transform system that implements an inverse wavelet transform. The inverse wavelet transform corresponds to the wavelet transform implemented by the wavelet transform system just described. As in the wavelet transform, the semi-orthogonal standard wavelets are used as the basic wavelets in the inverse wavelet transform and the related standard scaling functions are used as the basic scaling functions in the inverse wavelet transform.

The inverse wavelet transform system includes a reconstruction filter that reconstructs a set of dual scaling function coefficients at a resolution level $m=N$ and sets of dual wavelet coefficients at respective resolution levels $m=N$ to $M-1$ into a set of dual scaling function coefficients at a resolution level M . The reconstruction filter includes a corresponding reconstruction filter stage for each resolution level $m=N$ to $M-1$ at which a reconstruction is made.

The reconstruction filter stage for each resolution level m reconstructs a set of dual scaling function coefficients and a set of dual wavelet coefficients at the lower resolution level m into a set of dual scaling function coefficients at the next higher resolution level $m+1$. The reconstruction filter stage comprises a first mapping filter substage to apply a sequence of mapping coefficients to the set of dual scaling function coefficients at the resolution level m to generate a set of standard scaling function coefficients at the resolution level m . The reconstruction filter stage further includes a second mapping filter substage to apply the sequence of mapping coefficients to the set of dual wavelet coefficients at the lower resolution level m to generate a set of formatted wavelet coefficients at the resolution level m . Finally, the reconstruction filter stage includes a reconstruction filter substage to reconstruct the sets of standard scaling function and formatted wavelet coefficients at the lower resolution level m into the set of dual scaling function coefficients at the next higher resolution level $m+1$. This is done by applying a third finite sequence of reconstruction coefficients to the set of standard scaling function coefficients at the lower resolution level m and a fourth finite sequence of reconstruction coefficients to the set of formatted wavelet coefficients at the resolution level m . The third finite sequence of reconstruction coefficients is derived from the second finite sequence of reconstruction coefficients and the fourth finite sequence of reconstruction coefficients is derived from the first finite sequence of reconstruction coefficients.

The inverse wavelet transform system further includes a post-reconstruction filter. The post-reconstruction filter maps the set of dual scaling function coefficients at the resolution level M into a set of reconstructed data samples. The set of reconstructed data samples are also given by the first function $f_m(x)$, for the resolution level $m=M$.

As indicated previously, semi-orthogonal standard wavelets are used as the basic wavelets in the wavelet transform and the inverse wavelet transforms. Thus, the first and second finite sequences of reconstruction coefficients are not infinite sequences of coefficients that have been truncated.

Furthermore, in one embodiment of each decomposition filter stage of the wavelet transform system, downsampling is not used. Similarly, in one embodiment of each reconstruction filter stage of the inverse wavelet transform, upsampling is not used.

Additionally, in one embodiment of the pre-decomposition filter of the wavelet transform system, the pre-decomposition filter has a transfer function that is the sum of an all-zeros polynomial and an all-poles rational function. In this case, the pre-decomposition filter comprises parallel FIR and IIR filters. In another embodiment of the pre-decomposition filter, the pre-decomposition filter has a transfer function that is obtained from the orthogonal projection of the second function $f(x)$ to the first function $f_m(x)$, for the resolution level $m=M$. In the case where this embodiment of the pre-decomposition filter is used in the wavelet transform system, a corresponding embodiment of the post-reconstruction filter is used in the inverse wavelet transform.

BRIEF DESCRIPTION OF THE DRAWINGS

FIG. 1 shows a block diagram of an embodiment of a wavelet transform system in accordance with the present invention.

FIG. 2 shows a block diagram of each decomposition filter stage of the wavelet transform system of FIG. 1.

FIG. 3 shows a block diagram of an embodiment of an inverse wavelet transform system in accordance with the present invention.

FIG. 4 shows a block diagram of each reconstruction filter stage of the inverse wavelet transform system of FIG. 3.

FIG. 5 shows a block diagram of another embodiment of a wavelet transform system in accordance with the present invention.

FIG. 6 shows a block diagram of a pre-decomposition filter of the wavelet transform system of FIG. 5.

FIG. 7 shows a block diagram of each decomposition filter stage of the wavelet transform system of FIG. 5.

FIG. 8 shows a block diagram of another embodiment of an inverse wavelet transform system in accordance with the present invention.

FIG. 9 shows a block diagram of the first reconstruction filter stage of the inverse wavelet transform system of FIG. 8.

FIG. 10 shows a block diagram of every other reconstruction filter stage of the inverse wavelet transform system of FIG. 8.

FIG. 11 shows a block diagram of a data processing workstation according to the present invention.

FIG. 12 shows a block diagram of the operation of the data processing system of the data processing workstation of FIG. 11.

FIG. 13 shows a block diagram of an embodiment of a wavelet transform system for implementing a 1-D IWT according to present invention.

FIG. 14 shows a block diagram of an embodiment of an inverse wavelet transform system for implementing a 1-D inverse IWT according to present invention.

FIG. 15 shows a block diagram of each reconstruction filter stage of the inverse wavelet transform system of FIG. 14.

FIG. 16 shows a block diagram of a post-reconstruction filter of the inverse wavelet transform system of FIG. 14.

FIG. 17 shows a block diagram of an embodiment of a wavelet transform system for implementing a 2-D IWT according to present invention.

FIG. 18 shows a block diagram of a pre-decomposition filter of the wavelet transform system of FIG. 17.

FIG. 19 shows a block diagram of each decomposition filter substage of the wavelet transform system of FIG. 17.

FIG. 20 shows a block diagram of an embodiment of an inverse wavelet transform system for implementing a 2-D inverse IWT according to present invention.

FIG. 21 shows a block diagram of each reconstruction filter substage of the inverse wavelet transform system of FIG. 20.

FIG. 22 shows a block diagram of a post-reconstruction filter of the inverse wavelet transform system of FIG. 22.

FIG. 23 shows a block diagram of another embodiment of a pre-decomposition filter of the wavelet transform system of FIG. 13.

FIG. 24 shows a block diagram of still another embodiment of a pre-decomposition filter of the wavelet transform system of FIG. 13.

FIG. 25 shows a block diagram of another embodiment of a post-reconstruction filter of the inverse wavelet transform system of FIG. 14.

FIG. 26 shows a block diagram of another embodiment of each decomposition filter stage of the wavelet transform system of FIG. 13.

FIG. 27 shows a block diagram of another embodiment of each reconstruction filter substage of the inverse wavelet transform system of FIG. 14.

FIG. 28 shows a block diagram of pre-quantization filter substages of each decomposition filter stage of the wavelet transform system of FIG. 17.

FIG. 29 shows a block diagram of post-dequantization filter substages of each reconstruction filter stage of the inverse wavelet transform system of FIG. 20.

DETAILED DESCRIPTION OF THE INVENTION

Referring to FIG. 11, there is shown a data processing workstation 200 with a software embodiment of a data processing system 202. The data processing workstation 200 includes a memory 210. The memory stores 210 an operating system 212 and the data processing system 202. The operating system 212 and the data processing system 202 are run on the CPU 214 of the workstation 200. The operating system 212 controls and coordinates running of the data processing system 202 in response to commands issued by a user with the user interface 216 of the workstation 200.

Original data and encoded (i.e., compressed) data generated externally is received by the data processing workstation 200 from an external source (not shown) over external communications channels 218 via the communications interface 220. This data is then stored in the memory locations 204 and 206, respectively, by the operating system 212. Similarly, encoded data that is generated by the data processing system 202 is stored in the memory location 206 and may be retrieved by the operating system 212 and transmitted to external destinations (not shown). This is done over the communications channels 218 via the communications interface 220. These operations are all done in response to commands issued by the user with the user interface 216.

When the user wishes to compress original data, the user issues appropriate commands with the user interface 216 to invoke the data processing system 202 and select the data. This original data may be 1-D data, such as audio signals, and/or 2-D data, such as image data. The data is then compressed by the data processing system 202 in the manner discussed next.

Referring to FIG. 12, the data processing system 202 includes a wavelet transform system 230. The wavelet transform system 230 retrieves the original data from the memory location 206 and decomposes it to form decomposed data. As will be discussed shortly, the wavelet transform system 230 uses a novel implementation of 1-D and 2-D IWTs in order to do this.

Then, the decomposed data is quantized by the quantization system 224 of the data processing system 202 to generate quantized data. In doing so, the quantization system 224 quantizes the decomposed data by quantizing its data samples to predefined allowable integer values.

The encoding system 226 of the data processing system 202 then encodes the quantized data to generate encoded data that is compressed. This may be done by encoding the quantized data samples of the quantized data based on their

quantized integer values Using well known lossless and highly compact encoding techniques. The encoding subsystem then stores the encoded data in the memory location 206.

Referring back to FIG. 11, conversely when the user wishes to decompress encoded data, the user issues appropriate commands with the user interface 216 to invoke the data processing system 202 and select the encoded data. The data processing system 202 then decompresses the encoded data in the manner discussed next.

The decoding system 228 of the data processing system 202 retrieves the encoded data from the memory location 206 and decodes it to generate decoded data that is also quantized. The decoding system 228 does this by decoding the encoded data samples of the encoded data Using a decoding technique corresponding to the encoding technique described earlier.

Then, the decoded data is dequantized by the dequantization system 232 of the data processing system 202 to generate dequantized data that was decomposed by the wavelet transform system 230. This is done by dequantizing the quantized data samples of the decoded data from their predefined allowable integer values to dequantized values. In doing so, the dequantization system 232 uses a dequantization technique corresponding to the quantization technique mentioned earlier.

The inverse wavelet transform system 240 of the data processing system 202 reconstructs the dequantized data to generate reconstructed data. As will be described shortly, this is done using a novel implementation of the 1-D and 2-D inverse IWTs that correspond to the 1-D and 2-D IWTs implemented by the wavelet transform system 230. The reconstructed data is then stored in the memory location 208 by the inverse wavelet transform system 240.

1-D Wavelet Transform Using S.O. Standard Wavelets $\{\psi_{m,k_m}(x)\}$ as Basic Wavelets

Referring to FIG. 13, the wavelet transform system 230 is used to decompose original 1-D data that comprises a 1-D set of original data samples f_M at the original resolution level M. The set of original data samples f_M is decomposed by the wavelet transform system 230 into a set of dual scaling function coefficients \tilde{c}_N in an L frequency band at the resolution level M and sets of dual wavelet coefficients \tilde{d}_{M-1} to \tilde{d}_N in H frequency bands at respectively the resolution levels M-1 to N. This is done by using standard scaling functions $\{\phi_{m,k_m}(x)\}$ and s.o. standard wavelets $\{\psi_{m,k_m}(x)\}$ as the basic scaling functions and wavelets in the 1-D IWT in the same manner as discussed earlier for the wavelet transform system 140.

In order to do this, the wavelet transform system 230 includes the same pre-decomposition filter 142 as does the wavelet transform system 140 of FIG. 5. Moreover, the wavelet transform system 230 includes a decomposition filter 234. The decomposition filter 234 has a corresponding decomposition filter stage 236 for the last resolution level $m=N+1$ at which a decomposition is made and a corresponding decomposition filter stage 238 for every other resolution level $m=M$ to $N+2$ at which a decomposition is made. Each decomposition filter stage 236 and 238 includes a decomposition filter substage 148 for the corresponding resolution level m which is the same as the decomposition filter stage 148 shown in FIG. 7 for the wavelet transform system 140. However, in the wavelet transform system 230, alternative embodiments for the pre-decomposition filter 142 and each decomposition filter substage 148 may be used, as will be discussed later.

1-D Inverse Wavelet Transform Using S.O. Standard Wavelets $\{\psi_{m,k_m}(x)\}$ as Basic Wavelets

Turning to FIG. 14, the inverse wavelet transform system 240 is used to reconstruct a set of dual scaling function and wavelet coefficients \tilde{c}_N and \tilde{d}_{M-1} to \tilde{d}_N that were generated in the manner just described into a set of reconstructed data samples f_M . As mentioned earlier, the inverse wavelet transform system 240 uses a novel implementation of the 1-D inverse IWT that corresponds to the 1-D IWT implemented by the wavelet transform system 230. Like the 1-D inverse IWT performed by the inverse wavelet transform system 160 of FIG. 8, the 1-D inverse IWT implemented by the inverse wavelet transform system 240 uses the standard scaling functions $\{\phi_{m,k_m}(x)\}$ and the s.o. standard wavelets $\{\psi_{m,k_m}(x)\}$ as the basic scaling functions and wavelets in the 1-D inverse IWT.

In order to implement the 1-D inverse IWT, the inverse wavelet transform system 240 includes a reconstruction filter 242. The reconstruction filter 242 reconstructs the sets of dual scaling function and wavelet coefficients \tilde{c}_N and \tilde{d}_{M-1} to \tilde{d}_N into the set of dual scaling function coefficients \tilde{c}_M . The reconstruction filter 242 includes a corresponding reconstruction filter stage 243 for the first resolution level $m=N$ at which a reconstruction is made and a corresponding reconstruction filter stage 245 for every other resolution level $m=N+1$ to $M-1$ at which a reconstruction is made. Each reconstruction filter stage 243 and 245 includes a reconstruction filter substage 244 for the corresponding resolution level m. The reconstruction filter substage 244 has the transfer functions $P^*(z)\alpha(z)^{-1}$ and $Q^*(z)\alpha(z)^{-1}$ to reconstruct the sets of dual wavelet and scaling function coefficients \tilde{c}_m and \tilde{d}_m in the L and H frequency bands at the lower resolution level m into the set of dual scaling function coefficients \tilde{c}_{m+1} in the L frequency band at the next higher resolution level $m+1$.

In order to do so, the reconstruction filter substage 244 for each resolution level m includes two mapping filter substages 166 with the transfer function $\alpha(z)^{-1}$, as shown in FIG. 15. Each mapping filter substage 166 is the same as the mapping filter stage 166 shown in FIG. 8. One mapping filter substage 166 maps the set of dual scaling function coefficients $\tilde{c}_m = \{\tilde{c}_{m,k_m}\}$ to the set of standard scaling function coefficients $\{c_{m,k_m}\}$ according to Eq. (10) for the resolution level m. However, the other mapping filter substage 166 maps the set of dual wavelet coefficients $\tilde{d}_m = \{\tilde{d}_{m,k_m}\}$ to a set of formatted wavelet coefficients $\{\bar{d}_{m,k_m}\}$ at the resolution level m according to:

$$\bar{d}_{m,k_m} = \sum_n \alpha_n \bar{d}_{m,k_m-n}. \quad (15)$$

In performing these mappings, the sequence of mapping coefficients $\{\Delta_n\}$ described earlier are applied by the mapping filter substages 166 to the sets of dual scaling function and wavelet coefficients $\{\tilde{c}_{m,k_m}\}$ and $\{\tilde{d}_{m,k_m}\}$ to respectively generate the sets of standard scaling function and formatted wavelet coefficients $\{c_{m,k_m}\}$ and $\{\bar{d}_{m,k_m}\}$.

The reconstruction filter substage 244 for each resolution level m also includes a reconstruction filter substage 246. The reconstruction filter substage 246 reconstructs the sets of standard scaling function and formatted wavelet coefficients $\{c_{m,k_m}\}$ and $\{\bar{d}_{m,k_m}\}$ in the L and H frequency bands at the lower resolution level m into the set of dual scaling function coefficients $\tilde{c}_{m+1} = \{\tilde{c}_{m+1,k_{m+1}}\}$ in the L frequency band at the next higher resolution level $m+1$. In order to do

so, the transfer functions $P^*(z)$ and $Q^*(z)$ are used by the reconstruction filter substage **246**.

The transfer functions $P^*(z)$ and $Q^*(z)$ are obtained in view of the fact that the sets of standard scaling function and formatted wavelet coefficients $\{c_{m,k_m}\}$ and $\{\tilde{d}_{m,k_m}\}$ are related to two sequences of reconstruction coefficients $\{p_n^*\}$ and $\{q_n^*\}$ and the set of dual scaling function coefficients $\{\tilde{c}_{m+1,k_{m+1}}\}$ according to:

$$\tilde{c}_{m+1,k_{m+1}} = \sum_{k_m} (p_{k_{m+1}-2k_m}^* \tilde{d}_{m,k_m} + q_{k_{m+1}-2k_m}^* c_{m,k_m}). \quad (16)$$

Furthermore, the sequences of reconstruction coefficients $\{p_n^*\}$ and $\{q_n^*\}$ are derived from the corresponding sequences of reconstruction coefficients $\{p_n\}$ and $\{q_n\}$ according to:

$$\begin{aligned} p_n^* &= (-1)^n p_{n+1} \\ q_n^* &= (-1)^{n-1} q_{n-1}. \end{aligned} \quad (17)$$

Thus, the transfer functions $P^*(z)$ and $Q^*(z)$ of the reconstruction filter substage **246** are polynomials that respectively have the sequences of reconstruction coefficients $\{p_n^*\}$ and $\{q_n^*\}$ as their coefficients.

In view of this, the reconstruction filter substage **246** at each resolution level m has upsamplers **126**, like the reconstruction filter stage **124**. In this case, the upsamplers **126** respectively upsample the sets of scaling function and formatted wavelet coefficients $\{c_{m,k_m}\}$ and $\{\tilde{d}_{m,k_m}\}$ to respectively generate the sets of intermediate coefficients $\{c_{m,k_{m+1}}\}$ and $\{\tilde{d}_{m,k_{m+1}}\}$.

Furthermore, similar to the reconstruction filter stage **124** of FIGS. **4** and **9**, the reconstruction filter substage **246** at each resolution level m has FIR filters **247** and **249**. The filters **247** and **249** respectively perform the transfer functions $P^*(z)$ and $Q^*(z)$ by respectively applying the sequences of decomposition coefficients $\{p_n^*\}$ and $\{q_n^*\}$ to the sets of formatted wavelet and scaling function coefficients $\{\tilde{d}_{m,k_{m+1}}\}$ and $\{c_{m,k_{m+1}}\}$ to respectively generate the sets of intermediate coefficients $\{\tilde{z}_{m+1,k_{m+1}}\}$ and $\{\tilde{y}_{m+1,k_{m+1}}\}$. The sets of intermediate coefficients $\{\tilde{z}_{m+1,k_{m+1}}\}$ and $\{\tilde{y}_{m+1,k_{m+1}}\}$ are then component-wise summed by the summer **134** of the reconstruction filter substage **246** to generate the set of dual scaling function coefficients $\tilde{c}_{m+1} = \{\tilde{c}_{m+1,k_{m+1}}\}$. The summer **134** is the same as that in the reconstruction filter stage **124** of FIGS. **4** and **9**.

Referring back to FIG. **14**, the post-reconstruction filter **248** of the inverse wavelet transform system **240** has the transfer function $\alpha(z)^{-1}\phi(z)$. This transfer function $\alpha(z)^{-1}\phi(z)$ is used by the post-reconstruction filter **248** to map the set of dual scaling function coefficients \tilde{c}_M into the set of reconstructed data samples f_M .

As shown in FIG. **16**, the post-reconstruction filter **248** includes a mapping filter stage **166** that is the same as the mapping filter stage **166** shown in FIG. **8** for the inverse wavelet transform system **160**. Here, the mapping filter stage **166** performs the transfer function $\alpha(z)^{-1}$ by applying the sequence of coefficients $\{\Delta_n\}$ to the set of dual scaling function coefficients \tilde{c}_M to generate the set of standard scaling function coefficients c_M . The post-reconstruction filter **248** also includes as a mapping filter stage **136** the same post-reconstruction filter **136** as in the inverse wavelet transform system **120** of FIG. **3**. This mapping filter stage **136** uses the transfer function $\phi(z)$ to map the set of standard scaling function coefficients c_M into the set of reconstructed data samples f_M by applying the sequence of mapping

coefficients $\{\phi_n\}$ to the set of standard scaling function coefficients c_M .

Referring back to FIG. **15**, in each reconstruction filter substage **244**, the use of the two mapping filter substages **166** and the switching of the filters **130** and **132** in the reconstruction filter substage **246** provides several benefits to the inverse wavelet transform system **240**. First, this avoids having to change basis using the same complex mapping filter substage **168** as in each reconstruction filter stage **164** shown in FIG. **9** for the inverse wavelet transform system **160** of FIG. **8**. Second, this avoids having to use the truncated infinite sequences of coefficients $\{a_n\}$ and $\{b_n\}$ applied with the complex filters **108** and **110** shown in FIG. **2** for each decomposition filter stage **106** of the wavelet transform system **100** of FIG. **1**.

2-D Wavelet Transform Using Standard Wavelets $\{\psi_{m,k_m}(x)\}$ and $\{\psi_{m,j_m}(y)\}$ as Basic Wavelets

In a typical 2-D IWT, the dual scaling functions and wavelets $\{\phi_{m,k_m}(x)\}$, $\{\phi_{m,j_m}(y)\}$, $\{\psi_{m,k_m}(x)\}$ and $\{\psi_{m,j_m}(y)\}$ to the standard wavelets $\{\phi_{m,k_m}(x)\}$, $\{\phi_{m,j_m}(y)\}$, $\{\psi_{m,k_m}(x)\}$, and $\{\psi_{m,j_m}(y)\}$ are used as the basic scaling functions and wavelets in the 2-D IWT, where the indexes k_m and j_m correspond to the resolution level m . For each resolution level $m=M$ to $N+1$ at which a decomposition is made in this 2-D IWT, there exists the following relationship between the 2-D set of standard scaling function coefficients $c_m = \{c_{m,k_m,j_m}\}$ at the higher resolution level m and the 2-D sets of standard scaling function and wavelet coefficients $c_{m-1} = \{c_{m-1,k_{m-1},j_{m-1}}\}$, $d_{m-1}^1 = \{d_{m-1,k_{m-1},j_{m-1}}^1\}$, $d_{m-1}^2 = \{d_{m-1,k_{m-1},j_{m-1}}^2\}$, $d_{m-1}^3 = \{d_{m-1,k_{m-1},j_{m-1}}^3\}$ at the next lower resolution level $m-1$:

$$\begin{aligned} \sum_{k_m,j_m} c_{m,k_m,j_m} \phi_{m,k_m}(x) \phi_{m,j_m}(y) &= \sum_{k_{m-1},j_{m-1}} (d_{m-1,k_{m-1},j_{m-1}}^1 \phi_{m-1,k_{m-1}}(x) \psi_{m-1,j_{m-1}}(y) + \\ & d_{m-1,k_{m-1},j_{m-1}}^2 \psi_{m-1,k_{m-1}}(x) \phi_{m-1,j_{m-1}}(y) + \\ & d_{m-1,k_{m-1},j_{m-1}}^3 \psi_{m-1,k_{m-1}}(x) \psi_{m-1,j_{m-1}}(y) + \\ & c_{m-1,k_{m-1},j_{m-1}} \phi_{m-1,k_{m-1}}(x) \phi_{m-1,j_{m-1}}(y)). \end{aligned} \quad (18)$$

In view of Eq. (18), the standard scaling function and wavelet coefficients $c_{m-1,k_{m-1},j_{m-1}}$, $d_{m-1,k_{m-1},j_{m-1}}^1$, $d_{m-1,k_{m-1},j_{m-1}}^2$, and $d_{m-1,k_{m-1},j_{m-1}}^3$ at each resolution level $m-1$ may be obtained according to the decomposition sequences:

$$\begin{aligned} c_{m-1,k_{m-1},j_{m-1}} &= \sum_{k_m,j_m} a_{k_m-2k_{m-1}} a_{j_m-2j_{m-1}} c_{m,k_m,j_m} \\ d_{m-1,k_{m-1},j_{m-1}}^1 &= \sum_{k_m,j_m} a_{k_m-2k_{m-1}} b_{j_m-2j_{m-1}} c_{m,k_m,j_m} \\ d_{m-1,k_{m-1},j_{m-1}}^2 &= \sum_{k_m,j_m} b_{k_m-2k_{m-1}} a_{j_m-2j_{m-1}} c_{m,k_m,j_m} \\ d_{m-1,k_{m-1},j_{m-1}}^3 &= \sum_{k_m,j_m} b_{k_m-2k_{m-1}} b_{j_m-2j_{m-1}} c_{m,k_m,j_m}. \end{aligned} \quad (19)$$

This is further described in U.S. Pat. No. 5,262,958 referenced earlier. From Eq. (19), it is clear that the dual scaling functions and wavelets $\{\phi_{m,k_m}(x)\}$, $\{\phi_{m,j_m}(y)\}$, $\{\psi_{m,k_m}(x)\}$, and $\{\psi_{m,j_m}(y)\}$ are used as the basic wavelets in this other 2-D IWT since it is defined by:

$$\begin{aligned}
c_{m,k,j} &= \int_{-\infty}^{\infty} f(x,y) \tilde{\phi}_{m,k_m}(x) \tilde{\phi}_{m,j_m}(y) dx dy \\
d_{m,k,j}^1 &= \int_{-\infty}^{\infty} f(x,y) \tilde{\phi}_{m,k_m}(x) \tilde{\psi}_{m,j_m}(y) dx dy \\
d_{m,k,j}^2 &= \int_{-\infty}^{\infty} f(x,y) \tilde{\psi}_{m,k_m}(x) \tilde{\phi}_{m,j_m}(y) dx dy \\
d_{m,k,j}^3 &= \int_{-\infty}^{\infty} f(x,y) \tilde{\psi}_{m,k_m}(x) \tilde{\psi}_{m,j_m}(y) dx dy.
\end{aligned} \tag{20}$$

Conversely, in the 2-D inverse IWT that corresponds to the 2-D IWT just described, each standard scaling function coefficient c_{m,k_m,j_m} is given by:

$$\begin{aligned}
c_{m,k_m,j_m} &= \sum_{k_{m-1},j_{m-1}} (p_{k_m-2k_{m-1}} q_{j_m-2j_{m-1}} c_{m-1,k_{m-1},j_{m-1}} + \\
&\quad p_{k_m-2k_{m-1}} q_{j_m-2j_{m-1}} d_{m-1,k_{m-1},j_{m-1}}^1 + \\
&\quad q_{k_m-2k_{m-1}} p_{j_m-2j_{m-1}} d_{m-1,k_{m-1},j_{m-1}}^2 + \\
&\quad q_{k_m-2k_{m-1}} q_{j_m-2j_{m-1}} d_{m-1,k_{m-1},j_{m-1}}^3).
\end{aligned} \tag{21}$$

This is also described in U.S. Pat. No. 5,262,958 referenced earlier. Here, the standard scaling functions and wavelets $\{\phi_{m,k_m}(x)\}$, $\{\phi_{m,j_m}(y)\}$, $\{\psi_{m,k_m}(x)\}$, and $\{\psi_{m,j_m}(y)\}$ are used as the basic scaling functions and wavelets in the 2-D inverse IWT, as can be seen by combining Eqs. (18) and (21).

But, similar to the 1-D IWT discussed earlier, a change of bases may be made so that the standard scaling functions and wavelets $\{\phi_{m,k_m}(x)\}$, $\{\phi_{m,j_m}(y)\}$, $\{\psi_{m,k_m}(x)\}$, and $\{\psi_{m,j_m}(y)\}$ are used also as the basic scaling functions and wavelets in the 2-D IWT. In this case, the relationships in Eqs. (18) to (21) are switched with those for when the dual scaling functions and wavelets $\{\phi_{m,k_m}(x)\}$, $\{\phi_{m,j_m}(y)\}$, $\{\psi_{m,k_m}(x)\}$, and $\{\psi_{m,j_m}(y)\}$ are the basic wavelets in the 2-D IWT. As a result, at each resolution level m , the standard scaling functions and wavelets $\{\phi_{m,k_m}(x)\}$, $\{\phi_{m,j_m}(y)\}$, $\{\psi_{m,k_m}(x)\}$, and $\{\psi_{m,j_m}(y)\}$ are respectively switched with the corresponding dual scaling functions and wavelets $\{\phi_{m,k_m}(x)\}$, $\{\phi_{m,j_m}(y)\}$, $\{\psi_{m,k_m}(x)\}$, and $\{\psi_{m,j_m}(y)\}$, the sets of standard scaling function and wavelet coefficients c_{m-1} , d_{m-1}^1 , d_{m-1}^2 , and d_{m-1}^3 are switched with the corresponding sets of dual scaling function and wavelet coefficients \tilde{c}_{m-1} , \tilde{d}_{m-1}^1 , \tilde{d}_{m-1}^2 , and \tilde{d}_{m-1}^3 , and the sequences of coefficients $\{a_n\}$ and $\{b_n\}$ are respectively switched with the sequences of coefficients $\{p_n\}$ and $\{q_n\}$. Furthermore, if the standard wavelets $\{\psi_{m,k_m}(x)\}$, and $\{\psi_{m,j_m}(y)\}$ are s.o. wavelets in this case, then the sequences of coefficients $\{p_n\}$ and $\{q_n\}$ are finite and can be used in both decomposition and reconstruction, as in the 1-D IWT discussed earlier.

Referring to FIG. 17, the wavelet transform system **230** implements such a 2-D IWT with s.o. standard wavelets $\{\psi_{m,k_m}(x)\}$, and $\{\psi_{m,j_m}(y)\}$ as the basic wavelets. Like the 1-D original data discussed earlier, the original 2-D data comprises a 2-D set of original data samples f_M at the original resolution level M . The set of original data samples $f_M = \{f_{M,n,i}\} = f_M(2^{-M}n, 2^{-M}i)$ is given by a 2-D function $f_M(x, y)$, where $x=2^{-M}n$ and $y=2^{-M}i$. Similar to the 1-D function $f_M(x)$, the 2-D function $f_M(x,y)$ approximates another 2-D function $f(x,y)$ at the original resolution level M . But, in this case, the set of original data samples f_M extends in two spatial dimensions, the x and y directions.

The set of original data samples f_M is decomposed by the wavelet transform system **230** into a 2-D set of dual scaling function coefficients \tilde{c}_N in an LL (vertical and horizontal direction low) frequency band at the resolution level N , 2-D

sets of dual wavelet coefficients \tilde{d}_{M-1}^1 to \tilde{d}_N^1 in LH (horizontal direction low and vertical direction high) frequency bands at respectively the resolution levels $m=M-1$ to N , 2-D sets of dual wavelet coefficients \tilde{d}_{M-1}^2 to \tilde{d}_N^2 in HL (horizontal direction high and vertical direction low) frequency bands at respectively the resolution levels $m=M-1$ to N , and 2-D sets of dual wavelet coefficients \tilde{d}_{M-1}^3 to \tilde{d}_N^3 in HH (horizontal and vertical direction high) frequency bands at respectively the resolution levels $m=M-1$ to N . This is done in a similar manner to that in the 1-D IWT implemented by the wavelet transform system **230**.

Specifically, the pre-decomposition filter **270** of the wavelet transform system **230** has the transfer function $\phi(z_1)^{-1}\alpha(z_1)\phi(z_2)^{-1}\alpha(z_2)$ for mapping the set of original data samples f_M into the set of dual scaling function coefficients \tilde{c}_M in an LL frequency band at the resolution level $m=M$. In order to do this, the pre-decomposition filter **270** includes two pre-decomposition filter stages **142**, as shown in FIG. **18**. Each pre-decomposition filter stage **142** is the same pre-decomposition filter **142** as that shown in FIG. **6** for the 1-D IWT.

The first pre-decomposition filter stage **142** has the transfer function $\phi(z_1)^{-1}\alpha(z_1)$ for mapping in the horizontal direction each row i of the 2-D set of original data samples $f_M = \{f_{M,n,i}\}$ into a corresponding row i of a 2-D set of intermediate coefficients $\tilde{u}_M = \{\tilde{u}_{M,k_M,i}\}$. Similarly, the second pre-decomposition filter stage **142** has the transfer function $\phi(z_2)^{-1}\alpha(z_2)$ for mapping in the vertical direction each column k_M of the 2-D set of intermediate coefficients $\tilde{u}_M = \{\tilde{u}_{M,k_M,i}\}$ into a corresponding column k_M of the 2-D set of dual scaling function coefficients $\tilde{c}_M = \{\tilde{c}_{M,k_M,j_M}\}$. This is done by these two pre-decomposition filter stages **142** in the manner described earlier for the 1-D IWT.

Referring back to FIG. **17**, the wavelet transform system **230** further includes a decomposition filter **272** to implement the 2-D IWT. The decomposition filter **272** decomposes the set of dual scaling function coefficients \tilde{c}_M into the sets of dual scaling function and wavelet coefficients \tilde{c}_N , \tilde{d}_{M-1}^1 to \tilde{d}_N^1 , \tilde{d}_{M-1}^2 to \tilde{d}_N^2 , and \tilde{d}_{M-1}^3 to \tilde{d}_N^3 . In order to do so, the decomposition filter **272** includes a corresponding decomposition filter stage **273** for the last resolution level $m=N+1$ at which a decomposition is made and a decomposition filter stage **275** for every other resolution level $m=M$ to $N+2$ at which a decomposition is made.

Each decomposition filter stage **273** and **275** includes a decomposition filter substage **274** for the corresponding resolution level m which has the transfer functions $P(z_1)P(z_2)$, $P(z_1)Q(z_2)$, $Q(z_1)P(z_2)$, and $Q(z_1)Q(z_2)$. This enables the decomposition filter substage **274** to decompose the set of dual scaling function coefficients \tilde{c}_m in the LL frequency band at the higher resolution level m into the sets of dual wavelet and scaling function coefficients \tilde{c}_{m-1} , \tilde{d}_{m-1}^1 , \tilde{d}_{m-1}^2 , and \tilde{d}_{m-1}^3 in the LL, LH, HL, and HH frequency bands at the next lower resolution level $m-1$.

More specifically, as shown in FIG. **19**, the decomposition filter substage **274** for each resolution level m includes three decomposition filter substages **148**. Each decomposition filter substage **148** is the same decomposition filter stage **148** as that shown in FIG. **7** for the 1-D IWT.

The first decomposition filter substage **148** has the transfer functions $P(z_1)$ and $Q(z_1)$. This allows this decomposition filter substage **148** to decompose in the horizontal direction each row j_m of the 2-D set of dual scaling function coefficients $\tilde{c}_m = \{\tilde{c}_{m,k_m,j_m}\}$ into a corresponding row j_m of the 2-D set of intermediate coefficients $\tilde{s}_{m-1} = \{\tilde{s}_{m-1,k_{m-1},j_m}\}$ and a corresponding row j_m of the 2-D set of intermediate coef-

coefficients $\tilde{t}_{m-1}=\{\tilde{t}_{m-1,k_{m-1}j_m}\}$. This is done in the manner described earlier for the 1-D IWT.

The second and third decomposition filter substages **148** each have the transfer functions $P(z_2)$ and $Q(z_2)$. Using the transfer functions $P(z_2)$ and $Q(z_2)$, the second decomposition filter substage **148** decomposes in the vertical direction each column k_{m-1} of the 2-D set of intermediate coefficients $\tilde{s}_{m-1}=\{\tilde{s}_{m-1,k_{m-1}j_m}\}$ into a corresponding column k_{m-1} of the 2-D set of dual scaling function coefficients $\tilde{c}_{m-1}=\{\tilde{c}_{m-1,k_{m-1}j_{m-1}}\}$ and a corresponding column k_{m-1} of the 2-D set of dual wavelets coefficients $\tilde{d}_{m-1}^1=\{\tilde{d}_{m-1,k_{m-1}j_{m-1}}^1\}$. Similarly, the third decomposition filter substage **148** uses the transfer functions $P(z_2)$ and $Q(z_2)$ to decompose in the vertical direction each column k_{m-1} of the 2-D set of intermediate coefficients $\tilde{t}_{m-1}=\{\tilde{t}_{m-1,k_{m-1}j_m}\}$ into a corresponding column k_{m-1} of the 2-D set of dual wavelet coefficients $\tilde{d}_{m-1}^2=\{\tilde{d}_{m-1,k_{m-1}j_{m-1}}^2\}$ and a corresponding column k_{m-1} of the 2-D set of dual wavelets coefficients $\tilde{d}_{m-1}^3=\{\tilde{d}_{m-1,k_{m-1}j_{m-1}}^3\}$. This is done by these two decomposition filter substages **148** in the manner described earlier for the 1-D IWT.

2-D Inverse Wavelet Transform Using S.O. Standard Wavelets $\{\psi_{m,k_m}(x)\}$ and $\{\psi_{m,j_m}(y)\}$ as Basic Wavelets

Turning to FIG. **20**, the inverse wavelet transform system **240** uses a 2-D inverse IWT that corresponds to the 2-D IWT implemented by the wavelet transform system **230** of FIG. **17**. Here, the standard scaling functions $\{\phi_{m,k_m}(x)\}$ and $\{\phi_{m,j_m}(y)\}$ and the s.o. standard wavelets $\{\psi_{m,k_m}(x)\}$ and $\{\psi_{m,j_m}(y)\}$ that were used as the basic wavelets in the 2-D IWT are also used as the basic scaling functions and wavelets in the 2-D inverse IWT. Thus, the inverse wavelet transform system **240** is used to reconstruct sets of dual scaling function and wavelet coefficients $\tilde{c}_N, \tilde{d}_{M-1}^1$ to $\tilde{d}_N^1, \tilde{d}_{M-1}^2$ to \tilde{d}_N^2 , and \tilde{d}_{M-1}^3 to \tilde{d}_N^3 into a set of reconstructed data samples f_M .

In order to implement the 2-D inverse IWT, the inverse wavelet transform system **240** includes a reconstruction filter **250**. The reconstruction filter **250** reconstructs the sets of dual scaling function and wavelet coefficients $\tilde{c}_N, \tilde{d}_{M-1}^1$ to $\tilde{d}_N^1, \tilde{d}_{M-1}^2$ to \tilde{d}_N^2 , and \tilde{d}_{M-1}^3 to \tilde{d}_N^3 into the set of dual scaling function coefficients \tilde{c}_M . To do so, the reconstruction filter **250** includes a corresponding reconstruction filter stage **253** for the first resolution level $m=N$ at which a reconstruction is made and a corresponding reconstruction filter stage **252** for every other resolution level $m=N$ to $M-1$ at which a reconstruction is made.

Each reconstruction filter stage **252** and **253** includes a reconstruction filter substage **254** for the corresponding resolution level m . This reconstruction filter substage **254** has the transfer functions $Q^*(z_1)\alpha(z_1)Q^*(z_2)\alpha(z_2)$, $Q^*(z_1)\alpha(z_1)P^*(z_2)\alpha(z_2)$, $P^*(z_1)\alpha(z_1)Q^*(z_2)\alpha(z_2)$, and $P^*(z_1)\alpha(z_1)P^*(z_2)\alpha(z_2)$. As a result, the reconstruction filter substage **254** reconstructs the sets of dual wavelet and scaling function coefficients $\tilde{c}_m, \tilde{d}_m^1, \tilde{d}_m^2$, and \tilde{d}_m^3 in the LL, LH, HL, and HH frequency bands at the lower resolution level m into the set of dual scaling function coefficients \tilde{c}_{m+1} in the LL frequency band at the next higher resolution level $m+1$.

For doing so, the reconstruction filter substage **254** includes three reconstruction filter substages **246**, as shown in FIG. **21**. Each reconstruction filter substage **246** is the same reconstruction filter substage **246** as that shown in FIG. **15** for the 1-D inverse IWT.

The first and second reconstruction filter substages **246** each have the transfer functions $Q^*(z_2)\alpha(z_2)$ and $P^*(z_2)\alpha(z_2)$.

Using the transfer functions $Q^*(z_2)\alpha(z_2)$ and $P^*(z_2)\alpha(z_2)$, the first reconstruction filter substage **246** reconstructs in the vertical direction each column k_m of a 2-D set of intermediate coefficients $\tilde{s}_{m+1}=\{\tilde{s}_{m+1,k_mj_{m+1}}\}$ from a corresponding column k_m of the 2-D set of dual scaling function coefficients $\tilde{c}_m=\{\tilde{c}_{m,k_mj_m}\}$ and a corresponding column k_m of the 2-D set of dual wavelet coefficients $\tilde{d}_m^1=\{\tilde{d}_{m,k_mj_m}^1\}$. Similarly, the second reconstruction filter substage **246** uses the transfer functions $Q^*(z_2)\alpha(z_2)$ and $P^*(z_2)\alpha(z_2)$ to reconstruct in the vertical direction each column k_m of a 2-D set of intermediate coefficients $\tilde{t}_m=\{\tilde{t}_{m+1,k_mj_{m+1}}\}$ from a corresponding column k_m of the 2-D set of dual wavelet coefficients $\tilde{d}_m^2=\{\tilde{d}_{m,k_mj_m}^2\}$ and a corresponding column k_m of the set of dual wavelet coefficients $\tilde{d}_m^3=\{\tilde{d}_{m,k_mj_m}^3\}$. This is done by these reconstruction filter substages **246** in the same manner as described earlier for the 1-D inverse IWT.

Furthermore, with the transfer functions $Q^*(z_1)\alpha(z_1)$ and $P^*(z_1)\alpha(z_1)$, the third reconstruction filter substage **246** reconstructs in the horizontal direction each row j_{m+1} of the 2-D set of dual scaling function coefficients $\tilde{c}_{m+1}=\{\tilde{c}_{m+1,k_{m+1}j_{m+1}}\}$ from a corresponding row j_{m+1} of the 2-D set of intermediate coefficients $\tilde{s}_{m+1}=\{\tilde{s}_{m+1,k_mj_{m+1}}\}$ and a corresponding row j_{m+1} of the 2-D set of intermediate coefficients $\tilde{t}_{m+1}=\{\tilde{t}_{m+1,k_mj_{m+1}}\}$. This is also done in the same manner as described earlier for the 1-D inverse IWT.

Referring back to FIG. **20**, the post-reconstruction filter **256** of the inverse wavelet transform system **240** has the transfer function $\phi(z_1)\alpha(z_1)^{-1}\phi(z_2)\alpha(z_2)^{-1}$ for mapping the set of dual scaling function coefficients \tilde{c}_M into the set of reconstructed data samples f_M . In order to do this, the post-reconstruction filter **256** includes two post-reconstruction filter stages **248**, as shown in FIG. **22**. Each post-reconstruction filter stage **248** is the same post-reconstruction filter **248** as that shown in FIG. **16** for the 1-D inverse IWT.

The first post-reconstruction filter stage **248** has the transfer function $\phi(z_2)\alpha(z_2)^{-1}$ for mapping in the vertical direction each column k_M of the 2-D set of dual scaling function coefficients $\tilde{c}_M=\{\tilde{c}_{M,k_Mj_M}\}$ into a corresponding column k_M of the 2-D set of intermediate coefficients $\tilde{u}_M=\{\tilde{u}_{M,k_M,i}\}$. Similarly, the second post-reconstruction filter stage **248** uses the transfer function $\phi(z_1)\alpha(z_1)^{-1}$ to map in the horizontal direction each row i of the 2-D set of intermediate coefficients $\tilde{u}_M=\{\tilde{u}_{M,k_M,i}\}$ into a corresponding row i of the 2-D set of reconstructed data samples $f_M=\{f_{M,n,i}\}$. This is done by these two post-reconstruction filter stages **248** in the same manner as described earlier for the 1-D inverse IWT.

Specific Implementation of Wavelet and Inverse Wavelet Transform Systems

In a specific implementation of the wavelet transform system **230** of FIGS. **13** and **17** and the inverse wavelet transform system **240** of FIGS. **14** and **20**, the scaling function $\phi_{0,0}(x)$ may be a cubic spline. In this case, the sequence of mapping coefficients $\{\phi_n\}$ is given by:

$$\begin{aligned} \phi_n &= 0, \\ \text{for } n &\leq -2 \text{ and } n \geq 2 \\ \phi_{-1} &= 1/6 \\ \phi_0 &= 4/6 \\ \phi_1 &= 1/6 \end{aligned} \quad (22)$$

Thus, the mapping filter stage **136** of FIG. **15** performs a moving average operation according to:

$$y_k = \dots + \epsilon_{-1}x_{k+1} + \epsilon_0x_k + \epsilon_1x_{k-1} + \dots \quad (23)$$

where the set of inputs $\{x_k\}$, the set of outputs $\{y_k\}$, and the sequence of taps $\{\epsilon_k\}$ are respectively the set of standard scaling function coefficients $\{c_{M,k_M}\}$, the set of original data samples $\{f_{M,k_M}\}$, and the sequence of mapping coefficients $\{\phi_{k_M}\}$.

Conversely, the sequence of mapping coefficients $\{\Theta_n\}$ corresponding to the sequence of mapping coefficients $\{\phi_n\}$ is given by:

$$\Theta_n = 0,$$

for $n \leq -2$ and $n \geq 2$

$$\Theta_0 = 1/6(2 + \sqrt{3})$$

$$\Theta_1 = \Theta_{-1} = 2 - \sqrt{3} \quad (24)$$

In this case, the mapping filter stage **102** of FIG. **6** performs a recursive two-stage feedback differencing operation as follows:

$$y_k = x_k - \mu_1 y_{k-1} - \dots \quad (25)$$

$$w_k = y_k - \mu_{-1} w_{k+1} - \dots,$$

$$z_k = \frac{w_k}{\mu_0}$$

where the set of inputs $\{x_k\}$, the set of outputs $\{z_k\}$, and the sequence of taps $\{\mu_k\}$ are respectively the set of original data samples $\{f_{M,k_M}\}$, the set of standard scaling function coefficients $\{c_{M,k_M}\}$, and the sequence of mapping coefficients $\{\Theta_{k_M}\}$.

Furthermore, in this specific implementation, the sequence of mapping coefficients $\{\alpha_n\}$ is given by:

$$\alpha_n = 0,$$

for $n \leq -5$ and $n \geq 5$

$$\alpha_{-4} = \alpha_4 = 1/5040$$

$$\alpha_{-3} = \alpha_3 = 12/5040$$

$$\alpha_{-2} = \alpha_2 = 1191/5040$$

$$\alpha_{-1} = \alpha_1 = 2416/5040 \quad (26)$$

Thus, the mapping filter stage **144** of FIG. **6** performs a moving average operation according to Eq. (23) where the set of inputs $\{x_k\}$, the set of outputs $\{y_k\}$, and the sequence of taps $\{\epsilon_k\}$ are respectively the set of standard scaling function coefficients $\{c_{m,k_m}\}$, the set of dual scaling function coefficients $\{\tilde{c}_{m,k_m}\}$, and the sequence of mapping coefficients $\{\alpha_{k_m}\}$.

Similarly, the sequence of mapping coefficients $\{\Delta_n\}$ corresponding to the sequence of mapping coefficients $\{\alpha_n\}$ is given by:

$$\Delta_n = 0,$$

for $n \leq -4$ and $n \geq 4$

$$\Delta_0 = 5.289556603818$$

$$\Delta_1 = \Delta_{-1} = 0.666983740798$$

$$\Delta_2 = \Delta_{-2} = 0.071619419287$$

$$\Delta_3 = \Delta_{-3} = 0.000600164327 \quad (27)$$

The mapping filter substages **166** shown in FIGS. **15** and the mapping filter stage **166** shown in FIG. **16** each perform a recursive two-stage feedback differencing operation according to Eq. (25). Thus, for the mapping filter substages **166** of FIG. **15**, the sets of inputs $\{x_k\}$ are respectively the sets of dual scaling function and wavelet coefficients $\{\tilde{c}_{m,k_m}\}$ and $\{\tilde{d}_{m,k_m}\}$, the sets of outputs $\{z_k\}$ are respectively the sets of standard scaling function and formatted wavelet coefficients $\{c_{m,k_m}\}$ and $\{\tilde{d}_{m,k_m}\}$, and the sequence of taps $\{\mu_k\}$ is the sequence of mapping coefficients $\{\Delta_{k_m}\}$. Finally, for the mapping filter stage **166** of FIG. **16**, the set of inputs $\{x_k\}$ is the set of dual scaling function coefficients $\{\tilde{c}_{M,k_M}\}$, the set of outputs $\{z_k\}$ is the set of standard scaling function coefficients $\{c_{M,k_M}\}$, and the sequence of taps $\{\mu_k\}$ is the sequence of mapping coefficients $\{\Delta_{k_M}\}$.

Additionally, the sequence of decomposition coefficients $\{p_n\}$ is given by:

$$p_n = 0,$$

for $n \leq -3$ and $n \geq 3$

$$p_{-2} = p_2 = 1/(8\sqrt{2})$$

$$p_{-1} = p_1 = 4/(8\sqrt{2})$$

$$p_0 = 6/(8\sqrt{2}) \quad (28)$$

As a result, the filter **130** shown in FIG. **7** performs a moving average operation according to Eq. (23) where the set of inputs $\{x_k\}$, the set of outputs $\{y_k\}$, and the sequence of taps $\{\epsilon_k\}$ are respectively the set of dual scaling function coefficients $\{\tilde{c}_{m,k_m}\}$, the set of intermediate coefficients $\{\tilde{c}_{m-1,k_m}\}$, and the sequence of decomposition coefficients $\{p_{k_m}\}$. But, the sequence of reconstruction coefficients $\{p_n^*\}$ **56** is obtained from Eq. (28) using Eq. (17). Thus, the filter **247** shown in FIG. **15** also performs a moving average operation according to Eq. (23) but where the set of inputs $\{x_k\}$, the set of outputs $\{y_k\}$, and the sequence of taps $\{\epsilon_k\}$ are respectively the set of formatted wavelet coefficients $\{\tilde{d}_{m,k_{m+1}}\}$, the set of intermediate coefficients $\{\tilde{z}_{m,k_{m+1}}\}$, and the sequence of reconstruction coefficients $\{p_{k_{m+1}}^*\}$.

Finally, the sequence of decomposition coefficients $\{q_n\}$ is given by:

$$q_n = 0,$$

for $N \leq -5$ and $n \geq 7$

$$q_{-4} = q_6 = 1/(5040\sqrt{2})$$

$$q_{-3} = q_5 = -124/(5040\sqrt{2})$$

$$q_{-2} = q_4 = 1677/(5040\sqrt{2})$$

$$q_{-1} = q_3 = -7904/(5040\sqrt{2})$$

$$q_0 = q_2 = 18482/(5040\sqrt{2})$$

$$q_1 = -24264/(5040\sqrt{2}) \quad (29)$$

The filter **132** of FIG. **7** performs a moving average operation according to Eq. (23) where the set of inputs $\{x_k\}$, the set of outputs $\{y_k\}$, and the sequence of taps $\{\epsilon_k\}$ are respectively the set of dual wavelet coefficients $\{\tilde{d}_{m,k_m}\}$, the set of intermediate coefficients $\{\tilde{d}_{m-1,k_m}\}$, and the sequence of decomposition coefficients $\{q_{k_m}\}$. Similarly, the filter **249** of FIG. **15** also performs a moving average operation

according to Eq. (23) but with the sequence of reconstruction coefficients $\{q_n^*\}$ obtained from Eq. (29) using Eq. (17). In this case, the set of inputs $\{x_k\}$, the set of outputs $\{y_k\}$, and the sequence of taps $\{\epsilon_k\}$ are respectively the set of standard scaling function coefficients $\{c_{m,k_{m+1}}\}$, the set of intermediate coefficients $\{\tilde{y}_{m,k_{m+1}}\}$, and the sequence of reconstruction coefficients $\{q_{k_{m+1}}\}$.

First Alternative Embodiment for Pre- Decomposition Filter

Turning now to FIG. 23, there is shown an alternative embodiment for the pre-decomposition filter 142 of FIG. 6. In this case, the pre-decomposition filter 142 has the transfer function $\Omega(z)+\Lambda(z)$ which is equivalent to the transfer function $\phi(z)^{-1}\alpha(z)$ for the embodiment in FIG. 6.

In the embodiment of FIG. 23, the pre-decomposition filter 142 includes a mapping filter stage 260 and an mapping filter stage 262 that operate in parallel to each other on the set of original data samples f_M . The mapping filter stages 260 and 262 are respectively FIR and IIR filters. The transfer function $\Omega(z)$ of the mapping filter stage 260 is an all-zeros polynomial (i.e., it has no poles) with a sequence of mapping coefficients $\{\Omega_n\}$ as its coefficients. The transfer function $\Lambda(z)$ of the mapping filter stage 262 is an all-poles rational function (i.e., it has no zeros) that has a sequence of mapping coefficients $\{\Lambda_n\}$ as its poles. The sequences of mapping coefficients $\{\Omega_n\}$ and $\{\Lambda_n\}$ are obtained in the manner described shortly.

The mapping filters stages 260 and 262 respectively perform the transfer functions $\Omega(z)$ and $\Lambda(z)$ by respectively applying the sequences of mapping coefficients $\{\Omega_n\}$ and $\{\Lambda_n\}$ to the set of original data samples $f_M=\{f_{M,n}\}$ to respectively generate 1-D sets of intermediate coefficients $\{I_{M,k_M}\}$ and $\{O_{M,k_M}\}$. The sets of intermediate coefficients $\{I_{M,k_M}\}$ and $\{O_{M,k_M}\}$ are then component-wise summed by a summing stage 264 to obtain the 1-D set of dual scaling function coefficients $\tilde{c}_M=\{\tilde{c}_{M,k_M}\}$.

In the pre-decomposition filter 142 of the embodiment of FIG. 6, the IIR mapping filter stage 102 with transfer function $\phi(z)^{-1}$ and the FIR mapping filter stage 144 with transfer function $\alpha(z)$ are cascaded and used for the same purpose as just described. More specifically, the combined transfer function $\phi(z)^{-1}\alpha(z)$ of the mapping filter stages 102 and 144 and the combined transfer function $\Omega(z)+\Lambda(z)$ of the mapping filter stages 260 and 262 and the summer 264 are equal and have the same effect. This effect includes obtaining the set of standard scaling function coefficients $\{c_{M,k_M}\}$ by ensuring that the function $f_M(x)$ goes through the set of original data samples $\{f_{M,n}\}$. Furthermore, this effect includes mapping the set of standard scaling function coefficients $\{c_{M,k_M}\}$ to the set of dual scaling function coefficients $\{\tilde{c}_{M,k_M}\}$ so that a change of bases is made with the standard scaling functions $\{\phi_{m,k_m}(x)\}$ and s.o. standard wavelets $\{\psi_{m,k_m}(x)\}$ being used as the basic scaling functions and wavelets instead of the dual scaling functions and wavelets $\{\phi_{m,k_m}(x)\}$ and $\{\psi_{m,k_m}(x)\}$.

However, by using the transfer functions $\Lambda(z)$ and $\Omega(z)$ rather than the transfer functions $\phi(z)^{-1}$ and $\alpha(z)$, the complexity of the pre-decomposition filter 142 is much less. Specifically, the transfer function $\Lambda(z)$ has only poles and is of the same degree and proportional to the transfer function $\phi(z)^{-1}$. The transfer function $\Omega(z)$ has only zeros and has a degree less than the degree of the transfer function $\alpha(z)$. Thus, the pre-decomposition filter 142 of the embodiment of FIG. 23 is easier to implement and more efficient than that of the embodiment of FIG. 6.

For example, since the combined transfer function $\Omega(z)+\Lambda(z)$ is equal to the combined transfer function $\phi(z)^{-1}\alpha(z)$, the sequences of coefficients $\{\Omega_n\}$ and $\{\Lambda_n\}$ may be obtained using the sequences of mapping coefficients $\{\Theta_n\}$ and $\{\alpha_n\}$. Thus, for the specific implementation of the wavelet transform system 230 and the inverse wavelets transform system 240 discussed earlier, the sequences of mapping coefficients $\{\Theta_n\}$ and $\{\alpha_n\}$ given by Eqs. (23) and (25) are used to obtain the sequences of coefficients $\{\Omega_n\}$ and $\{\Lambda_n\}$ such that:

$$\begin{aligned} \Omega_n &= 0, \\ \text{for } n &\leq -3 \text{ and } n \geq 3 \\ \Omega_{-2} &= \Omega_2 = 1/840 \\ \Omega_{-1} &= \Omega_1 = 116/840 \\ \Omega_0 &= 726/840 \\ \Lambda_n &= 0, \\ \text{for } n &\leq -2 \text{ and } n \geq 2 \\ \Lambda_0 &= -(8+7\sqrt{3})/6 \\ \Lambda_{-1} &= \Lambda_1 = -2+\sqrt{3} \end{aligned} \quad (30)$$

In this case, the complete transfer function $\Omega(z)+\Lambda(z)$ is the simple sum of a polynomial with five coefficients in the numerator and no poles and a rational function with 3 poles in the denominator and no zeros. However, with the sequences of mapping coefficients $\{\Theta_n\}$ and $\{\alpha_n\}$ given by Eqs. (23) and (25), the complete transfer function $\phi(z)^{-1}\alpha(z)$ of the pre-decomposition filter 142 of the embodiment of FIG. 6 is a much more complex rational function with seven coefficients in the numerator and three poles in the denominator.

Finally, the transfer function $\Omega(z)$ is performed by the mapping filter stage 260 with a moving average operation according to Eq. (23) where the set of inputs $\{x_k\}$, the set of outputs $\{y_k\}$, and the sequence of taps $\{\epsilon_k\}$ are respectively the set of original data samples $\{f_{M,k_M}\}$, the set of intermediate coefficients $\{I_{M,k_M}\}$, and the sequence of mapping coefficients $\{\Omega_{k_M}\}$. Similarly, the transfer function $\Lambda(z)$ is performed by the mapping filter stage 260 with a recursive two stage feedback differencing operation according to Eq. (25) where the set of inputs $\{x_k\}$, the set of outputs $\{z_k\}$, and the sequence of taps $\{\mu_k\}$ are respectively the set of original data samples $\{f_{M,k_M}\}$, the set of intermediate coefficients $\{O_{M,k_M}\}$, and the sequence of mapping coefficients $\{\Lambda_{k_M}\}$.

Second Alternative Embodiment for Pre- Decomposition Filter

FIG. 24 shows another embodiment of the pre-decomposition filter 142. In this embodiment, the pre-decomposition filter 142 is an FIR filter and has the transfer function $\lambda\phi(z)$ for mapping the set of original data samples $f_M=\{f_{M,n}\}$ into the set of dual scaling function coefficients $\tilde{c}_M=\{\tilde{c}_{M,k_M}\}$. In doing so, the transfer function $\lambda\phi(z)$ provides an orthogonal projection of the function $f(x)$ to the function $f_M(x)$ and a change of bases. As indicated earlier, this change of bases results in the standard scaling functions $\{\phi_{m,k_m}(x)\}$ and the s.o. standard wavelets $\{\psi_{m,k_m}(x)\}$ being used as the basic scaling functions and wavelets rather than the dual scaling functions and wavelets $\{\phi_{m,k_m}(x)\}$ and $\{\psi_{m,k_m}(x)\}$.

More specifically, the orthogonal projection of the function $f(x)$ to a function $f_m(x)$ that approximates the function $f(x)$ at the resolution level m is defined as:

$$\int_{-\infty}^{\infty} f(x)\phi_{m,k_m}(x)dx = \int_{-\infty}^{\infty} f_m(x)\phi_{m,k_m}(x)dx \quad (31)$$

Using the strategy of optimal polynomial exactness, Eq. (31) can be re-written as follows:

$$\int_{-\infty}^{\infty} f(x)\phi_{m,k_m}(x)dx = \sum_n \lambda_n \phi_n f_{m,k_m-n} \quad (32)$$

Here, the 1-D sequence of weighting coefficients $\{\lambda_n\}$ is selected for the highest degree that the function $f(x)$ can have for which Eq. (32) will be exact.

As alluded to earlier, when the standard scaling functions $\{\phi_{m,k_m}(x)\}$ are used as the basic scaling functions in the 1-D IWT, the set of standard scaling function coefficients $\{c_{m,k_m}\}$ is switched with the set of dual scaling function coefficients $\{\tilde{c}_{m,k_m}\}$ and the dual scaling functions $\{\phi_{m,k_m}(x)\}$ are switched with the standard scaling functions $\{\phi_{m,k_m}(x)\}$ in Eq. (6). Thus, in view of Eqs. (6) and (32) for this case, each dual scaling function coefficient \tilde{c}_{m,k_m} is given by:

$$\tilde{c}_{m,k_m} = \sum_n \lambda_n \phi_n f_{m,k_m-n} \quad (33)$$

Thus, for $m=M$, the set of dual scaling function coefficients $\tilde{c}_M = \{\tilde{c}_{M,k_M}\}$ can be obtained according to Eq. (33).

In order to do so, the z -transform of Eq. (33), for $m=M$, is taken so that the pre-decomposition filter **142** has the transfer function $\lambda\phi(z)$. Here, the transfer function $\lambda\phi(z)$ is a polynomial that has the sequence of mapping coefficients $\{\lambda_n\}$ as its coefficients. Thus, the pre-decomposition filter **142** performs the transfer function $\lambda\phi(z)$ by applying the sequence of mapping coefficients $\{\lambda_n\}$ to the set of original data samples $f_M = \{f_{M,n}\}$ to generate the set of dual scaling function coefficients $c_M = \{c_{M,k_M}\}$.

Furthermore, in the specific implementation of the wavelet transform system **230** and inverse wavelet transform system **240** discussed earlier, the scaling function $\phi_{0,0}(x)$ is a cubic spline and the sequence of mapping coefficients $\{\phi_n\}$ are given by Eq. (22). In this case, the sequence of weighting coefficients $\{\lambda_n\}$ is given by:

$$\lambda_n = 0,$$

for $n \leq -2$ and $n \geq 2$

$$\lambda_{-1} = \lambda_0 = \lambda_1 = 1 \quad (34)$$

and is selected so that Eq. (32) is exact for when the function $f(x)$ has a degree that is not greater than four. Here, the pre-decomposition filter **142** performs a moving average operation according to Eq. (23) where the set of inputs $\{x_k\}$, the set of outputs $\{y_k\}$, and the sequence of taps $\{\epsilon_k\}$ are respectively the set of original data samples $\{f_{M,k_M}\}$, the set of dual scaling function coefficients $\{\tilde{c}_{M,k_M}\}$, and the sequence of mapping coefficients $\{\lambda_{k_M}\phi_{k_M}\}$.

Alternative Embodiment for Post-Reconstruction Filter

FIG. **25** shows another embodiment of the post-reconstruction filter **248**. In this embodiment, the post-

reconstruction filter **248** is an IIR filter and has the transfer function $\lambda\phi(z)^{-1}$. The transfer function $\lambda\phi(z)^{-1}$ is the inverse of the transfer function $\lambda\phi(z)$ of the pre-decomposition filter **142** in the embodiment of FIG. **24**. Thus, the post-reconstruction filter **248** is used in the inverse wavelet transform system **240** when the pre-decomposition filter **142** of the embodiment of FIG. **24** is used in the wavelet transform system **230**.

Here, the transfer function $\lambda\phi(z)^{-1}$ is used to map the set of dual scaling function coefficients $\tilde{c}_M = \{\tilde{c}_{M,k_M}\}$ into the set of original data samples $f_M = \{f_{M,n}\}$. The transfer function $\lambda\phi(z)^{-1}$ is a rational function that has a sequence of mapping coefficients $\{\omega_n\}$ as its poles. Thus, the post-reconstruction filter **248** performs the transfer function $\lambda\phi(z)^{-1}$ by applying the sequence of mapping coefficients $\{\omega_n\}$ to the set of original dual scaling function coefficients $\tilde{c}_M = \{\tilde{c}_{M,k_M}\}$ to generate the set of original data samples $f_M = \{f_{M,n}\}$.

Additionally, in the specific implementation of the wavelet transform system **230** and inverse wavelet transform system **240** discussed earlier where the sequences of mapping and weighting coefficients $\{\phi_n\}$ and $\{\lambda_n\}$ are given in Eqs. (21) and (33), the sequence of mapping coefficients $\{\omega_n\}$ is the same as the sequence of mapping coefficients $\{\Theta_n\}$ given in Eq. (24). Thus, the post-reconstruction filter **248** performs a recursive two-stage feedback differencing operation according to Eq. (25) where the set of inputs $\{x_k\}$, the set of outputs $\{z_k\}$, and the sequence of taps $\{\mu_k\}$ are respectively the set of dual scaling function coefficients $\{\tilde{c}_{M,k_M}\}$, the set of original data samples $\{f_{M,k_M}\}$, and the sequence of mapping coefficients $\{\omega_{k_M}\}$.

Alternative Embodiment for Decomposition Filter Substage

Turning now to FIG. **26**, there is shown another embodiment for the decomposition filter substage **148** for each resolution level m in the wavelet transform system **230**. Like the embodiment of the decomposition filter substage **148** of FIG. **7**, the decomposition filter substage **148** in this embodiment has the transfer functions $P(z)$ and $Q(z)$ for decomposing the set of dual scaling function coefficients \tilde{c}_m at the higher resolution level m into the sets of dual scaling function and wavelet coefficients \tilde{c}_{m-1} and \tilde{d}_{m-1} at the next lower resolution level $m-1$. However, as will be described next, these transfer functions $P(z)$ and $Q(z)$ are implemented in a different and novel manner.

Specifically, the decomposition filter substage **148** has a deinterleaver **280**. The deinterleaver **280** deinterleaves the 1-D set of dual scaling function coefficients $\tilde{c}_m = \{\tilde{c}_{m,k_m}\}$ into a 1-D subset of dual scaling function coefficients $\{\tilde{c}_{m,2k_{m-1}}\}$ with even indexes $k_m = 2k_{m-1}$ and a 1-D subset of dual scaling function coefficients $\{\tilde{c}_{m,2k_{m-1}+1}\}$ with odd indexes $k_m = 2k_{m-1}+1$. The deinterleaver may be a demultiplexer or hardwired connections.

The decomposition filter substage **148** also has FIR filters **282** and **284** that respectively have the transfer functions $P_e(z)$ and $P_o(z)$. The transfer functions $P_e(z)$ and $P_o(z)$ are polynomials that respectively have the subsequences of decomposition coefficients $\{p_{2i}\}$ and $\{p_{2i+1}\}$ as coefficients. Here, the subsequences of decomposition coefficients $\{p_{2i}\}$ and $\{p_{2i+1}\}$ are subsequences of the sequence of decomposition coefficients $\{p_n\}$ that respectively have even indexes $n=2i$ and odd indexes $n=2i+1$. Thus, the transfer functions $P_e(z)$ and $P_o(z)$ are performed by respectively applying the subsequences of decomposition coefficients $\{p_{2i}\}$ and $\{p_{2i+1}\}$ to the subsets of dual scaling function coefficients $\{\tilde{c}_{m,2k_{m-1}}\}$ and $\{\tilde{c}_{m,2k_{m-1}+1}\}$ to respectively generate the sets of intermediate coefficients $\{\hat{y}_{m-1,2k_{m-1}}\}$ and $\{\hat{v}_{m-1,2k_{m-1}+1}\}$.

Similarly, the decomposition filter substage **148** also includes FIR filter stages **286** and **288** that respectively have the transfer functions $Q_e(z)$ and $Q_o(z)$. Like the sequence of decomposition coefficients $\{p_n\}$, the sequence of decomposition coefficients $\{q_n\}$ has subsequences of decomposition coefficients $\{q_{2i}\}$ and $\{q_{2i+1}\}$ that respectively have even indexes $n=2i$ and odd indexes $n=2i+1$. Thus, the transfer functions $Q_e(z)$ and $Q_o(z)$ are polynomials that respectively have the subsequences of decomposition coefficients $\{q_{2i}\}$ and $\{q_{2i+1}\}$ as coefficients. In order to perform the transfer functions $Q_e(z)$ and $Q_o(z)$, the subsequences of decomposition coefficients $\{q_{2i}\}$ and $\{q_{2i+1}\}$ are respectively applied to the subsets of dual scaling function coefficients $\{\tilde{c}_{m,2k_{m-1}+1}\}$ and $\{\tilde{c}_{m,2k_{m-1}}\}$ to respectively generate the sets of intermediate coefficients $\{\hat{w}_{m-1,2k_{m-1}+1}\}$ and $\{\hat{z}_{m-1,2k_{m-1}}\}$.

The decomposition filter stage **148** further includes two summers **290**. The sets of intermediate coefficients $\{\hat{y}_{m-1,2k_{m-1}}\}$ and $\{\hat{v}_{m-1,2k_{m-1}}\}$ are then component-wise summed together by one summer **289** to obtain the set of dual scaling function coefficients $\tilde{c}_{m-1}=\{\tilde{c}_{m-1,k_{m-1}}\}$. The sets of intermediate coefficients $\{\hat{w}_{m-1,2k_{m-1}+1}\}$ and $\{\hat{z}_{m-1,2k_{m-1}}\}$ are then component-wise summed together by the other summer **289** to obtain the set of dual wavelet coefficients $\tilde{d}_{m-1}=\{\tilde{d}_{m-1,k_{m-1}}\}$.

It should be noted here that, in the embodiment of FIG. 7 of the decomposition filter substage **148**, the transfer functions $P(z)$ and $Q(z)$ are performed by operating the entire sequences of decomposition coefficients $\{p_n\}$ and $\{q_n\}$ on the entire set of dual scaling function coefficients $\{\tilde{c}_{m,k_m}\}$ to generate the resulting sets of intermediate coefficients $\{\tilde{c}_{m-1,k_m}\}$ and $\{\tilde{d}_{m-1,k_m}\}$. Only then are the resulting sets of intermediate coefficients $\{\tilde{c}_{m-1,k_m}\}$ and $\{\tilde{d}_{m-1,k_m}\}$ down-sampled to generate the sets of dual scaling function and wavelet coefficients $\{\tilde{c}_{m-1,k_{m-1}}\}$ and $\{\tilde{d}_{m-1,k_{m-1}}\}$. This may be done so that those of the intermediate coefficients $\{\tilde{c}_{m-1,k_m}\}$ with even indexes $\{k_m\}$ are discarded and those of the intermediate coefficients $\{\tilde{c}_{m-1,k_m}\}$ with odd indexes $\{k_m\}$ are the set of dual scaling function coefficients $\{\tilde{c}_{m-1,k_{m-1}}\}$. And those of the intermediate coefficients $\{\tilde{d}_{m-1,k_m}\}$ with odd indexes $\{k_m\}$ would be discarded so that those of the intermediate coefficients $\{\tilde{d}_{m-1,k_m}\}$ with even indexes $\{k_m\}$ are the set of dual wavelet coefficients $\{\tilde{d}_{m-1,k_{m-1}}\}$. As a result, the operation of the even indexed decomposition coefficients $\{p_{2i}\}$ and the odd indexed decomposition coefficients $\{q_{2i+1}\}$ on the odd indexed dual scaling function coefficients $\{\tilde{c}_{m,2k_{m-1}+1}\}$ is wasteful and timing consuming. Furthermore, the operation of the odd indexed decomposition coefficients $\{p_{2i+1}\}$ and the even indexed decomposition coefficients $\{q_{2i}\}$ on the even indexed dual scaling function coefficients $\{\tilde{c}_{m,2k_{m-1}}\}$ is similarly wasteful and timing consuming.

But, in the embodiment of FIG. 26 of the decomposition filter substage **148**, only the subsequence of even indexed decomposition coefficients $\{p_{2i}\}$ and the subsequence of odd indexed decomposition coefficients $\{q_{2i+1}\}$ are operated on the subset of even indexed dual scaling function coefficients $\{\tilde{c}_{m,2k_{m-1}}\}$. Similarly, only the subsequence of odd indexed decomposition coefficients $\{p_{2i+1}\}$ and the subsequence of even indexed decomposition coefficients $\{q_{2i}\}$ are operated on the subset of odd indexed dual scaling function coefficients $\{\tilde{c}_{m,2k_{m-1}+1}\}$. Thus, the embodiment of FIG. 26 of the decomposition filter substage **148** is more efficient than that of the embodiment of FIG. 7.

Additionally, in the specific implementation of the wavelet transform system **230** and inverse wavelet transform

system **240** discussed earlier, the sequences of decomposition coefficients $\{p_n\}$ and $\{q_n\}$ are given in Eqs. (27) and (28). Thus, the FIR filters **282**, **284**, **286**, and **288** perform respective moving average operations according to Eq. (23). The filters **282** and **284** have sets of inputs $\{x_k\}$ that are respectively the subsets of dual scaling function coefficients $\{\tilde{c}_{m,2k_{m-1}}\}$ and $\{\tilde{c}_{m,2k_{m-1}+1}\}$, sets of outputs $\{y_k\}$ that are respectively the sets of intermediate coefficients $\{\hat{y}_{m-1,2k_{m-1}}\}$ and $\{\hat{v}_{m-1,2k_{m-1}+1}\}$, and sequences of taps $\{\epsilon_k\}$ that are respectively the subsequences of decomposition coefficients $\{p_{2k_{m-1}}\}$ and $\{p_{2k_{m-1}+1}\}$. Similarly, for the filters **288** and **286**, the sets of inputs $\{x_k\}$ are respectively the subsets of dual scaling function coefficients $\{\tilde{c}_{m,2k_{m-1}}\}$ and $\{\tilde{c}_{m,2k_{m-1}+1}\}$, the sets of outputs $\{y_k\}$ are respectively the sets of intermediate coefficients $\{\hat{z}_{m-1,2k_{m-1}}\}$ and $\{\hat{w}_{m-1,2k_{m-1}+1}\}$, and the sequences of taps $\{\epsilon_k\}$ are respectively the subsequences of decomposition coefficients $\{q_{2k_{m-1}+1}\}$ and $\{q_{2k_{m-1}}\}$.

As those skilled in the art will recognize, the concepts developed for the embodiment of FIG. 26 for the decomposition filter substage **148** are not limited to that just described. They may be applied to any decomposition filter stage that uses two parallel transfer functions to decompose a 1-D set of input coefficients in an L frequency band at a higher resolution level into two 1-D sets of output coefficients in respective H and L frequency bands at a next lower resolution level. For example, these concepts may be applied to the decomposition filter **104** shown in FIG. 2. Thus, in FIG. 26 for this example, the set of dual scaling function coefficients $\{\tilde{c}_{m,k_m}\}$ would be replaced by the set of standard scaling function coefficients $\{c_{m,k_m}\}$, the subsets of even and odd indexed dual scaling function coefficients $\{\tilde{c}_{m,2k_{m-1}}\}$ and $\{\tilde{c}_{m,2k_{m-1}+1}\}$ would be respectively replaced by the subsets of even and odd indexed standard scaling function coefficients $\{c_{m,2k_{m-1}}\}$ and $\{c_{m,2k_{m-1}+1}\}$, the transfer functions $P_e(z)$, $P_o(z)$, $Q_e(z)$, and $Q_o(z)$ would be respectively replaced by the transfer functions $A_e(z)$, $A_o(z)$, $B_e(z)$, and $B_o(z)$, and the sets of dual scaling function and wavelet coefficients $\{\tilde{c}_{m-1,k_{m-1}}\}$ and $\{\tilde{d}_{m-1,k_{m-1}}\}$ would be replaced by the sets of standard scaling function and wavelet coefficients $\{c_{m-1,k_{m-1}}\}$ and $\{d_{m-1,k_{m-1}}\}$.

Alternative Embodiment for Reconstruction Filter Stage

Turning now to FIG. 27, there is shown another embodiment for the reconstruction filter substage **246** for each resolution level m in the inverse wavelet transform system **240**. As described earlier for the reconstruction filter substage **246** of FIG. 15, the reconstruction filter substage **246** has the transfer functions $P^*(z)$ and $Q^*(z)$ for reconstructing the sets of formatted wavelet and standard scaling function coefficients $\{\bar{d}_{m,k_m}\}$ and $\{c_{m,k_m}\}$ at the lower resolution level m into the set of dual scaling function coefficients $\tilde{c}_{m+1}=\{\tilde{c}_{m+1,k_{m+1}}\}$ at the next higher resolution level $m+1$. These transfer functions $P^*(z)$ and $Q^*(z)$ are implemented in a different and novel manner that is similar to the way in which these transfer functions were implemented in the decomposition filter substage **148** of FIG. 26.

Similar to the decomposition filter substage **148** of FIG. 26, the reconstruction filter substage **246** has FIR filters **281**, **283**, **285**, and **287** and summers **289**. The respective transfer functions $P_e^*(z)$ and $P_o^*(z)$ of the filters **281** and **283** are performed by respectively applying the subsequences of reconstruction coefficients $\{p_{2i}^*\}$ and $\{p_{2i+1}^*\}$ to the set of formatted wavelet coefficients $\{\bar{d}_{m,k_m}\}$ to respectively generate the sets of intermediate coefficients $\{\bar{y}_{m+1,k_m}\}$ and $\{\bar{z}_{m+1,k_m}\}$. Similarly, the respective transfer functions $Q_e^*(z)$

and $Q_o^*(z)$ of the filters **285** and **287** are performed by respectively applying the subsequences of reconstruction coefficients $\{q_{2i}^*\}$ and $\{q_{2i+1}^*\}$ to the set of standard scaling function coefficients $\{c_{m,k_m}\}$ to respectively generate the sets of intermediate coefficients $\{\bar{v}_{m+1,k_m}\}$ and $\{\bar{w}_{m+1,k_m}\}$.

The sets of intermediate coefficients $\{\bar{y}_{m+1,k_m}\}$ and $\{\bar{w}_{m+1,k_m}\}$ are summed by the first summer **289** to generate the set of intermediate coefficients $\{\bar{c}_{m+1,2k_m}\}$ with even indexes $2k_m$. Similarly, the sets of intermediate coefficients $\{\bar{v}_{m+1,k_m}\}$ and $\{\bar{z}_{m+1,k_m}\}$ are summed by the second summer **289** to generate the set of intermediate coefficients $\{\bar{c}_{m+1,2k_m+1}\}$ with odd indexes $2k_m+1$.

The reconstruction filter substage **248** also includes an interleaver **290**. The interleaver **290** interleaves the 1-D set of intermediate coefficients $\{\bar{c}_{m+1,2k_m}\}$ with even indexes $k_{m+1}=2k_m$ and the 1-D set of intermediate coefficients $\{\bar{c}_{m+1,2k_m+1}\}$ with odd indexes $k_{m+1}=2k_m+1$ into the 1-D set of dual scaling function coefficients $\tilde{c}_{m+1}=\{\tilde{c}_{m+1,k_{m+1}}\}$. The interleaver may be a multiplexer or hardwired connections.

However, in the embodiment of FIG. **15** of the reconstruction filter substage **246**, the sets of formatted wavelet and standard scaling function coefficients $\{\bar{d}_{m,k_m}\}$ and $\{c_{m,k_m}\}$ are respectively upsampled into the sets of intermediate coefficients $\{\bar{d}_{m,k_{m+1}}\}$ and $\{c_{m,k_{m+1}}\}$. This may be done so that those of the intermediate coefficients $\{c_{m,k_{m+1}}\}$ with odd indexes $\{k_{m+1}\}$ have zero values and those of the intermediate coefficients $\{c_{m,k_{m+1}}\}$ with even indexes $\{k_{m+1}\}$ are the set of standard scaling function coefficients $\{c_{m,k_m}\}$. Similarly, those of the intermediate coefficients $\{\bar{d}_{m,k_{m+1}}\}$ with even indexes $\{k_{m+1}\}$ would have zero values and those of the intermediate coefficients $\{\bar{d}_{m,k_{m+1}}\}$ with odd indexes $\{k_{m+1}\}$ would be the set of formatted wavelet coefficients $\{\bar{d}_{m,k_m}\}$. Only then are the transfer functions $P^*(z)$ and $Q^*(z)$ respectively performed by applying the entire sequences of decomposition coefficients $\{p_n^*\}$ and $\{q_n^*\}$ to respectively the entire sets of intermediate coefficients $\{\bar{d}_{m,k_{m+1}}\}$ and $\{c_{m,k_{m+1}}\}$ to respectively generate the sets of intermediate coefficients $\{\tilde{z}_{m+1,k_{m+1}}\}$ and $\{\tilde{y}_{m+1,k_{m+1}}\}$. The sets of sets of intermediate coefficients $\{\tilde{z}_{m+1,k_{m+1}}\}$ and $\{\tilde{y}_{m+1,k_{m+1}}\}$ are then component-wise summed together to generate the set of dual scaling function coefficients $\{\tilde{c}_{m+1,k_{m+1}}\}$. As a result, the operation of the entire sequence of decomposition coefficients $\{p_n^*\}$ on the even indexed intermediate coefficients $\{\bar{d}_{m,k_{m+1}}\}$ and the operation of the entire sequence of decomposition coefficients $\{q_n^*\}$ on the odd indexed intermediate coefficients $\{c_{m,k_{m+1}}\}$ is wasted.

But, in the embodiment of FIG. **27** of the reconstruction filter substage **246**, the subsequences of even and odd indexed decomposition coefficients $\{p_{2i}^*\}$ and $\{p_{2i+1}^*\}$ are operated only on the set of formatted wavelet coefficients $\{\bar{d}_{m,k_n}\}$. Similarly, the subsequences of even and odd indexed decomposition coefficients $\{q_{2i}^*\}$ and $\{q_{2i+1}^*\}$ are operated only on the set of standard scaling function coefficients $\{c_{m,k_m}\}$. Thus, the embodiment of FIG. **27** of the reconstruction filter substage **246** is more efficient than that of the embodiment of FIG. **15**.

For the specific implementation of the wavelet transform system **230** and inverse wavelet transform system **240** discussed earlier, the FIR filters **281**, **283**, **285**, and **287** perform respective moving average operations according to Eq. (23) using the sequences of decomposition coefficients $\{p_n^*\}$ and $\{q_n^*\}$ given in Eqs. (28) and (29). In this case, the filters **281** and **283** have each the set of formatted wavelet coefficients $\{\bar{d}_{m,k_m}\}$ as a set of inputs $\{x_k\}$, the sets of intermediate coefficients $\{\bar{y}_{m+1,k_m}\}$ and $\{\bar{z}_{m+1,k_m}\}$ as respective sets of outputs $\{y_k\}$, and the subsequences of recon-

struction coefficients $\{p_{2k_m}^*\}$ and $\{p_{2k_m+1}^*\}$ as respective sequences of taps $\{\epsilon_k\}$. Similarly, the filters **285** and **287** have each the set of standard scaling function coefficients $\{c_{m,k_m}\}$ as a set of inputs $\{x_k\}$, the sets of intermediate coefficients $\{\bar{v}_{m+1,k_m}\}$ and $\{\bar{w}_{m+1,k_m}\}$ as respective sets of outputs $\{y_k\}$, and the subsequences of reconstruction coefficients $\{q_{2k_m}^*\}$ and $\{q_{2k_m+1}^*\}$ as respective sequences of taps $\{\epsilon_k\}$.

As with the embodiment of FIG. **26** for the decomposition filter substage **148**, the concepts developed here for the embodiment of FIG. **27** for the reconstruction filter stage **246** are not limited to that just described. They may be applied to any reconstruction filter stage that uses two transfer functions to reconstruct two 1-D sets of input coefficients in respective H and L frequency bands at a lower resolution level into a 1-D set of output coefficients in an L frequency band at a next higher resolution level. For example, these concepts may be applied to the reconstruction filter stage **124** shown in FIG. **4**. Thus, in FIG. **27** for this example, the sets of formatted wavelet and standard scaling function coefficients $\{\bar{d}_{m,k_m}\}$ and $\{c_{m,k_m}\}$ would be respectively replaced by the sets of standard scaling function and wavelet coefficients $\{c_{m,k_m}\}$ and $\{d_{m,k_m}\}$ and the set of dual scaling function coefficients $\{\bar{c}_{m+1,k_{m+1}}\}$ would be replaced by the set of standard scaling function coefficients $\{c_{m+1,k_{m+1}}\}$.

Preparation for Uniform Quantization in 1-D Wavelet Transform

Turning back to FIG. **13**, for the 1-D IWT implemented by the wavelet transform system **230**, the decomposition filter **234** also maps the sets of dual scaling function and wavelet coefficients \tilde{c}_N and \tilde{d}_{M-1} to \tilde{d}_N respectively into the sets of pre-quantized scaling function and wavelet coefficients \hat{c}_N and \hat{d}_{M-1} to \hat{d}_N . This is done so that the sets of pre-quantized scaling function and wavelet coefficients \hat{c}_N and \hat{d}_{M-1} to \hat{d}_N are prepared for uniform quantization by the quantization system **224**.

In order to do so, each decomposition filter stage **238** and **236** of the decomposition filter **234** includes a pre-quantization filter substage **292**. Thus, there is a corresponding pre-quantization filter substage **292** for each resolution level $m=M$ to $N+1$ at which a decomposition is made. The pre-quantization filter substage **292** for each resolution level m has the transfer function $G(z)$ for mapping the set of dual wavelet coefficients \tilde{d}_{m-1} that was generated from the decomposition at the resolution level m into the set of pre-quantized wavelet coefficients \hat{d}_{m-1} . Furthermore, the decomposition filter stage **236** includes a second pre-quantization filter substage **294**. This pre-quantization filter substage **294** has the transfer function $R(z)^{-1}$ for mapping the set of dual scaling function coefficients \tilde{c}_N that was generated from the decomposition at the resolution level $N+1$ into the set of pre-quantized scaling function coefficients \hat{c}_N .

The transfer function $G(z)$ is a polynomial that has a sequence of mapping coefficients $\{g_n\}$ as its coefficients. Thus, the pre-quantization filter substage **292** for each resolution level $m=M$ to $N+1$ at which a decomposition is made is an FIR filter. This pre-quantization filter substage **292** performs the transfer function $G(z)$ by applying the sequence of mapping coefficients $\{g_n\}$ to the set of dual wavelet coefficients \tilde{d}_{m-1} to generate the set of pre-quantized wavelet coefficients \hat{d}_{m-1} . In the specific implementation of the wavelet transform system **230** and the inverse wavelet transform system **240** discussed earlier, the set of mapping coefficients $\{g_n\}$ is given by:

$$\begin{aligned}
&g_n=0, \\
&\text{for } n \leq -3 \text{ and } n \geq 3 \\
&g_{-2}=g_2=0.030933 \\
&g_{-1}=g_1=0.0488054 \\
&g_0=0.6874
\end{aligned} \tag{35}$$

Thus, the pre-quantization filter substage **292** for each resolution level m performs a moving average according to Eq. (23) where the set of inputs $\{x_k\}$, the set of outputs $\{y_k\}$, and the sequence of taps $\{\epsilon_k\}$ are respectively the set of dual wavelet coefficients $\tilde{d}_{m-1}=\{\tilde{d}_{m-1,k_{m-1}}\}$, the set of pre-quantized wavelet coefficients $\hat{d}_{m-1}=\{\hat{d}_{m-1,k_{m-1}}\}$, and the sequence of mapping coefficients $\{g_{k_{m-1}}\}$.

However, the transfer function $R(z)^{-1}$ is a rational function that has a sequence of mapping coefficients $\{e_n\}$ as its poles. Thus, the pre-quantization filter substage **294** for the last resolution level $N+1$ at which a reconstruction is made is an IIR filter. The pre-quantization filter substage **294** performs the transfer function $R(z)^{-1}$ by applying the sequence of mapping coefficients $\{e_n\}$ to the set of dual scaling function coefficients \tilde{c}_N to generate the set of pre-quantized wavelet coefficients \hat{c}_N . In the specific implementation of the wavelet transform system **230** and the inverse wavelet transform system **240** discussed earlier, the set of mapping coefficients $\{e_n\}$ is given by:

$$\begin{aligned}
&e_n=0, \\
&\text{for } n \leq -3 \text{ and } n \geq 3 \\
&e_{-2}=e_2=0.009486309340 \\
&e_{-1}=e_1=0.340509034999 \\
&e_0=0.573751762833
\end{aligned} \tag{36}$$

Thus, the pre-quantization filter substage **294** performs a recursive two stage feedback differencing operation according to Eq. (25) where the set of inputs $\{x_k\}$, the set of outputs $\{z_k\}$, and the sequence of taps $\{\mu_k\}$ are respectively the set of dual scaling function coefficients $\tilde{c}_N=\{\tilde{c}_{N,k_N}\}$, the set of pre-quantized scaling function coefficients $\hat{c}_N=\{\hat{c}_{N,k_N}\}$, and the sequence of mapping coefficients $\{e_{k_N}\}$.

Post Dequantization in 1-D Inverse Wavelet Transform

Turning again to FIG. 14, in the 1-D inverse IWT implemented by the inverse wavelet transform system **240**, the dequantization system **232** first performs a uniform dequantization to generate the sets of dequantized scaling function and wavelet coefficients \hat{c}_N and \hat{d}_{M-1} to \hat{d}_N . The reconstruction filter **242** then maps the sets of dequantized scaling function and wavelet coefficients \hat{c}_N and \hat{d}_{M-1} to \hat{d}_N respectively into the sets of dual scaling function and wavelet coefficients \tilde{c}_N and \tilde{d}_{M-1} to \tilde{d}_N for reconstruction in the manner described earlier.

In order to do so, the reconstruction filter stage **243** of the reconstruction filter **242** includes a post-dequantization filter substage **298**. This post-dequantization filter substage **298** has the transfer function $R(z)$ for mapping the set of dequantized scaling function coefficients \hat{c}_N into the set of dual scaling function coefficients \tilde{c}_N at the first resolution level $m=N$ at which a reconstruction is made. However, the transfer function $R(z)$ is the inverse of the transfer function $R(z)^{-1}$ and is a polynomial that has a sequence of mapping

coefficients $\{r_n\}$ as its coefficients. Thus, the post-dequantization filter substage **298** is an FIR filter that performs the transfer function $R(z)$ by applying the sequence of mapping coefficients $\{r_n\}$ to the set of dequantized wavelet coefficients \hat{c}_N to generate the set of dual scaling function coefficients \tilde{c}_N .

In the specific implementation of the wavelet transform system **230** and the inverse wavelet transform system **240** discussed earlier, the set of mapping coefficients $\{r_n\}$ is given by:

$$\begin{aligned}
&r_n=0, \\
&\text{for } n \leq -3 \text{ and } n \geq 3 \\
&r_{-2}=r_2=0.005442786707 \\
&r_{-1}=r_1=0.197220977140 \\
&r_0=0.640327847858
\end{aligned} \tag{37}$$

Here, the post-dequantization filter substage **298** performs a moving average according to Eq. (23) where the set of inputs $\{x_k\}$, the set of outputs $\{y_k\}$, and the sequence of taps $\{\epsilon_k\}$ are respectively the set of dequantized scaling function coefficients $\hat{c}_N=\{\hat{c}_{N,k_N}\}$, the set of dual scaling function coefficients $\tilde{c}_N=\{\tilde{c}_{N,k_N}\}$, and the sequence of mapping coefficients $\{e_{k_N}\}$.

Furthermore, each reconstruction filter stage **243** and **244** of the reconstruction filter **242** includes a post-dequantization filter substage **296**. Thus, for each resolution level $m=N$ to $M-1$ at which a reconstruction is made, there is a corresponding post-dequantization filter substage **296**. The post-dequantization filter substage **296** for each resolution level m is an IIR filter that has the transfer function $G(z)^{-1}$ for mapping the set of dequantized wavelet coefficients \hat{d}_m into the set of dual wavelet coefficients \tilde{d}_m . This is done by applying a sequence of mapping coefficients $\{h_n\}$ to the set of dequantized wavelet coefficients \hat{d}_m to generate the set of dual wavelet coefficients \tilde{d}_m . The transfer function $G(z)^{-1}$ is the inverse of the transfer function $G(z)$ and is a rational function that has the sequence of mapping coefficients $\{h_n\}$ as its poles.

In the specific implementation of the wavelet transform system **230** and the inverse wavelet transform system **240** discussed earlier, the set of mapping coefficients $\{h_n\}$ is given by:

$$\begin{aligned}
&h_n=0, \\
&\text{for } n \leq -3 \text{ and } n \geq 3 \\
&h_{-2}h_2=0.045392766814 \\
&h_{-1}h_1=0.06837990082 \\
&h_0=0.682806105800
\end{aligned} \tag{38}$$

The post-dequantization filter substage **296** for each resolution level m therefore performs a recursive two stage feedback differencing operation according to Eq. (25) where the set of inputs $\{x_k\}$, the set of outputs $\{z_k\}$, and the sequence of taps $\{\mu_k\}$ are respectively the set of dequantized wavelet coefficients $\hat{d}_m=\{\hat{d}_{m,k-di_m}\}$, the set of dual wavelet coefficients $\tilde{d}_m=\{\tilde{d}_{m,k_m}\}$, and the sequence of mapping coefficients $\{h_{k_m}\}$.

Preparation for Uniform Quantization in 2-D Wavelet Transform

Turning back to FIG. 17, for the 2-D IWT implemented by the wavelet transform system **230**, the sets of pre-quantized

scaling function and wavelet coefficients $\hat{c}_N, \hat{d}_{M-1}^1$ to \hat{d}_N^1 , \hat{d}_{M-1}^2 to \hat{d}_N^2 , and \hat{d}_{M-1}^3 to \hat{d}_N^3 are generated in preparation for uniform quantization by the quantization system 224. This is done by the decomposition filter 272 which maps the sets of dual scaling function and wavelet coefficients $\tilde{c}_N, \tilde{d}_{M-1}^1$ to \tilde{d}_N^1 , \tilde{d}_{M-1}^2 to \tilde{d}_N^2 , and \tilde{d}_{M-1}^3 to \tilde{d}_N^3 respectively into the sets of pre-quantized scaling function and wavelet coefficients $\hat{c}_N, \hat{d}_{M-1}^1$ to \hat{d}_N^1 , \hat{d}_{M-1}^2 to \hat{d}_N^2 , and \hat{d}_{M-1}^3 to \hat{d}_N^3 .

In order to do so, each decomposition filter stage 273 and 275 of the decomposition filter 272 includes three pre-quantization filter substages 300, 302, and 304. Thus, there are three corresponding pre-quantization filter substages 300, 302, and 304 for each resolution level $m=M$ to $N+1$ at which a decomposition is made.

The pre-quantization filter substage 300 for each resolution level m has the transfer function $R(z_1)^{-1}G(z_2)$ for mapping the set of dual wavelet coefficients \tilde{d}_{m-1}^1 that was generated from the decomposition at the resolution level m into the set of pre-quantized wavelet coefficients \hat{d}_{m-1}^1 . Referring to FIG. 28, in order to do so, the pre-quantization filter substage 300 includes the same filter substages 294 and 292 as that described earlier. Here, the filter substage 294 has the transfer function $R(z_1)^{-1}$ for mapping in the horizontal direction each row j_{m-1} of the 2-D set of dual wavelet coefficients $\tilde{d}_{m-1}^1 = \{\tilde{d}_{m-1, k_{m-1}, j_{m-1}}^1\}$ into a corresponding row j_{m-1} of a 2-D set of intermediate coefficients $\tilde{y}_{m-1}^1 = \{\tilde{y}_{m-1, k_{m-1}, j_{m-1}}^1\}$. The filter substage 292 then maps each column k_{m-1} of the 2-D set of intermediate coefficients $\tilde{y}_{m-1}^1 = \{\tilde{y}_{m-1, k_{m-1}, j_{m-1}}^1\}$ into a corresponding column k_{m-1} of the 2-D set of pre-quantized wavelet coefficients $\hat{d}_{m-1}^1 = \{\hat{d}_{m-1, k_{m-1}, j_{m-1}}^1\}$ in the vertical direction with the transfer function $G(z_2)$.

The pre-quantization filter substage 302 for each resolution level m maps the set of dual wavelet coefficients \tilde{d}_{m-1}^2 that was generated from the decomposition at the resolution level m into the set of pre-quantized wavelet coefficients \hat{d}_{m-1}^2 using the transfer function $G(z_1)R(z_2)^{31}$. Turning again to FIG. 28, like the pre-quantization filter substage 300, the pre-quantization filter substage 302 includes the same filter substages 292 and 294 as that described earlier. However, in this case, the filter substage 292 has the transfer function $G(z_1)$ for mapping in the horizontal direction each row j_{m-1} of the 2-D set of dual wavelet coefficients $\tilde{d}_{m-1}^2 = \{\tilde{d}_{m-1, k_{m-1}, j_{m-1}}^2\}$ into a corresponding row j_{m-1} of a 2-D set of intermediate coefficients $\tilde{y}_{m-1}^2 = \{\tilde{y}_{m-1, k_{m-1}, j_{m-1}}^2\}$. The filter substage 294 then maps each column k_{m-1} of the 2-D set of intermediate coefficients $\tilde{y}_{m-1}^2 = \{\tilde{y}_{m-1, k_{m-1}, j_{m-1}}^2\}$ into a corresponding column k_{m-1} of the 2-D set of pre-quantized wavelet coefficients $\hat{d}_{m-1}^2 = \{\hat{d}_{m-1, k_{m-1}, j_{m-1}}^2\}$ in the vertical direction with the transfer function $R(z_2)^{31}$.

Similarly, the pre-quantization filter substage 304 for each resolution level m has the transfer function $G(z_1)G(z_2)$ for mapping the set of dual wavelet coefficients \tilde{d}_{m-1}^3 that was generated from the decomposition at the resolution level m into the set of pre-quantized wavelet coefficients \hat{d}_{m-1}^3 . Turning again to FIG. 28, in this case, the pre-quantization filter substage 304 includes two of the filter substages 292 that were described earlier. The first filter substage 292 has the transfer function $G(z_1)$ for mapping in the horizontal direction each row j_{m-1} of the 2-D set of dual wavelet coefficients $\tilde{d}_{m-1}^3 = \{\tilde{d}_{m-1, k_{m-1}, j_{m-1}}^3\}$ into a corresponding row j_{m-1} of a 2-D set of intermediate coefficients $\tilde{y}_{m-1}^3 = \{\tilde{y}_{m-1, k_{m-1}, j_{m-1}}^3\}$. The second filter substage 292 then maps each column k_{m-1} of the 2-D set of intermediate coefficients $\tilde{y}_{m-1}^3 = \{\tilde{y}_{m-1, k_{m-1}, j_{m-1}}^3\}$ into a corresponding column k_{m-1} of

the 2-D set of pre-quantized wavelet coefficients $\hat{d}_{m-1}^3 = \{\hat{d}_{m-1, k_{m-1}, j_{m-1}}^3\}$ in the vertical direction with the transfer function $G(z_2)$.

Finally, the decomposition filter stage 273 includes a pre-quantization filter substage 306 for the last resolution level $m=N+1$ at which a decomposition is made. The pre-quantization filter substage 306 maps the 2-D set of dual scaling function coefficients \tilde{c}_N generated by the decomposition at the resolution level $m=N+1$ into the 2-D set of pre-quantized scaling function coefficients \hat{c}_N . In doing so, each dual scaling function coefficient \tilde{c}_{N, k_N, j_N} in the 2-D set of dual scaling function coefficients $\tilde{c}_N = \{\tilde{c}_{N, k_N, j_N}\}$ is multiplied by a constant K to generate a corresponding pre-quantized scaling function coefficient $\hat{c}_N = \{\hat{c}_{N, k_N, j_N}\}$ in the 2-D set of pre-quantized scaling function coefficients $\hat{c}_N = \{\hat{c}_{N, k_N, j_N}\}$. In the specific implementation of the wavelet transform system 230 and the inverse wavelet transform system 240 discussed earlier, this constant K is equal to 4.965.

Post Dequantization in 2-D Inverse Wavelet Transform

Turning once more to FIG. 20, in the 2-D inverse IWT implemented by the inverse wavelet transform system 240, the sets of dequantized scaling function and wavelet coefficients $\hat{c}_N, \hat{d}_{M-1}^1$ to \hat{d}_N^1 , \hat{d}_{M-1}^2 to \hat{d}_N^2 , and \hat{d}_{M-1}^3 to \hat{d}_N^3 are generated by the dequantization system 232 during uniform dequantization. The reconstruction filter 250 then maps the sets of dequantized scaling function and wavelet coefficients $\hat{c}_N, \hat{d}_{M-1}^1$ to \hat{d}_N^1 , \hat{d}_{M-1}^2 to \hat{d}_N^2 , and \hat{d}_{M-1}^3 to \hat{d}_N^3 respectively into the sets of dual scaling function and wavelet coefficients $\tilde{c}_N, \tilde{d}_{M-1}^1$ to \tilde{d}_N^1 , \tilde{d}_{M-1}^2 to \tilde{d}_N^2 , and \tilde{d}_{M-1}^3 to \tilde{d}_N^3 for reconstruction in the manner described earlier.

To do so, the reconstruction filter stage 253 includes a post-dequantization filter substage 308 for the first resolution level $m=N$ at which a reconstruction is made. The post-dequantization filter substage 308 maps the set of dequantized scaling function coefficients \hat{c}_N into the set of dual scaling function coefficients \tilde{c}_N . This is done by multiplying each dequantized scaling function coefficient \hat{c}_{N, k_N, j_N} in the set of dequantized scaling function coefficients $\hat{c}_N = \{\hat{c}_{N, k_N, j_N}\}$ by a constant $1/K$ in order to generate a corresponding dual scaling function coefficient \tilde{c}_{N, k_N, j_N} in the set of dual scaling function coefficients $\tilde{c}_N = \{\tilde{c}_{N, k_N, j_N}\}$. In the specific implementation of the wavelet transform system 230 and the inverse wavelet transform system 240 discussed earlier, this constant is the inverse of the constant K given earlier and is equal to $1/4.965$.

Furthermore, each reconstruction filter stage 252 and 253 of the reconstruction filter 250 includes three post-dequantization filter substages 310, 312, and 314. Thus, there are three corresponding post-dequantization filter substages 310, 312, and 314 for each resolution level $m=N$ to $M-1$ at which a reconstruction is made.

The post-dequantization filter substage 310 for each resolution level m has the transfer function $R(z_1)G(z_2)^{-1}$ for mapping the set of dequantized wavelet coefficients \hat{d}_m^1 into the set of dual wavelet coefficients \tilde{d}_m^1 . Referring to FIG. 29, in order to do so, the pre-quantization filter substage 310 includes the same filter substages 296 and 298 as that described earlier. The filter substage 296 maps each column k_m of the 2-D set of dequantized wavelet coefficients $\hat{d}_m^1 = \{\hat{d}_{m, k_m, j_m}^1\}$ into a corresponding column k_m of the 2-D set of intermediate coefficients $\tilde{y}_m^1 = \{\tilde{y}_{m, k_m, j_m}^1\}$ in the vertical direction with the transfer function $G(z_2)^{-1}$. Then, the filter

substage **298** uses the transfer function $R(z_1)$ to map in the horizontal direction each row j_m of the 2-D set of intermediate coefficients $\tilde{y}_m^1 = \{\tilde{y}_{m,k_m^1 j_m}\}$ into a corresponding row j_m of the 2-D set of dual wavelet coefficients $\tilde{d}_m^1 = \{\tilde{d}_{m,k_m^1 j_m}\}$.

The post-dequantization filter substage **312** for each resolution level m maps the set of pre-quantized wavelet coefficients \hat{d}_m^2 into the set of dual wavelet coefficients \tilde{d}_m^2 using the transfer function $G(z_1)^{-1}R(z_2)$. Turning again to FIG. **29**, the post-dequantization filter substage **312** includes the same filter substages **298** and **296** as that described earlier. However, in this case, the filter substage **298** maps each column k_m of the 2-D set of dequantized wavelet coefficients $\hat{d}_m^2 = \{\hat{d}_{m,k_m^2 j_m}\}$ into a corresponding column k_m of the 2-D set of intermediate coefficients $\tilde{y}_m^2 = \{\tilde{y}_{m,k_m^2 j_m}\}$ in the vertical direction with the transfer function $R(z_2)$. Furthermore, the filter substage **296** uses the transfer function $G(z_1)^{-1}$ to map in the horizontal direction each row j_m of the 2-D set of intermediate coefficients $\tilde{y}_m^2 = \{\tilde{y}_{m,k_m^2 j_m}\}$ into a corresponding row j_m of the 2-D set of dual wavelet coefficients $\tilde{d}_m^2 = \{\tilde{d}_{m,k_m^2 j_m}\}$.

Finally, the post-dequantization filter substage **314** for each resolution level m has the transfer function $G(z_1)^{-1}G(z_2)^{-1}$ for mapping the set of pre-quantized wavelet coefficients $\hat{d}_m^3 = \{\hat{d}_{m,k_m^3 j_m}\}$ into the set of dual wavelet coefficients $\tilde{d}_m^3 = \{\tilde{d}_{m,k_m^3 j_m}\}$. Turning once more to FIG. **29**, the post-dequantization filter substage **314** includes two of the filter substages **296** that were described earlier. The first filter substage **296** maps each column k_m of the set of pre-quantized wavelet coefficients $\hat{d}_m^3 = \{\hat{d}_{m,k_m^3 j_m}\}$ into a corresponding column k_m of the set of intermediate coefficients $\tilde{y}_m^3 = \{\tilde{y}_{m,k_m^3 j_m}\}$ into in the vertical direction with the transfer function $G(z_2)^{-1}$. The second filter substage **296** has the transfer function $G(z_1)^{-1}$ for mapping in the horizontal direction each row j_m of the 2-D set of intermediate coefficients $\tilde{y}_m^3 = \{\tilde{y}_{m,k_m^3 j_m}\}$ into a corresponding row j_m of the 2-D set of dual wavelet coefficients $\tilde{d}_m^3 = \{\tilde{d}_{m,k_m^3 j_m}\}$.

Other Alternative Embodiments

Referring to FIG. **11**, the wavelet transform system **230** has been described for use with an encoding system **226** and quantization system **224** for compression of original data. Similarly, the wavelet inverse transform system **240** has been described for use with a decoding system **228** and dequantization system **232** for decompression of encoded data. But, those skilled in the art will recognize that the 1-D and 2-D IWTs and inverse IWTs described herein and performed by the wavelet and inverse wavelet transform systems **230** and **240** may be used for other purposes. In such a case, a digital signal processor (DSP) could be used to post-process and/or pre-process the these 1-D and 2-D IWTs and inverse IWTs.

Moreover, in FIG. **11**, the wavelet transform system **230** and the inverse wavelet transform system **240** are shown in a software implementation. However, a hardware implementation of the wavelet transform system **230** and the inverse wavelet transform system **240** would also take advantage of the benefits of the 1-D and 2-D IWTs and inverse IWTs just described.

Finally, while the present invention has been described with reference to a few specific embodiments, the description is illustrative of the invention and is not to be construed as limiting the invention. Various modifications may occur to those skilled in the art without departing from the true spirit and scope of the invention as defined by the appended claims.

What is claimed is:

1. A decomposition filter stage for decomposing a set of input coefficients into a first set of output coefficients and a second set of output coefficients by applying a first sequence of decomposition coefficients and a second sequence of decomposition coefficients to the set of input coefficients, the decomposition filter stage comprising:

- a deinterleaver that deinterleaves the set of input coefficients into a first subset of the set of input coefficients and a second subset of the set of input coefficients;
- a first filter that applies a first subsequence of the first sequence of coefficients to the first subset of the set of input coefficients to generate a first set of intermediate coefficients;
- a second filter that applies a first subsequence of the second sequence of coefficients to the second subset of the set of input coefficients to generate a second set of intermediate coefficients;
- a first summer that sums the first and second sets of intermediate coefficients to generate the first set of output coefficients;
- a third filter that applies a second subsequence of the first sequence of coefficients to the first subset of the set of input coefficients to generate a third set of intermediate coefficients;
- a fourth filter that applies a second subsequence of the second sequence of coefficients to the second subset of the set of input coefficients to generate a fourth set of intermediate coefficients; and
- a second summer that sums the third and fourth sets of intermediate coefficients to generate the second set of output coefficients.

2. The decomposition filter stage of claim **1** wherein the first, second, third, and fourth filters comprise FIR filters that respectively apply the first subsequence of the first sequence of coefficients, the first subsequence of the second sequence of coefficients, the second subsequence of the first sequence of coefficients, and the second subsequence of the second sequence of coefficients in respective moving average operations.

- 3.** The decomposition filter stage of claim **2** wherein:
- the first and second subsets of the set of input coefficients respectively comprises those of the input coefficients in the set of input coefficients with even and odd indexes;
 - the first and second subsequences of the first sequence of coefficients respectively comprises those of the coefficients in the first sequence of coefficients with even and odd indexes; and
 - the first and second subsequences of the second sequence of coefficients respectively comprises those of the coefficients in the second sequence of coefficients with odd and even indexes.

4. A method of decomposing a set of input coefficients into a first set of output coefficients and a second set of output coefficients by applying a first sequence of decomposition coefficients and a second sequence of decomposition coefficients to the set of input coefficients, the method comprising the steps of:

- deinterleaving the set of input coefficients into a first subset of the set of input coefficients and a second subset of the set of input coefficients;
- applying a first subsequence of the first sequence of coefficients to the first subset of the set of input coefficients to generate a first set of intermediate coefficients;

applying a first subsequence of the second sequence of coefficients to the second subset of the set of input coefficients to generate a second set of intermediate coefficients;

summing the first and second sets of intermediate coefficients to generate the first set of output coefficients;

applying a second subsequence of the first sequence of coefficients to the first subset of the set of input coefficients to generate a third set of intermediate coefficients;

applying a second subsequence of the second sequence of coefficients to the second subset of the set of input coefficients to generate a fourth set of intermediate coefficients; and

summing the third and fourth sets of intermediate coefficients to generate the second set of output coefficients.

5. The method of claim 4 wherein the first, second, third, and fourth applying steps respectively include applying the first subsequence of the first sequence of coefficients, the first subsequence of the second sequence of coefficients, the second subsequence of the first sequence of coefficients, and the second subsequence of the second sequence of coefficients in respective moving average operations.

6. The method of claim 5 wherein:

the first and second subsets of the set of input coefficients respectively comprises those of the input coefficients in the set of input coefficients with even and odd indexes;

the first and second subsequences of the first sequence of coefficients respectively comprises those of the coefficients in the first sequence of coefficients with even and odd indexes; and

the first and second subsequences of the second sequence of coefficients respectively comprises those of the coefficients in the second sequence of coefficients with odd and even indexes.

7. A reconstruction filter stage for reconstructing a first set of input coefficients and a second set of input coefficients into a set of output coefficients by applying a first sequence of coefficients and a second sequence of coefficients to the first and second sets of input coefficients, the reconstruction filter stage comprising:

a first filter that applies a first subsequence of the first sequence of coefficients to the first set of input coefficients to generate a first set of intermediate coefficients;

a second filter that applies a first subsequence of the second sequence of coefficients to the second set of input coefficients to generate a second set of intermediate coefficients;

a first summer that sums the first and second sets of intermediate coefficients to generate a first subset of the set of output coefficients;

a third filter that applies a second subsequence of the first sequence of coefficients to the first set of input coefficients to generate a third set of intermediate coefficients;

a fourth filter that applies a second subsequence of the second sequence of coefficients to the second set of input coefficients to generate a fourth set of intermediate coefficients;

a second summer that sums the third and fourth sets of intermediate coefficients to generate a second subset of the set of output coefficients; and

an interleaver that interleaves the first and second subsets of the set of output coefficients to generate the set of output coefficients.

8. The reconstruction filter stage of claim 7 wherein the first, second, third, and fourth filters comprise FIR filters that

respectively apply the first subsequence of the first sequence of coefficients, the first subsequence of the second sequence of coefficients, the second subsequence of the first sequence of coefficients, and the second subsequence of the second sequence of coefficients in respective moving average operations.

9. The reconstruction filter stage of claim 8 wherein:

the first and second subsets of the set of input coefficients respectively comprises those of the input coefficients in the set of input coefficients with even and odd indexes;

the first and second subsequences of the first sequence of coefficients respectively comprises those of the coefficients in the first sequence of coefficients with even and odd indexes; and

the first and second subsequences of the second sequence of coefficients respectively comprises those of the coefficients in the second sequence of coefficients with odd and even indexes.

10. A method of reconstructing a first set of input coefficients and a second set of input coefficients into a set of output coefficients by applying a first sequence of coefficients and a second sequence of coefficients to the first and second sets of input coefficients, the method comprising the steps of:

applying a first subsequence of the first sequence of coefficients to the first set of input coefficients to generate a first set of intermediate coefficients;

applying a first subsequence of the second sequence of coefficients to the second set of input coefficients to generate a second set of intermediate coefficients;

summing the first and second sets of intermediate coefficients to generate a first subset of the set of output coefficients;

applying a second subsequence of the first sequence of coefficients to the first set of input coefficients to generate a third set of intermediate coefficients;

applying a second subsequence of the second sequence of coefficients to the second set of input coefficients to generate a fourth set of intermediate coefficients;

summing the third and fourth sets of intermediate coefficients to generate a second subset of the set of output coefficients; and

interleaving the first and second subsets of the set of output coefficients to generate the set of output coefficients.

11. The method of claim 10 wherein the first, second, third, and fourth applying steps respectively include applying the first subsequence of the first sequence of coefficients, the first subsequence of the second sequence of coefficients, the second subsequence of the first sequence of coefficients, and the second subsequence of the second sequence of coefficients in respective moving average operations.

12. The method of claim 11 wherein:

the first and second subsets of the set of input coefficients respectively comprises those of the input coefficients in the set of input coefficients with even and odd indexes;

the first and second subsequences of the first sequence of coefficients respectively comprises those of the coefficients in the first sequence of coefficients with even and odd indexes; and

the first and second subsequences of the second sequence of coefficients respectively comprises those of the coefficients in the second sequence of coefficients with odd and even indexes.