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Arribat
[11] 3,918,365
[45] Nov. 11, 1975

[54] NEW AND USEFUL IMPROVEMENTS IN PROPERGOLS OR PROPELLANTS

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[73] Assignee: **The Republic of France**, Paris, France

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[21] Appl. No.: **442,849**

Related U.S. Application Data
[63] Continuation of Ser. No. 181,909, Sept. 20, 1971, abandoned.

[52] **U.S. Cl.** **102/101**; 60/250
[51] **Int. Cl.**^{2} **F42B 1/00**
[58] **Field of Search** 102/101; 60/250, 254

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Primary Examiner—Samuel Feinberg
Attorney, Agent, or Firm—Brooks Haidt Haffner & Delahunty

[57] **ABSTRACT**

The present invention relates to propergols or propellants in the form of blocks which comprise two propergols or propellants having different speeds of combustion and, in accordance with the invention, the blocks have cross-sections according to which the inner contour of the propergol or propellant having the faster speed of combustion, has the shape of a star, and there is a separatrix between the two propergols or propellants also having the shape of a star, the number of branches of which is at least equal to the number of branches of the star shape forming the inner contour. The invention also relates to a method for calculating the shape of said blocks.

13 Claims, 23 Drawing Figures

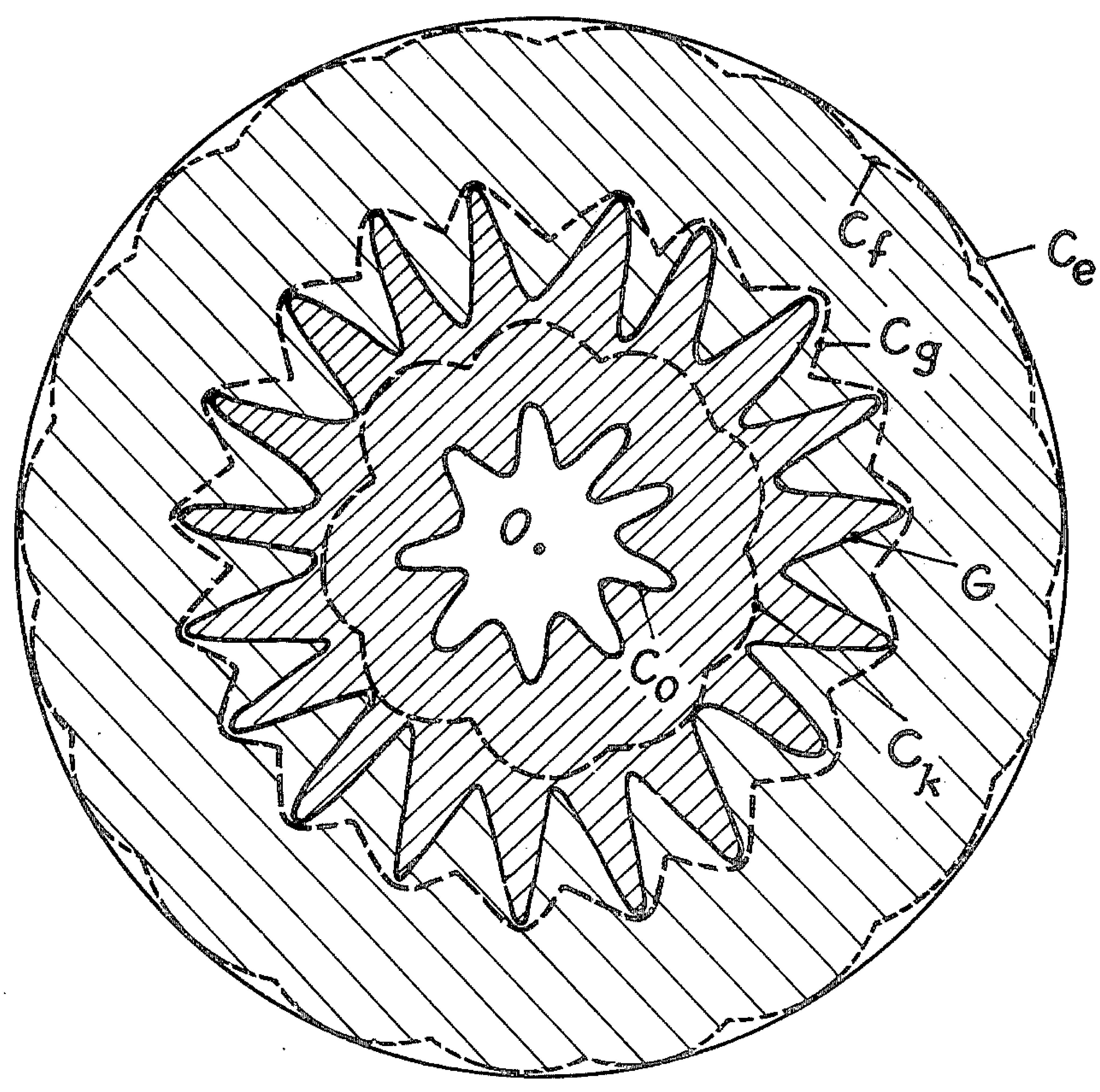


FIG. 1

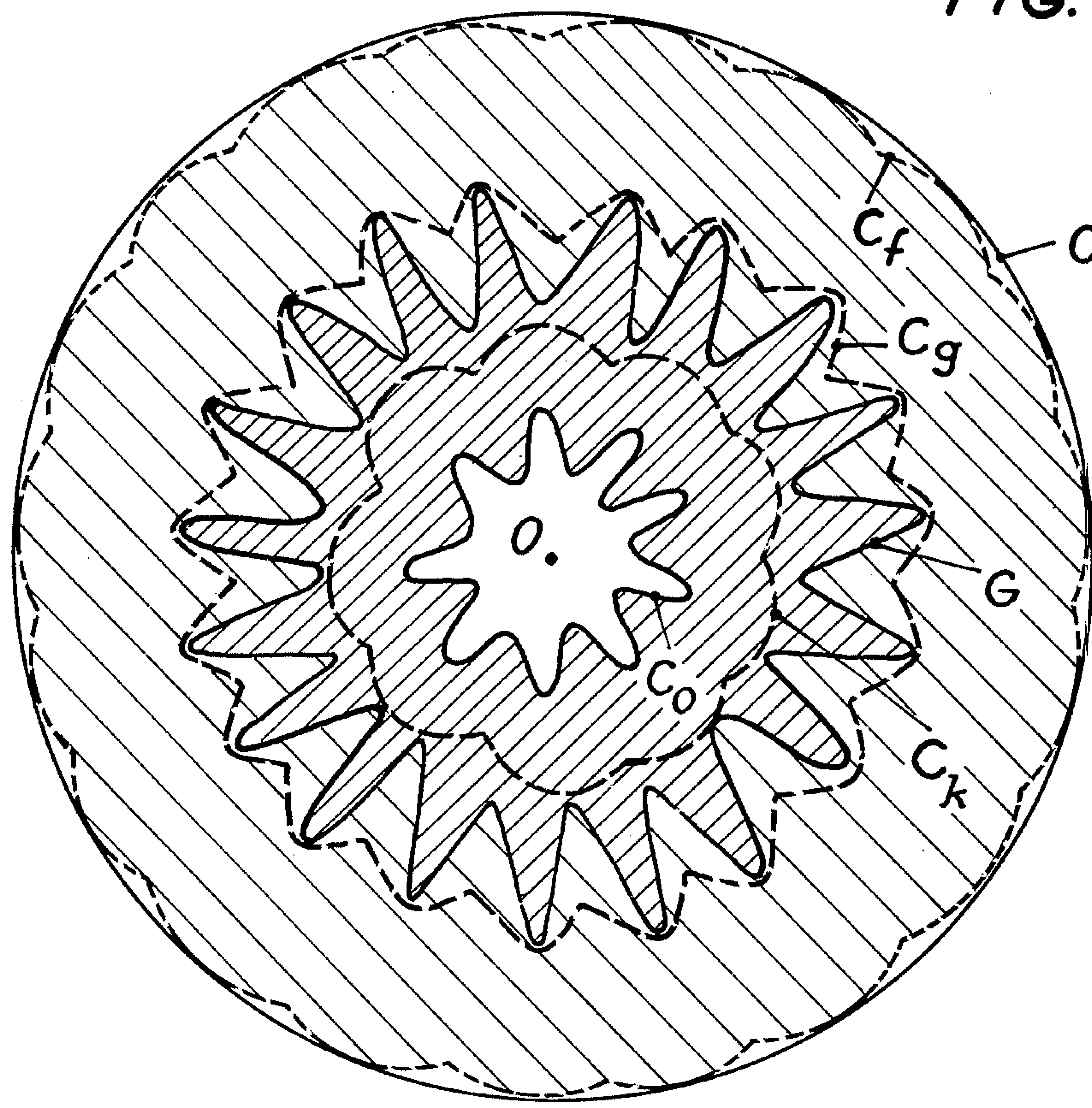
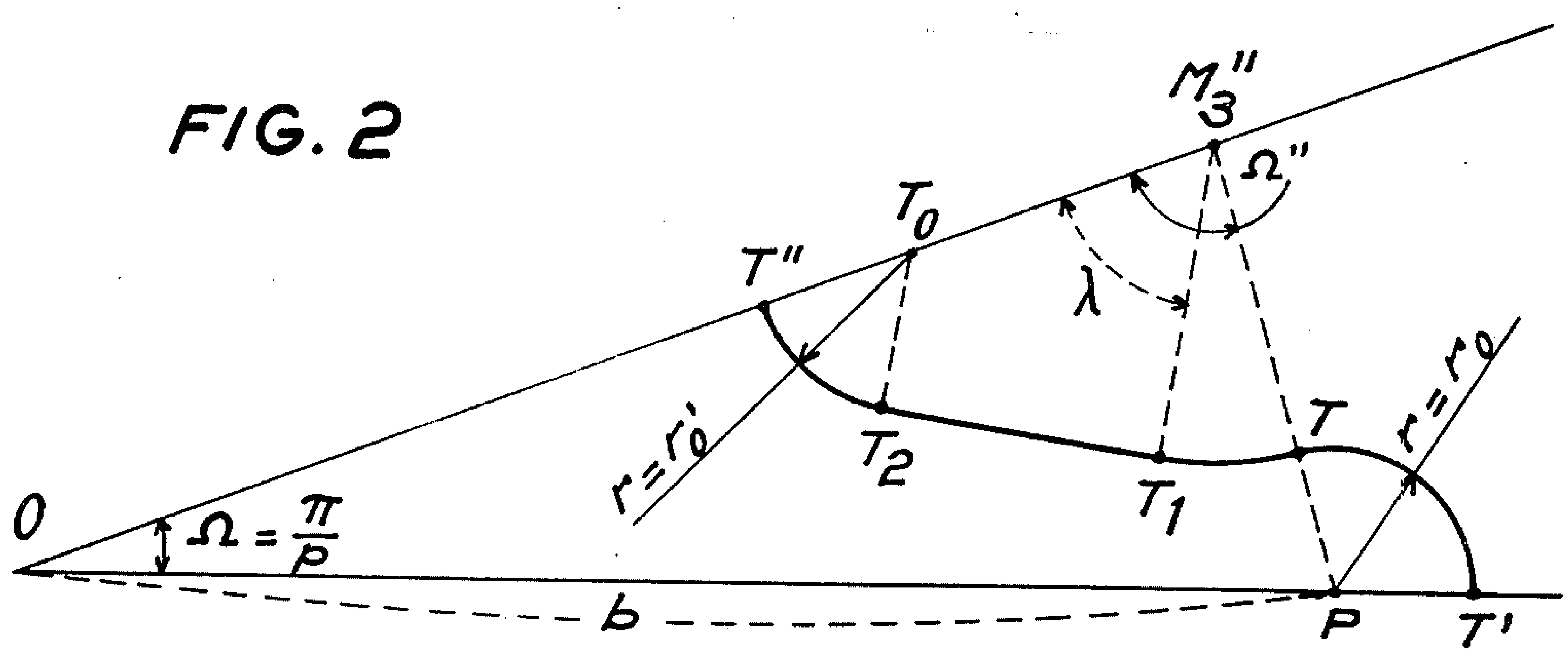


FIG. 2



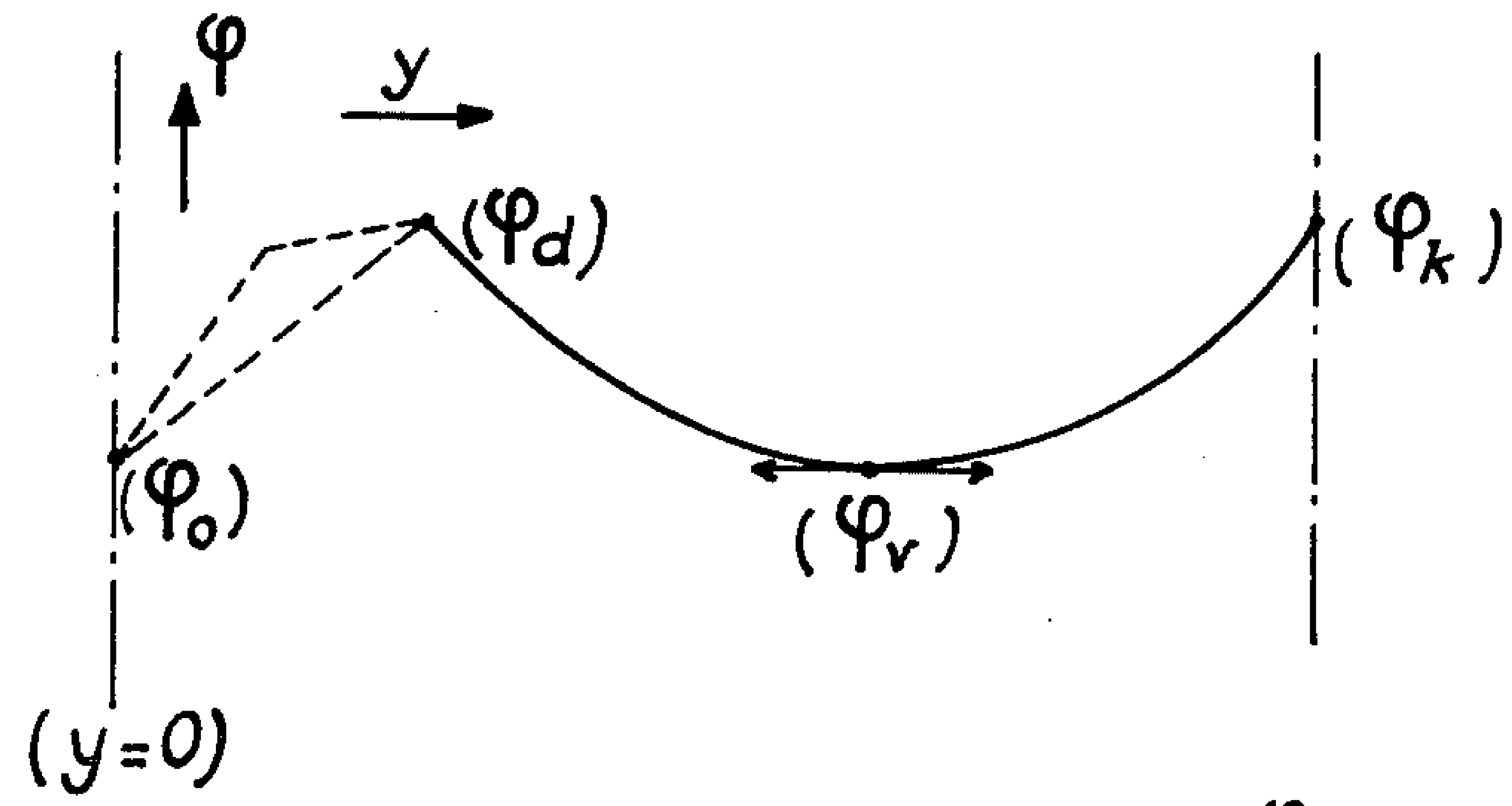


FIG. 3

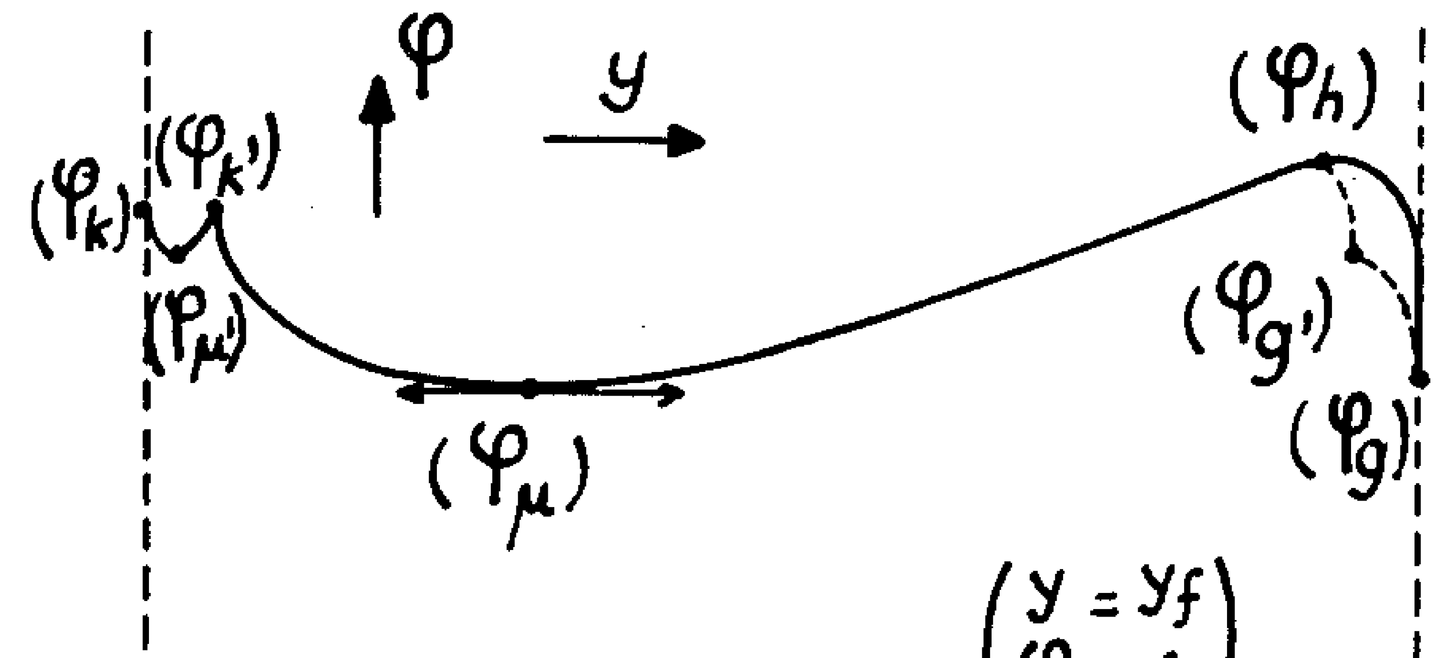


FIG. 4

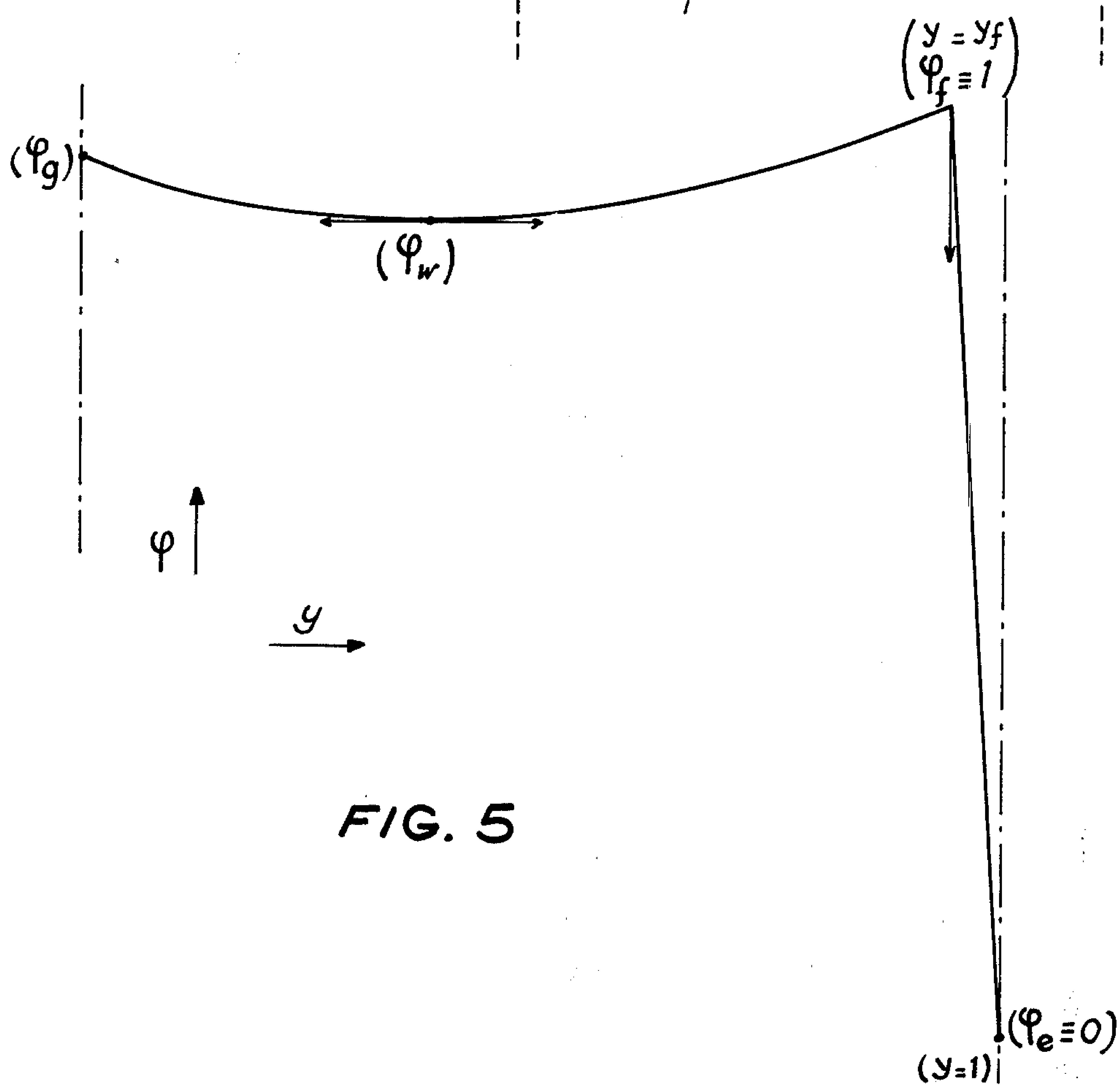


FIG. 5

FIG. 6

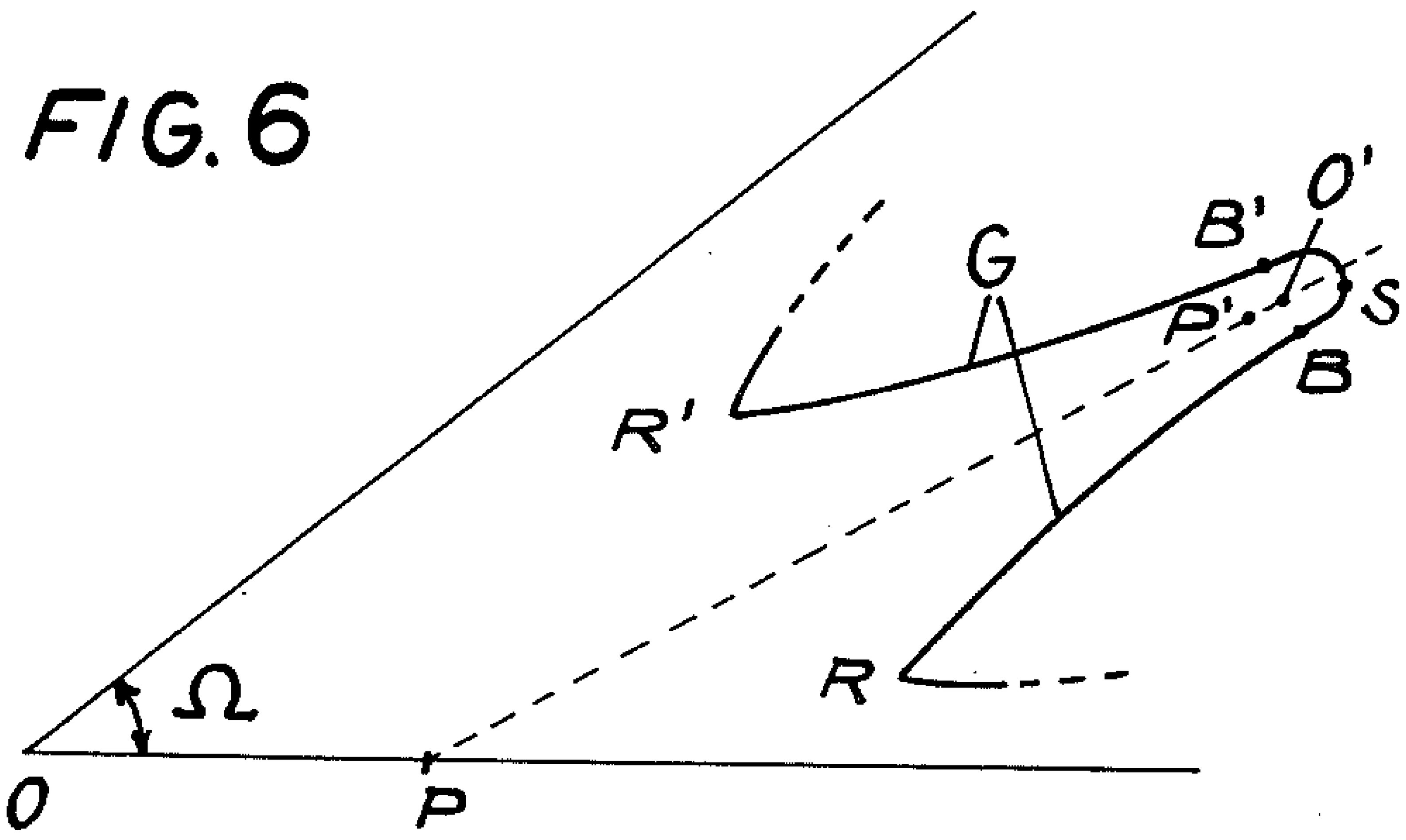


FIG. 7

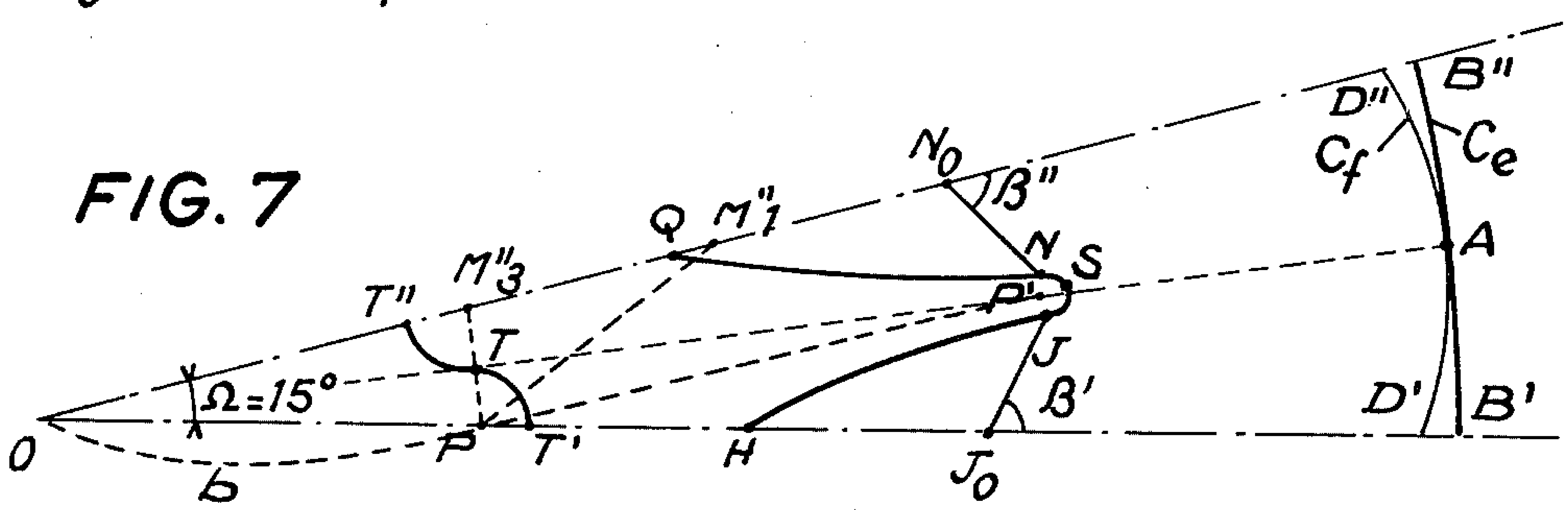


FIG. 8

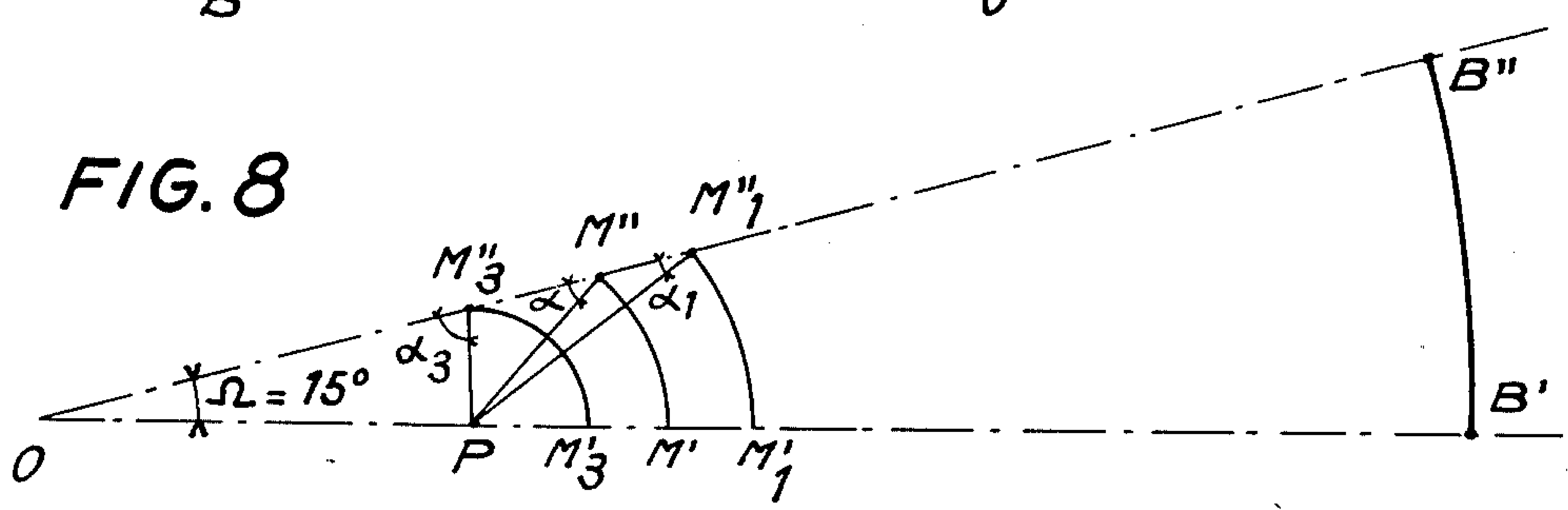
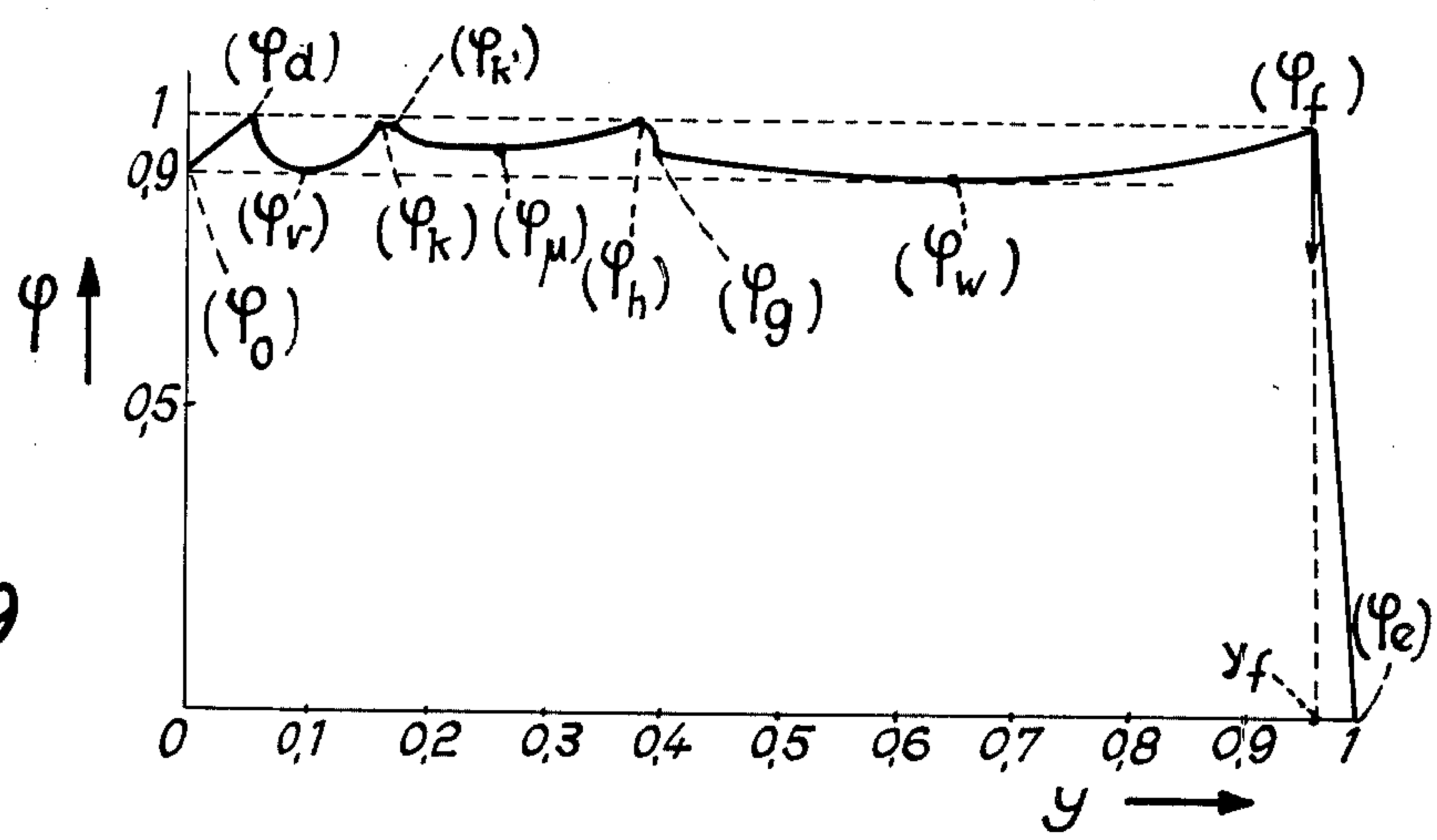


FIG. 9



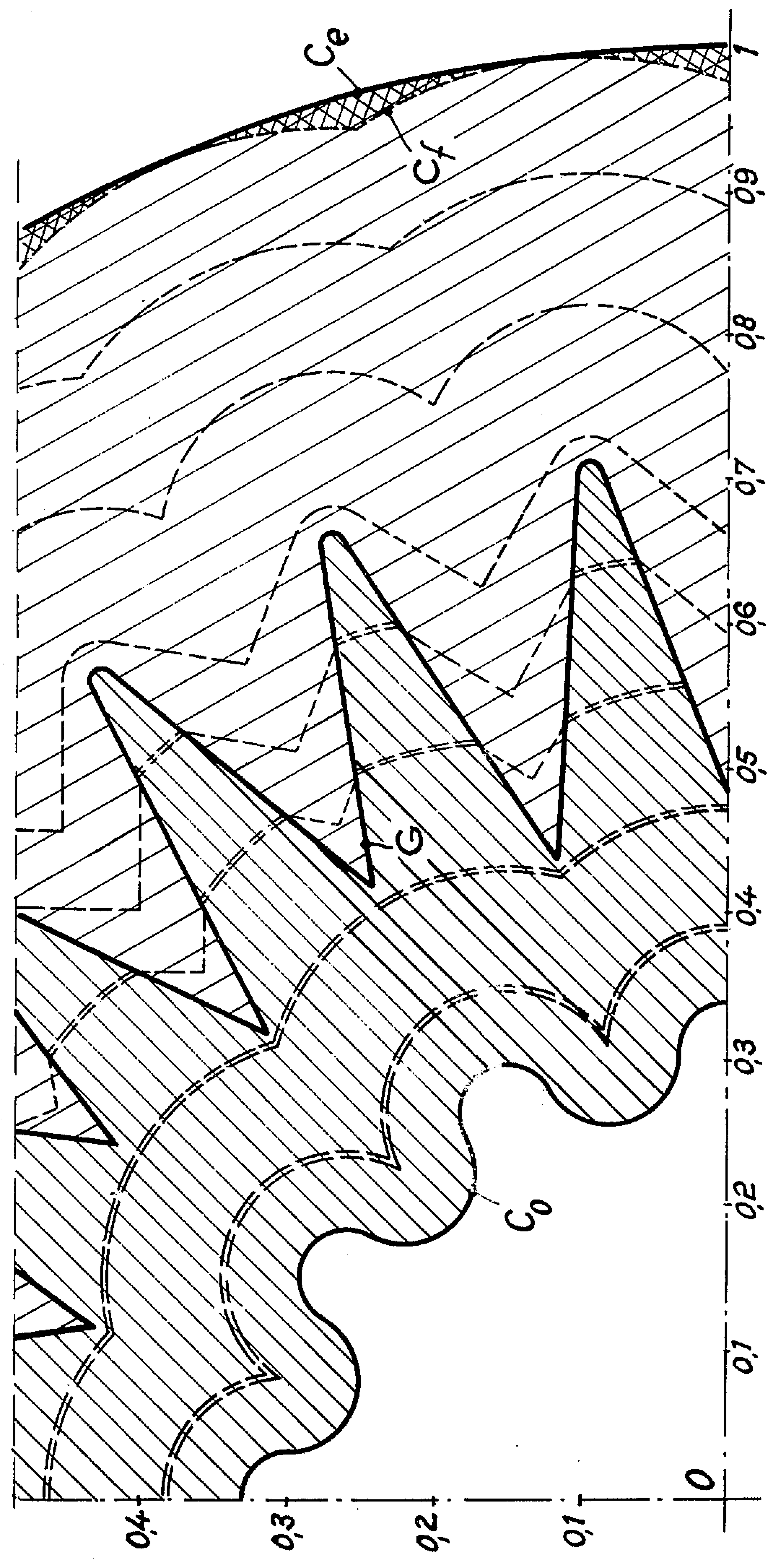


FIG. 10

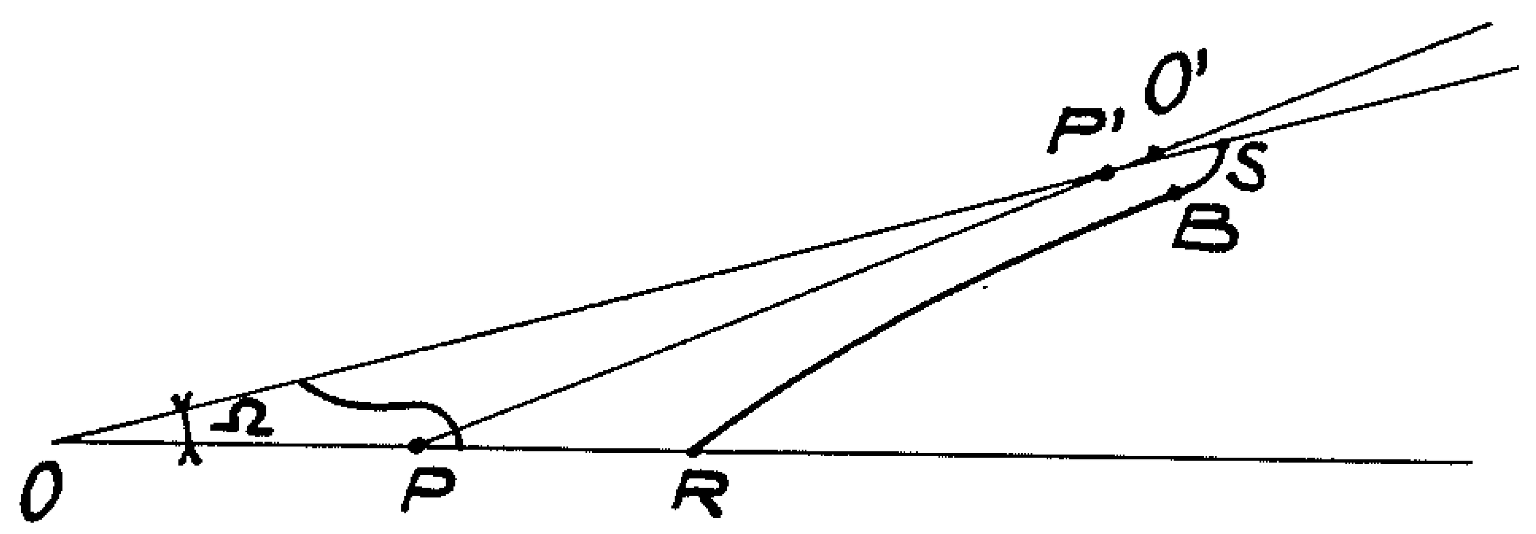


FIG. 11

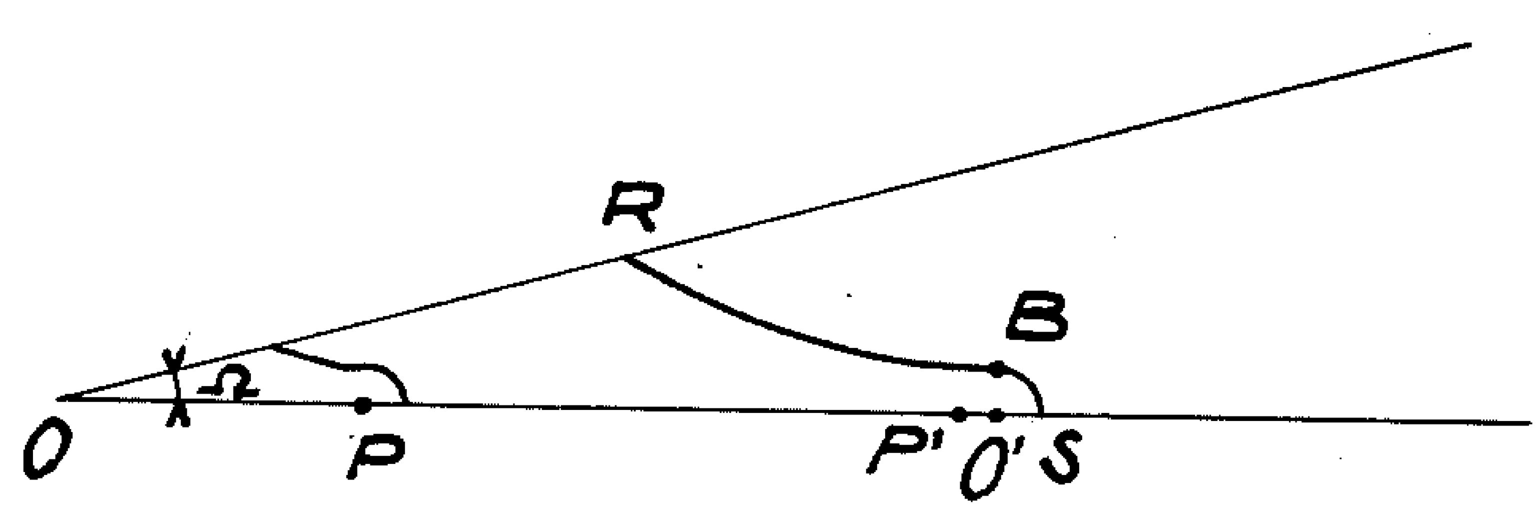


FIG. 12

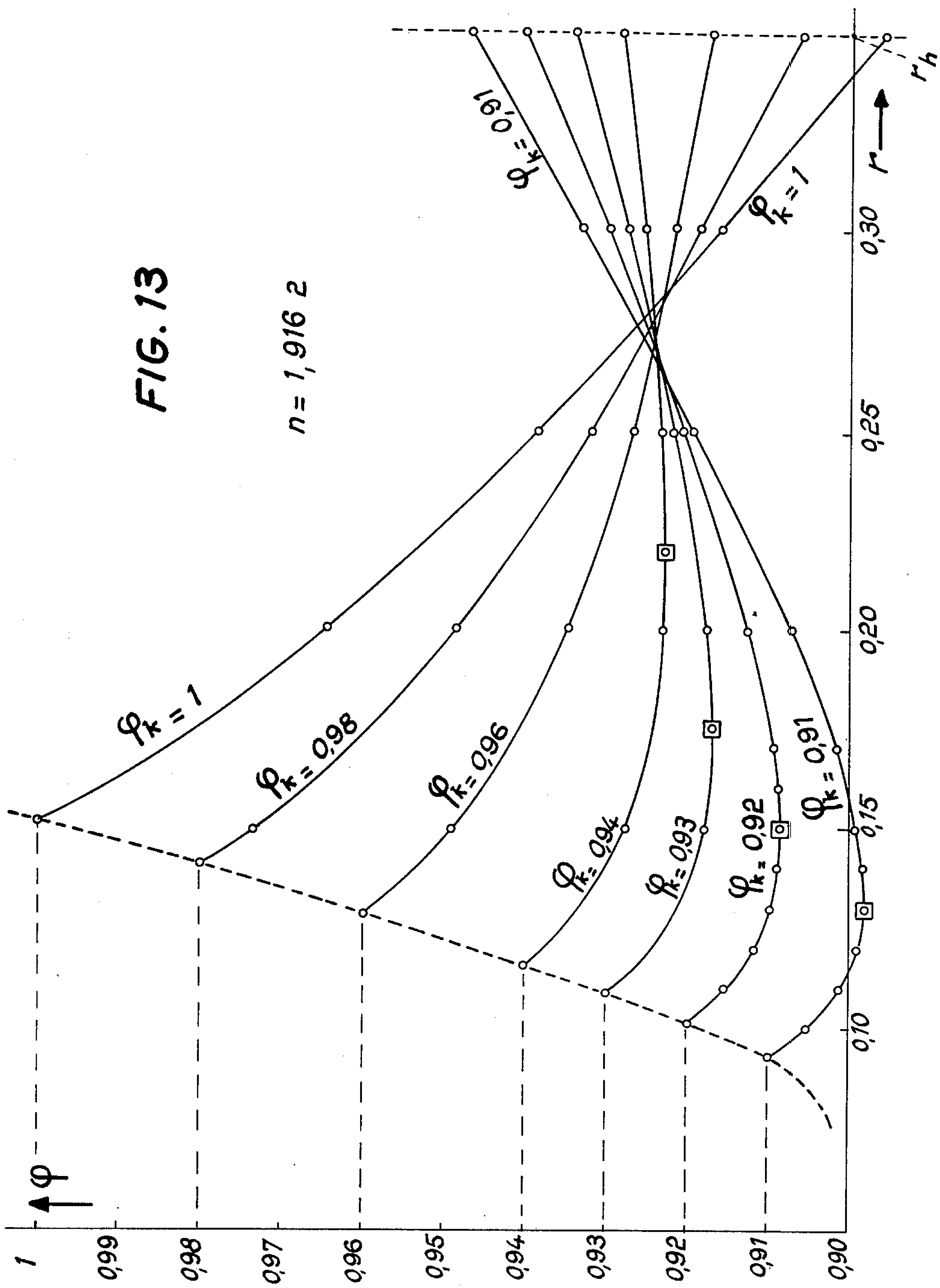

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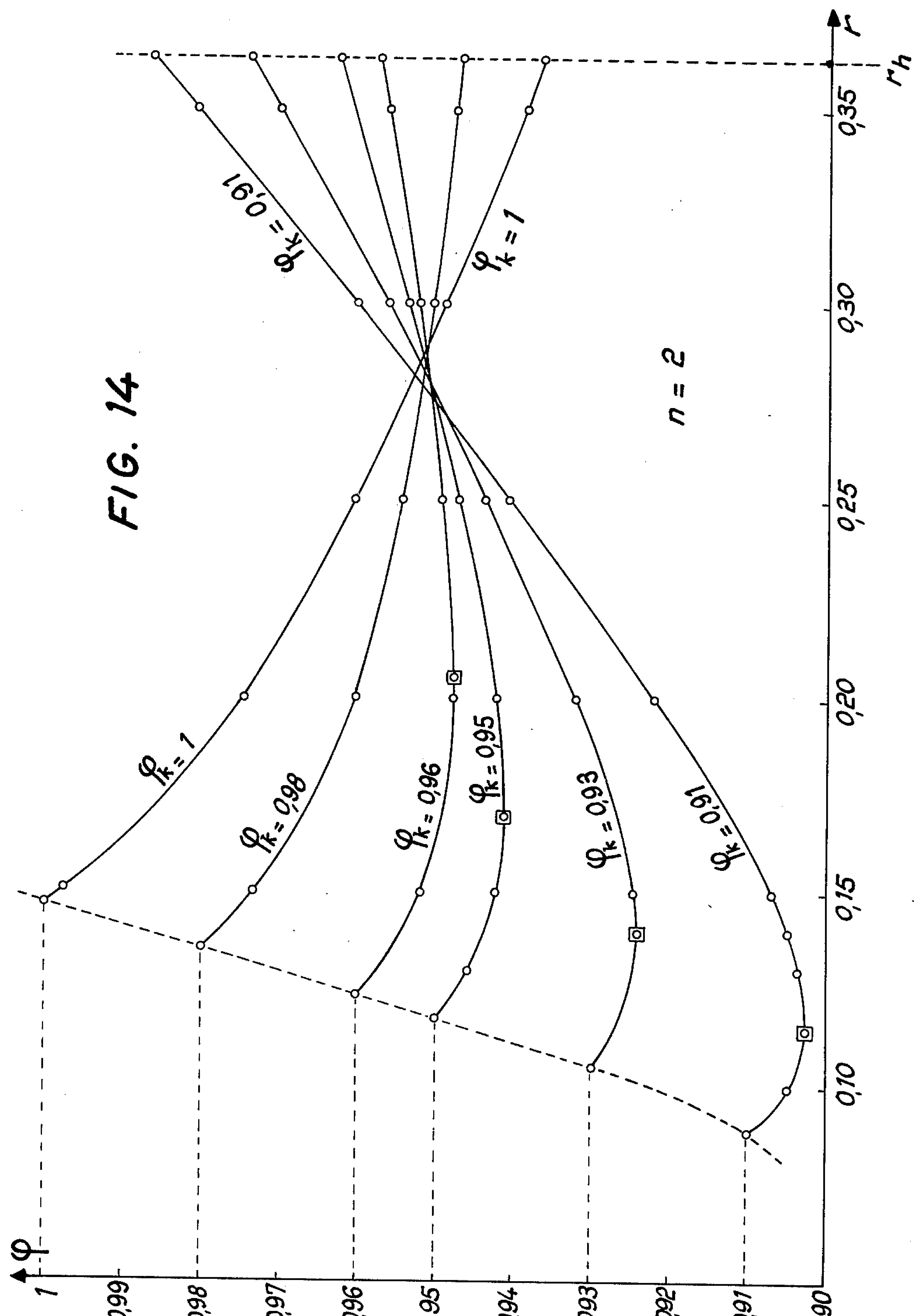
FIG. 13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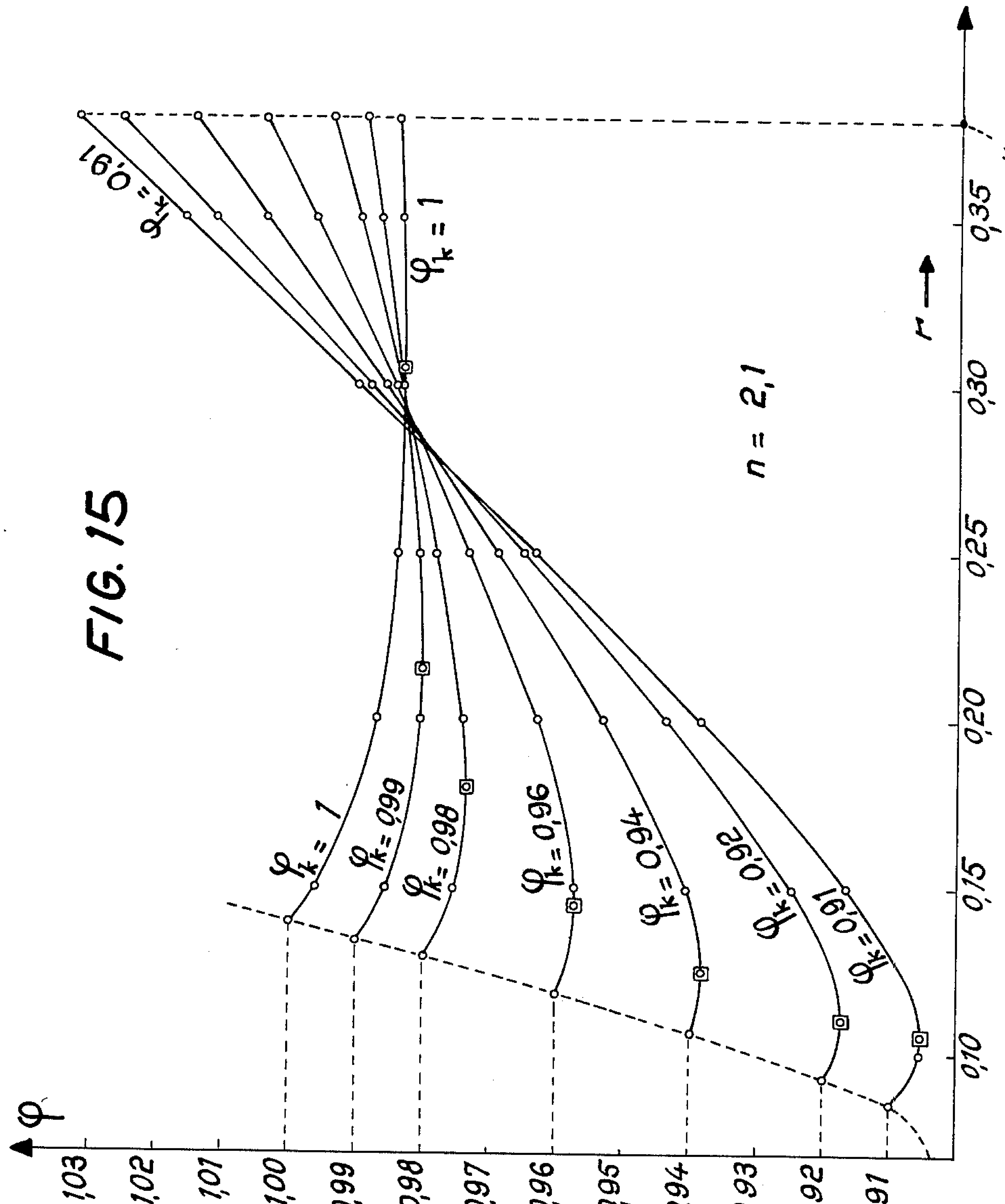
0,90

FIG. 14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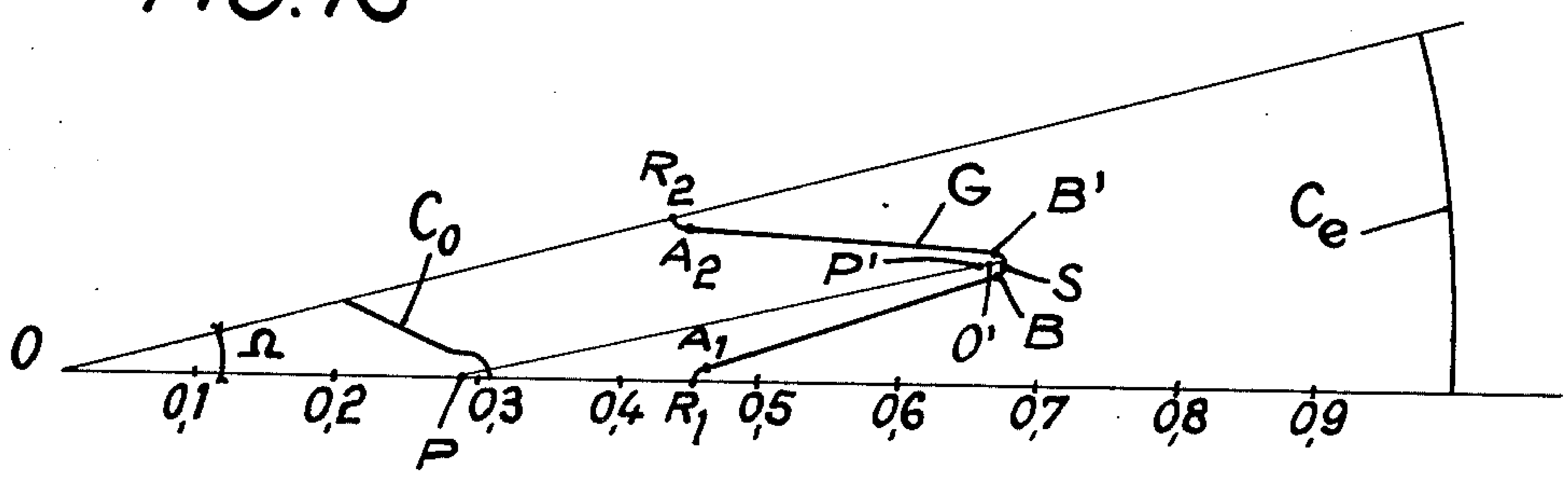
φ_k = 0,91

FIG. 15

FIG. 16



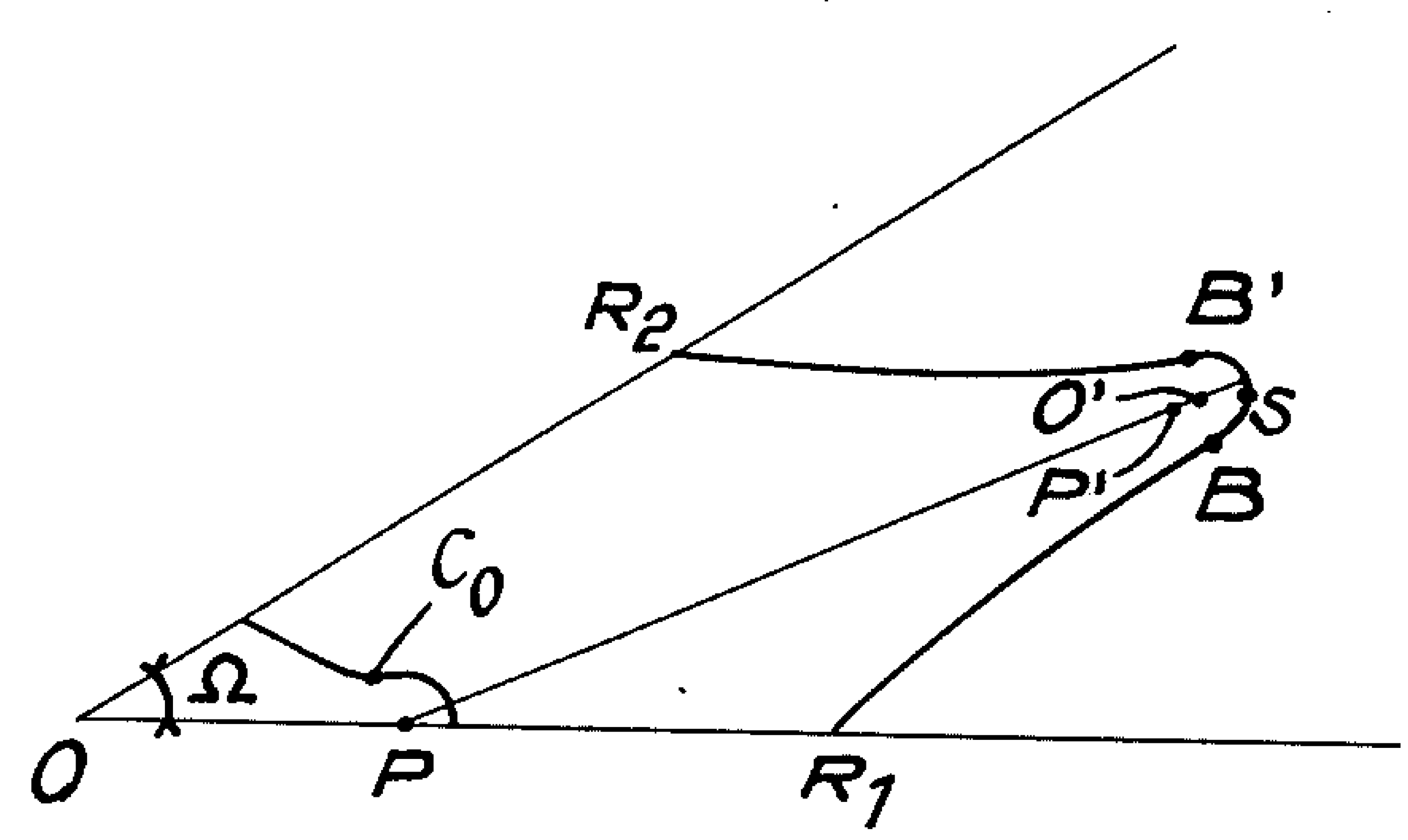


FIG. 17

FIG. 18

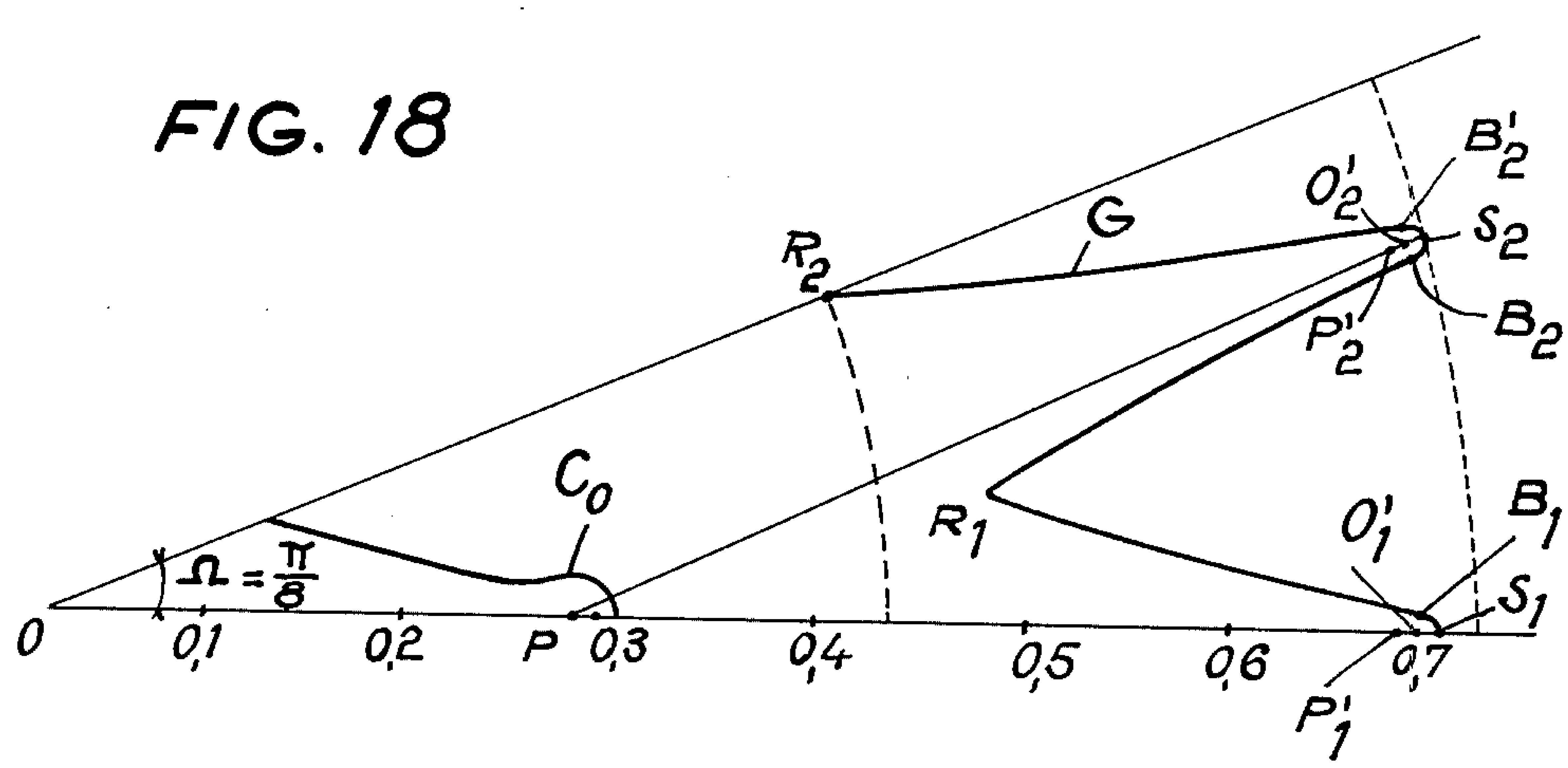
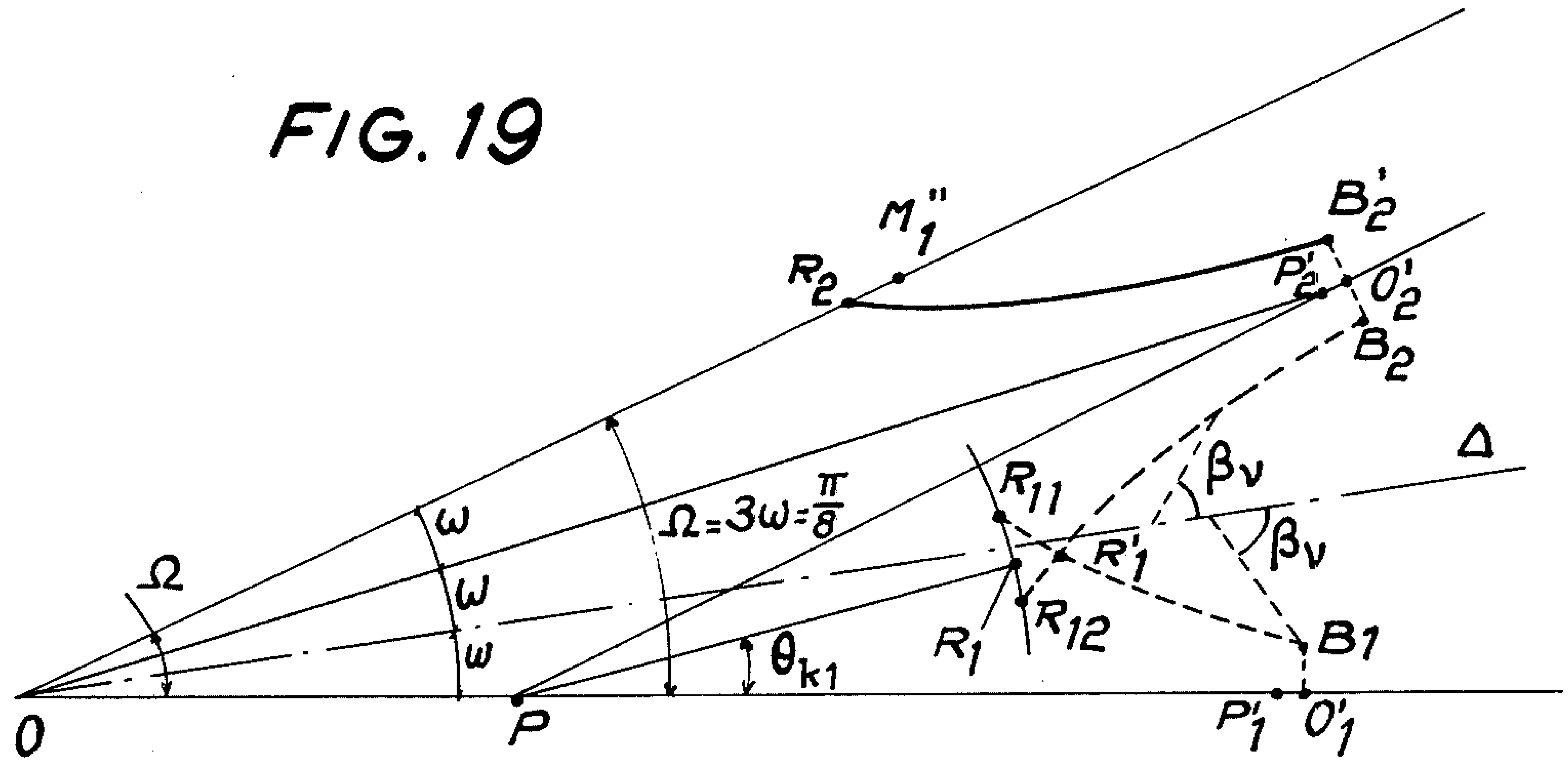


FIG. 19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P'2
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R11
βν
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R'1
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O
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P'1
O'1

FIG. 20

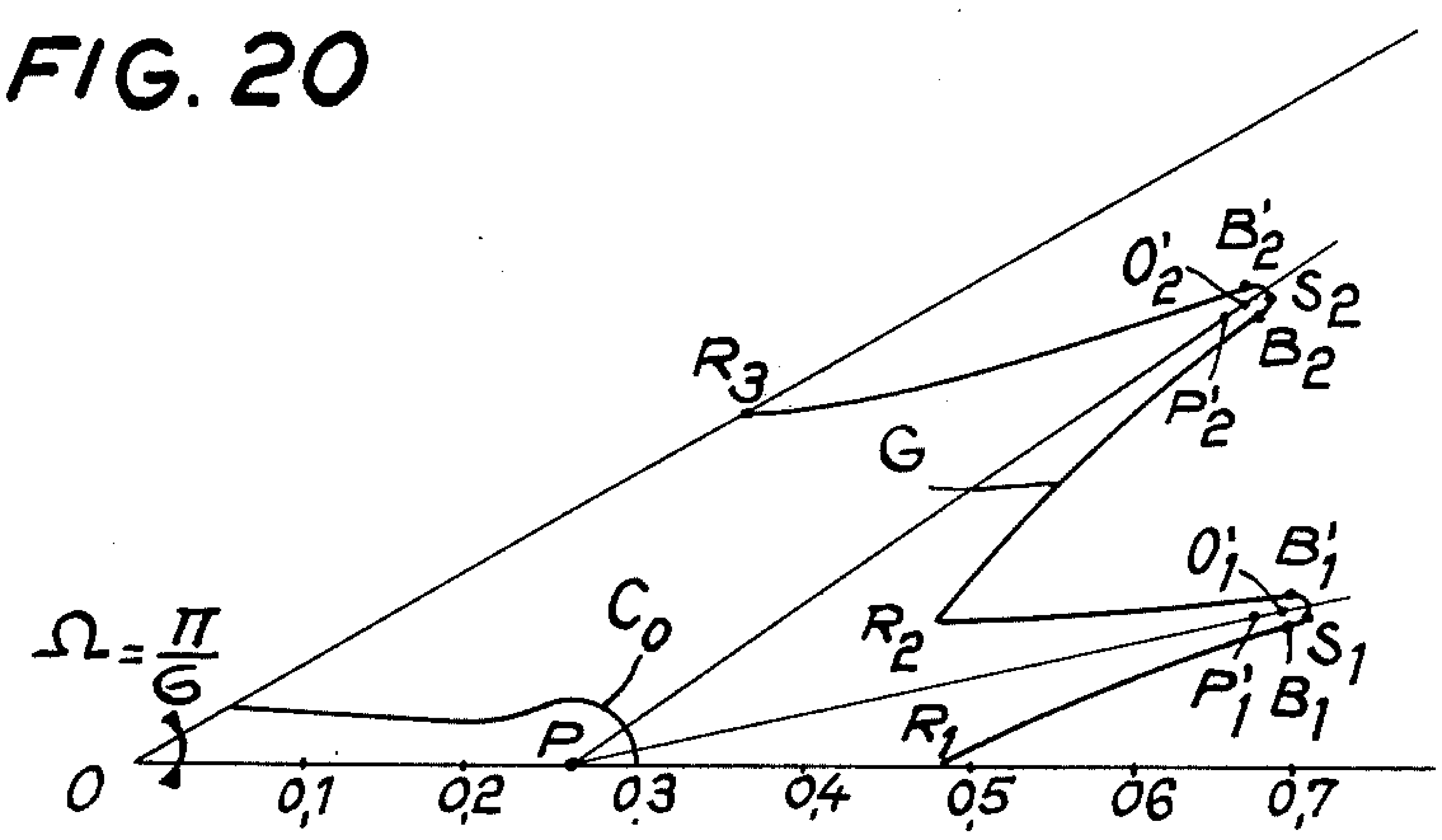
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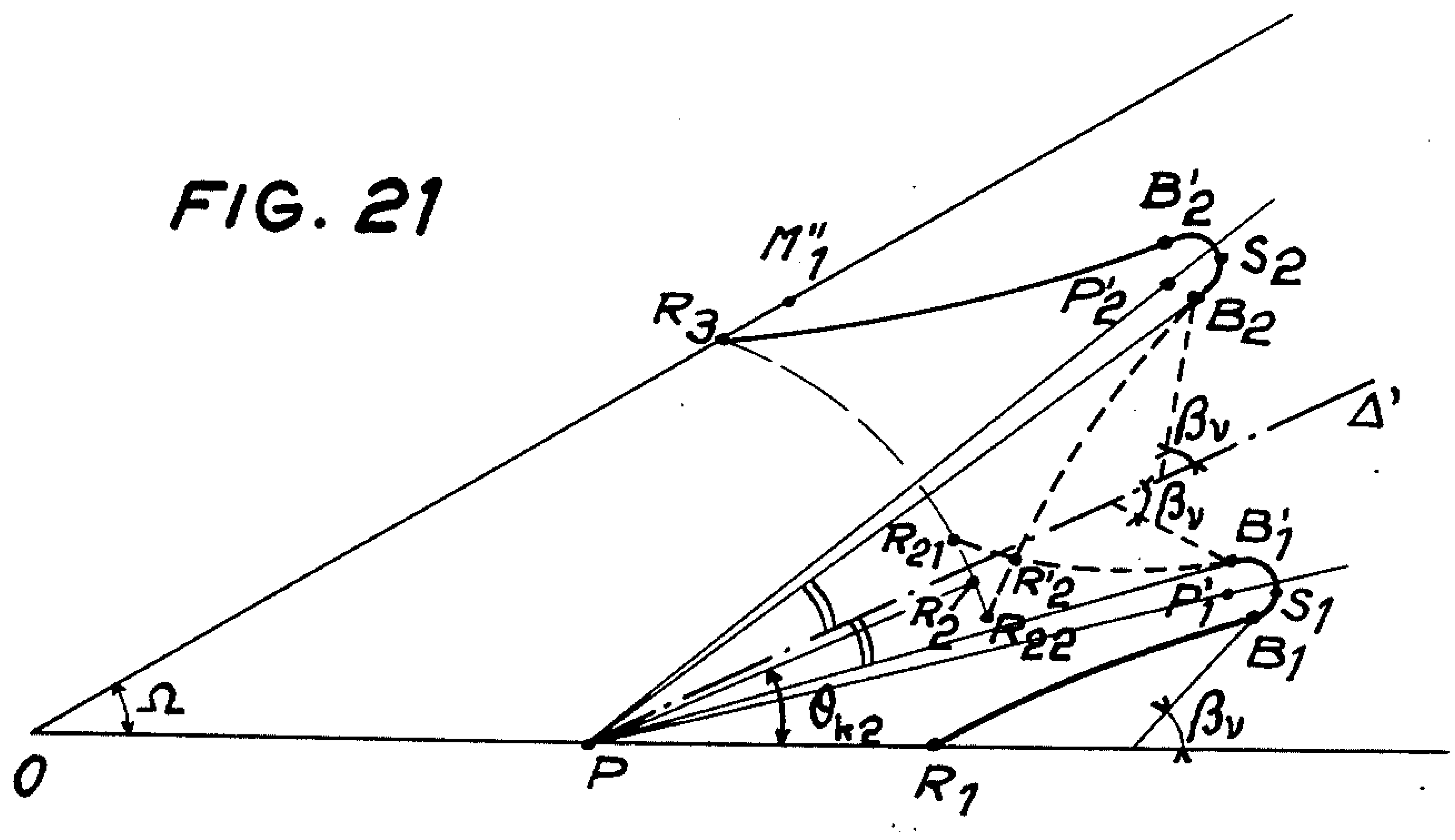
FIG. 21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βv
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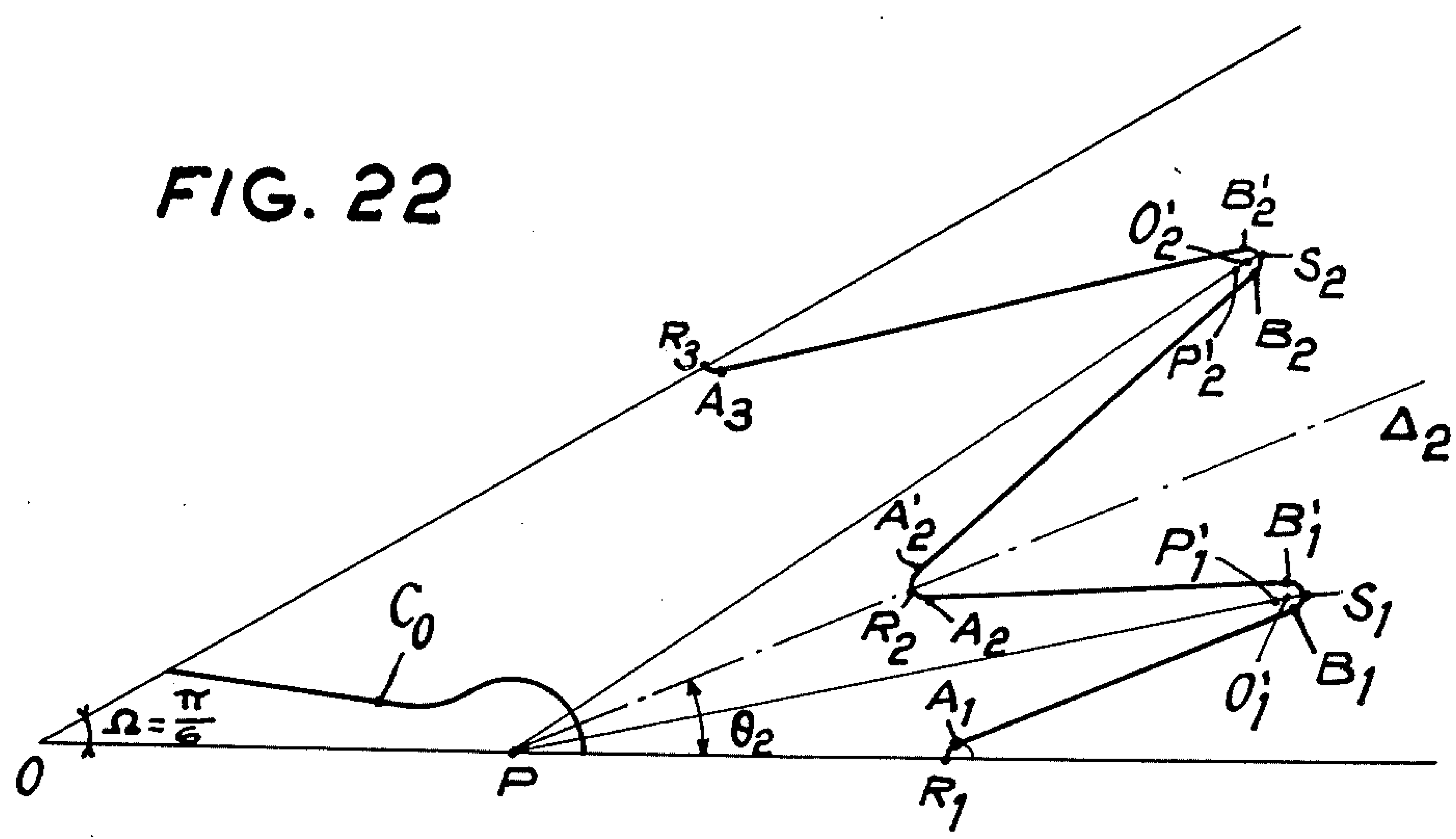
P
R1

FIG. 22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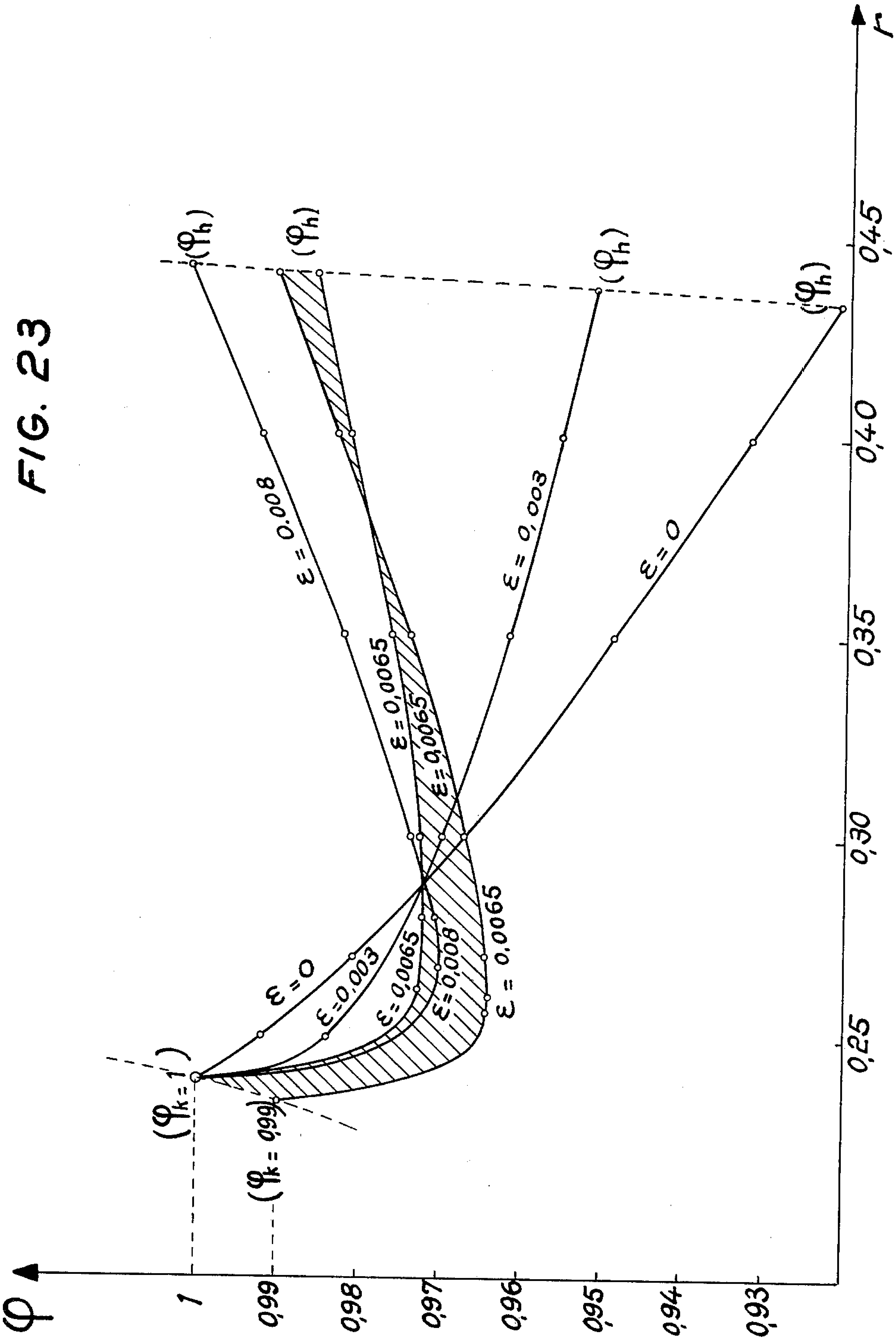
O
P
R1

FIG. 23



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0,40
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(φh)
(φh)
(φh)
(φh)
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ε = 0,0065

NEW AND USEFUL IMPROVEMENTS IN PROPERGOLS OR PROPELLANTS

This is a continuation of copending U.S. patent application Ser. No. 181,909, filed Sept. 20, 1971 now abandoned.

The present invention relates to products of block form of specific shapes, made by assembling two materials called propergols (i.e. propellants) and capable of being converted into gas, more especially usable for the reaction propulsion of civil or military vehicles in space or in a gaseous or liquid medium. The invention also relates to a method for determining the characteristic surfaces of the propergols manufactured in accordance with the invention.

These blocks are intended for combustion in chambers which are provided with one or more apertures, in which the gases produced during the combustion become pressurised with respect to the exterior, and which are generally cylindrical.

The outer lateral surface of a block in accordance with the invention has substantially the shape of a cylinder of revolution, and it is either adhered to the wall of the chamber, or inhibited, so as not to participate in the combustion. Along this surface and about its axis, the block has a central cavity, of elongated shape in the direction of this axis, set alight at the start of the firing; this cavity opens out at at least one of the ends of the block.

The block is manufactured with two separate homogeneous propergols, intimately coupled one to the other with interruption along a surface of separation all in one piece, surrounding the central cavity.

The present invention is further illustrated and described by reference to the accompanying drawings, wherein

FIG. 1 is a cross-section of a solid propergol prepared according to the invention;

FIG. 2 represents the relationships obtain in one sector of a typical propellant according to the invention;

FIGS. 3, 4 and 5 illustrate the relationships between the shape functions of a propellant and the reduced thickness during various phases of combustion;

FIGS. 6, 7 and 8 show relationships in typical sectors of propellants according to the invention;

FIG. 9 shows relationships between the shape function and reduced thickness of an embodiment during combustion;

FIG. 10 shows a partial cross-section of initial conditions and flame fronts during combustion of an embodiment of the invention;

FIGS. 11 and 12 show relationships in sectors of propellants according to the invention;

FIGS. 13, 14 and 15 show families of curves representing flame fronts or surfaces of combustion in certain embodiments;

FIGS. 16, 17, 18, 19, 20, 21 and 22 illustrate the relationships in sectors of various embodiments of the invention; and

FIG. 23 shows relationships between the shape functions and radial distances in certain embodiments.

The propergols each burn, both separately and simultaneously, in substantially parallel layers, but their combustion speeds are different, and the quotient n of the rate of combustion of the rapid propergol divided by that of the slow propergol is substantially constant. The rapid propergol occupies the entire volume of the block between the central cavity and the surface of separation, and the slow propergol occupies the whole volume of the block between the surface of separation and the outer surface.

On a section of the block made in its cylindrical portion perpendicularly to the directions of the generatrices, and referring to FIG. 1 of the accompanying drawings, the following terms are applicable:

"outer contour" C_e, is the outline of the outer surface it is constituted by a circle whose centre is marked O and whose radius has a length taken conventionally, in all that follows, equal to unity;

"inner contour" C_o, is the outline of the inner surface; it is constituted by a closed curve, without double point, surrounding O, and within C_e;

"separatrix" G, is the outline of the surface of separation; it is constituted by a closed curve, without double point, surrounding C_o, within c_e, not touching either C_o or C_e;

"flame front" $C(t)$, is the outline of the surface formed by the points under ignition at a given instant t of the firing; at the instant of the ignition, $C(t)$ is blended with C_o; at the instant when the combustion reaches the outer contour for the first time, $C(t)$ is a curve C_f touching C_e at at least one point; beyond C_o and as far as C_f inclusive, $C(t)$ is a closed curve, initially within G, then cutting G, finally outside G; in the course of the firing, the first flame front touching G is marked C_k, and the last one having one point at least in common with G is marked C_g.

The flame fronts or surfaces of combustion C_k, C_g, and C_f are shown as dashed lines, and the drawing is hatched to indicate that the section represented by the sectional area between inner contour C_o and separatrix G comprises a first faster-burning propergol and the section at area between separatrix G and outer curve C_e represents a slower-burning propergol.

Each inner contour C_o has the general aspect of a star having p branches, with p being at least equal to 3; this star is more or less deformed, but it is still selected such that at the end of the "rapid phase" of the combustion (the phase when the rapid propergol alone burns), the flame front C_k is formed only of p consecutive arches which all turn their concavity towards the centre O, which are each an element of curve parallel to a portion of the end of a diffeent branch of the star, and which are themselves substantially composed of arcs of circles whose centres, called "main centres of curvature", are situated on p radii of C_e called "main sides" and each intersecting the inner contour at one point of the end of a different star branch.

The separatrix G has the general aspect of a star having ν branches, with ν at least equal to p; this new star, more or less deformed, is still chosen and positioned in such a way that, on the one hand, at the start of the "slow phase" (the phase when the slow propergol alone burns), the flame front C_g itself also has the aspect of a star having ν branches envelopping as it were the separatrix (FIG. 1), and that, on the other hand, the flame front C_f is formed only of ν consecutive arches, turning their concavities towards O, all being substantially tangential to C_e, each being located opposite the end of a different branch of G, and being composed substantially of arcs of circles whose centres, called "image-summits", are all situated in the vicinities of the ends of the ν branches of G.

Due to the shapes of the curves referred to above, the blocks in accordance with the invention are called "bistellar blocks", whilst the conventional blocks having a single propergol and having an inner contour in the form of a star will be called, in contradistinction, "monostellar blocks" when reference is hereinafter made to them.

A first group of bistellar blocks is characterised in that the circle C_e can be cut up fictitiously into a certain number p' (at least equal to p) of sectors subtending equal angles at the centre, and all containing portions of C_k substantially superimposable one on the other and portions of G substantially superimposable one on the other; these blocks have the advantage of an easier study of the successive flame fronts as from the "mixed phase" (when the two propergols burn together).

However, such a study is also facilitated on a second group of bistellar blocks, called "symmetrical bistellars", characterised as follows by calling a sector of C_e limited by two consecutive main sides, the "main sector"; the portions of C_k and of G contained in any main sector each allow the inner bisectrix of the main sector to be the axis of symmetry; the main sides all intersect G at points situated on the ends of the star branches.

In this way, the evolution of the flame fronts in the blocks of the second group is effected without reciprocal influence of the different main sectors, and in each of these, it is also simplified by the existence of a symmetry in relation to the bisectrix of the main sector in question.

In a general manner, as from a main sector possessing such a symmetry, a sector of C_e limited by one of the sides and by the inner bisectrix of the main sector in question is referred to as an "elementary sector". This bisectrix is the "second side" of the elementary sector, and the portions of C_k and of G contained in an elementary sector constitute an "elementary motif".

The study and the performance of a symmetrical bistellar block are deduced immediately, and in evident manner, from the properties of the "main motifs", that is to say from the outlines of C_k and of G in the main sectors. However, these main motifs have properties which vary in a continuous manner with the value 2 Ω for the angle at the centre of the main sector. In this way, it is possible in practice to limit the study to main sectors capable of producing blocks belonging to a third group, which is constituted by definition of the blocks of the second group having their main motifs all substantially superimposable one on the other.

In order to assist in considering the description of the invention, there follows a legend of symbols and abbreviations used in disclosing the invention and embodiments thereof:

LEGEND OF SYMBOLS

n = Combustion quotient, the ratio of rate of combustion in rapid propergol : rate of combustion in slow propergol
C_e = Outer contour. Its radius is taken as 1.
O = Center of outer contour C_e.
C_o = Outline of inner surface
G = Separatrix or surface of separation between the two propergols — Does not touch C_e or C_o
$C(t)$ = Curve of flame front surface at time t
C_f = Curve of first contact of flame front with C_e — touches C_e at at least one point
p = Number of branches of star forming C_o. Is ≥ 3
C_k = Curve of first flame front touching G
ν = Number of branches of star forming separatrix G
Ω = Angle at center of main sector = π/p
Shape A = Shape of Co composed of circular arcs and straight line portions
Shape N = An inner contour of Shape A
Shape (or Form) D composed of straight-line segments = Final form of G
ϵ = Radius of arcs constituting roundoffs at G segments
Shape E = First outline of G
c = Class = (ν/p) which is ≥ 1 and at most = 4
Cg = Last flame front touching G
$l'(t)$ = Perimeter of $C(t)$ in rapid propergol
$l''(t)$ = Perimeter of $C(t)$ in slow propergol
$u'(t)$ = Flame radius in rapid propergol
$u''(t)$ = Flame radius in slow propergol
$u(t) = u'(t) + n\,u''(t)$ = corrected distance of $C(t)$
$l(t) = n'l(t) + l''(t)$ = corrected perimeter of $C(t)$
n' = Selected coefficient chosen so $l(t)$ is $C(t)$ if situated solely in slow propergol — Generally $n' = n$
ρ = Filling coefficient = surface between C_o and C_e/inner surface of C_e; preferably $\rho \geq 0.9$
σ = Ratio of theroretical residual = surface between C_f and C_e/surface between C_o and C_e; At most 0.05
ϕ = Shape function = corrected perimeter of $C(t)$/corrected perimeter of C_f
y = Reduced thickness = corrected distance of $C(t)/C_e$.
ϕ_o = Shape function of initial flame front, i.e., on C_o
ϕ_a = First maximum value
ϕ_v = Minimum value in fast propergol
ϕ_k = Terminal value in fast propergol
$\phi_{k'}$, $_{k''}$, . . . = Values at $C_{k'}$, $C_{k''}$ . . . in mixed phase, i.e., both fast and slow propergols
ϕ_μ = Median value
ϕ_h = Value at C_h, first of final median phase
ϕ_w = Final minimum value
ϕ_f = Value at C_f = 1
ϕ_e = Value at C_e = 0
$1-y_f$ = Residual reduced thickness
b = Common value of distances to O from main centers of curvatures
a = Arithmetic means of distances to O from apeximages
r_k = Shortest distance from G to a main center of curvature
a_M = Greatest distance of G to O = minimal shaping $\angle$ 0.75
r_o = Radius of curvature of outer apices of C_o
r_n = Minimal value of r_o = 0.06/bp ϵ = Radius of curvature at points of G situated in vicinity of outer apex
$\epsilon_n = 0.15/\nu$
Parameters of construction p, c, r_o, ϵ, b, a, r_k
Parameters of utilization r_o, ϵ, a_M, ρ, σ, ϕ, (various values)
Shape A = of C_o
$\lambda = \angle\ OM_3''T_1$
Shape N = Normalised inner contour
ω_ν = Angle of neutrality relative to ω
$2_\omega = 2\pi/\nu$
j = Number of apex-images and geometrical centers of Shape D or E
P = Main center of curvature of sector
a_i = Distance of ith apex-imagine from O
B_i = Inner apices of G
r_k = Radius of arcs of circles comprising C_k
π = Plane normal to axis
K_M = Maximum "inherent locking"
m = Number of different contours

z = Lengths in direction of gas flow

s = Elementary inner perimeter

ζ = Minimum value of shortest distance to main side.

Such blocks have, moreover, industrial interest, since it is generally not best for the main motifs to be different. However, if these motifs are all equal and each have their axis of symmetry, it is not absolutely necessary, for the convenience of the calculations, that the main sides intersect G at the ends of star branches.

For these reasons, the bistellar blocks whose description is detailed below are those of a fourth group, characterised in that: first, the main motifs are substantially all equal one with another and each allow the bisectrix of the corresponding main sector to be the axis of symmetry; then, the two conditions below, aiming at simplifying industrial manufacture, are satisfied:

on the one hand, the inner contour C_o is made solely of portions of straight lines or of circles, whereby, like C_k and G, the bisectrices of the main sectors may be axes of symmetry;

on the other hand, the separatrix G is, in its final form, called form D, composed solely of straight-line segments constituting the sides of the star branches and small arcs of circle, of common radius ϵ constituting the round-offs at the junctions of these sides.

However, it has been found that, for this shape D, the portions of flame fronts situated in the slow propergol were very close to lines made of portions of straight lines or of circles, and that conversely, if G had been traced so as to make the flame fronts in the slow propergol strictly identical to such lines, there would have been obtained a separatrix shape very close to the shape D, the sides of the branches therefore being very stretched hyperbolic arcs, and the contours of the ends of the branches becoming portions of small Descartes ovals.

It has been found then that it was possible, with a view to simplifying the calculation of the perimeters of flame fronts as from C_k, to make a first outline of G, called "shape E", defined in this way:

the sides of the branches are constituted by very stretched hyperbolic arcs, which are each constructed so as to transform the circular flame fronts of the rapid propergol into rectilinear flame fronts in the slow propergol, which meet two by two in order to create on G angular points between neighbouring branches, and which are connected by portions of small ellipses to the ends of the branches;

these ellipses confrom to an approximation of the Descartes ovals and are, to this end, constructed so as to change the flame fronts in the rapid propergol which arrive at the ends of the branches and which can be likened for this purpose to small straight-line segments, into circular flame fronts in the slow propergol;

the connection betwen an hyperbolic arc and an elliptical arc is effected either tangentially or (to simplify the calculations, in the case of a general study or of a preliminary plan) on an apex of the small axis of the ellipse; unless there are indications to the contrary, this is the mode of connection which will be used in the examples which follow;

finally, all the ellipses used in the construction of G are equal one to another.

However, a shape E leads to straight-line or circular flame fronts in the slow propergol only for a single value of n, namely precisely the one which has been used upon the geometrical definition of the ellipses and of the hyperbolas.

However, once the outline of the separatrix has been perfected in an E shape, it is easy to deduce from this latter a shape D which is very close thereto. There has, therefore, been obtained, with the minimum of trial and error, an outline of G which can be manufactured industrially and from which can be made without additional difficulties, if so desired, calculations of flame fronts for other values of n, more especially for values close to that retained at the start.

Finally, the shapes D and E of the separatrix can both be used in the course of the study of the blocks of the fourth group. This group is called that of the "symmetrical bistellar blocks of order p and of class c" (or more briefly "blocks of order p and of class c"), the class c being the entire quotient (ν/p), at least equal to 1.

For the shape E, each apex-image is one of the focusses, called "main focus", of the ellipses forming the contour of G at the end of a branch; if this end is inside, or intersects the main side, of an elementary motif, there is only a single elliptical arc connecting the two sides of the star branch; if the same end intersects the second side of an elementary motif, it is then formed from two small elliptical arcs symmetrical in relation to the second side of the sector and having the same main focus, situated on this second side.

By definition, a point of any closed curve surrounding O is called "outer apex" or "inner apex" if its distance to O is greater or smaller, respectively, than that of the points situated in its vicinity, before and after it on the curve.

For the shape D, the apex-images are substantially the centres of curvatures at the outer apices of C_g, or C_f, or of any flame front between C_g and C_f.

In all the cases, the separatrix G is called "regular" if the apex-images are all equidistant from O, and "not regular" in the contrary case; it is called "of first sort" if it has outer apices on the main sides, and "of second sort" in the contrary case.

The whole of the flame fronts $C(t)$ between C_o and C_f constitutes a family of parallel curves having for orthogonal trajectories lines each formed by two segments of straight lines (or exceptionally by a single one), all leaving from C_o, and called "flame radii". In other words: $l'(t)$ and $l''(t)$, respectively, the portions of perimeter of $C(t)$ situated in the rapid propergol and in the slow propergol; $u'(t)$ and $u''(t)$, respectively, the lengths situated in the rapid propergol and in the slow propergol of a flame radius going from C_o to $C(t)$. The one or the other of the lengths $l'(t)$, $l''(t)$ can obviously be nil, and the same holds true for the one or the other of the lengths $u'(t)$, $u''(t)$.

The sum $u(t) = u'(t) + n\, u''(t)$ is called "corrected distance" of the flame front $C(t)$.

The expression "corrected perimeter" of $C(t)$ is used to refer to the sum $l(t) = n'\, l'(t) + l''(t)$, where n' is a coefficient chosen in such a way that $l(t)$ represents the perimeter which $C(t)$ ought to have, if it were situated solely in the slow propergol, so that at the instant t of the firing the pressure or the thrust obtained are substantially the same as with this flame front. Generally, n' is equal to the ratio of the speeds n, and it is this value which will be adopted in the following; however, the conduct ot the calculations is absolutely similar if $n' \neq n$.

It is obviously desirable that inside C_e the surface occupied by the propergols be the greatest possible, taking into account the space necessary for the normal flow of the burned gases through the central cavity; it is also necessary that the portion of this surface between C_f and C_e be the smallest possible since it is the domain of the discontinuous flame fronts, having a very rapidly decreasing perimeter, therefore of poor yield (the final flame front of the firing called "punctiliar flame front" and marked $\mathring{C}_e$ is reduced to a finite number of points of C_e). By definition:

the "filling coefficient ρ" is the quotient of the surface between C_o and C_e divided by the inner surface of C_e; a high value of ρ is sought, generally at least equal to 0.9;

the "theoretical residual rate σ" is the quotient of the surface between C_f and C_e divided by the surface between C_o and C_e; a low value of σ is sought, generally at the most equal to 0.05;

the "shape function ϕ" is the quotient of the corrected perimeter of any flame front $C(t)$ divided by the corrected perimeter of C_f; ϕ can be considered as a function of "the reduced thickness y", quotient of the corrected distances of $C(t)$ divided by that of the punctiliar flame front C_e; generally, it is desired that the graph of ϕ (y) does not deviate too much (generally not by more than 10%) from a standard predetermined curve, and constituted more often than not in its major part by one or more straight-line segments which are horizontal or slightly inclined.

For all the blocks which are the objects of the invention, the graph of variation of ϕ as a function of y has three successive portions, each of which has a characteristic shape.

The first is that corresponding to the rapid phase; it comprises two periods, as shown in FIG. 3 of the accompanying drawings.

The initial period (lines in dashes of the graph) concerns the flame fronts which are not made solely of arcs of circles, whose centres are on the main centres of curvature; the following are to be noted:

ϕ_o the value of ϕ on the initial flame front, that is to say on C_o;

ϕ_d a first maximum, after an outline start which is generally rectilinear or polygonal.

The following period relates to flame front constituted solely by arcs of circles having centres on the main centres of curvature. The corresponding graph is a curve convex downwards; therein:

ϕ_r is the ordinate of the minimum (flame front C_r);

ϕ_k is the terminal ordinate, which is more often than not a second maximum (flame front C_k);

In certain cases, ϕ_r can merge either with ϕ_d or with ϕ_k.

It is to be noted that it is a question, in this phase as in the others, of maxima and of minima which are relative.

The second portion corresponds to the mixed phase and goes from C_k to C_g; three periods are distinguished there (see FIG. 4).

The commencement period commences with the first flame front noted C_k which touches G, and ends with the first flame front which reaches G in the vicinities of all its inner apices; it can be reduced to the flame front C_k, more especially when the elementary motif comprises only one inner apex of G; if it comprises more than one flame front, the inner apices contained in this motif can be touched successively by flame fronts which are then marked C_k, $C_{k'}$, $C_{k''}$, etc. . . . . The values of ϕ relative to these flame fronts are marked ϕ_k, $\phi_{k'}$, $\phi_{k''}$, etc., and the corresponding points of the graph are angular points, of ordinates either less than or higher than ϕ_k, capable or not of being separated by points where ϕ is minimum and whose ordinates are then marked $\phi_{\mu'}$, $\phi_{\mu''}$, etc.

Then comes a median period, characterised in that the number of the points of intersection of the flame fronts and of the separatrix remains equal to 2 ν. The graph is then a curve convex downwards, with a minimum, of ordinate ϕ_μ, situated generally somewhat close to the start of the period. However, sometimes ϕ_μ is shifted to one of the ends of the period and constantly increasing or constantly decreasing.

The final period of the mixed phase commences with the first flame front, marked C_h, which reaches a point of connection on the separatrix between a side and a round-off of an outer apex. The value of ϕ relative to C_h, marked ϕ_h, is quite often a maximum; it always presages a change in the aspect of the variation of ϕ. This period comprises only the flame front C_h in the extreme case where all the outer apices of G are angular and all belong to one and the same flame front which is then C_h. In the general case, it comprises other flame fronts subsequent to C_h, intersecting or touching the separatrix but capable of having with it less than 2 common points; its final flame front is C_g, and those for which there occurs a discontinuity of the number of the points common with G are, ascending as from G, marked $C_{g'}$, $C_{g''}$, etc; on the graph, the corresponding ordinates are, in the order of the increasing y, ϕ_h . . . etc., $\phi_{g''}$, $\phi_{g'}$, ϕ_g; these ordinates decrease rather suddenly, from ϕ_h to ϕ_g, and the representative points are angular points of the graph.

The third portion corresponds to the slow phase as shown in FIG. 5 of the accompanying drawings. The noteworthy ordinates are:

ϕ_w, final minimum of the curve (y, ϕ), situated generally towards the middle of the phase;

ϕ_f, identically equal to 1, for the flame front C_f;

ϕ_e, identically equal to **0**, for the punctiliar flame front C_e.

The minimum ϕ_w can slip either towards ϕ_g or towards ϕ_f, but the graph is always either rectilinear or convex downwards.

The drop of ϕ, from $\phi_f = 1$ to $\phi_e = 0$, has to be effected in a time which is as short as possible in order to avoid a rupture of the chamber through excessive heating in the case of the moulded and adhered charges. It is, therefore, sought to make the difference $1 - y_f$, referred to as "residual reduced thickness", very small, where y_f is the thickness reduced relative to C_f.

For example ϕ arranged to be between 0.85 approximately and 1 during the entire firing.

Three geometrical magnitudes, drawn directly from C_o and from G, play an important part in the evolution of ϕ; these are:

the common value b of the distance to the centre O of the main centres of curvature;

the arithmetical mean a of the distances to the centre O of the apex-images;

the shortest distance r_k from the separatrix G to a main centre of curvature.

Three other geometrical magnitudes $a_M r_o$, ϵ, concern the manufacture of the propergol:

a_M is the greatest distance of the separatrix G to the centre O, and is called the "minimal shaping" of the block; in fact, when this latter is moulded directly in a propellent by means of cores of the conventional type (that is to say non-retractable), a_M is the minimal radius possible of the aperture of the rear bottom. For the large propellents, it is generally necessary that $a_M < 0.75$;

r_o is the radius of curvature at the outer apices of C_o; it is equal to zero if these apices are angular points; the value of r_o is closely tied to the admissible level of the internal tensions in the propergol mass at the ends of the star branches of the inner contour, and has to be chosen large enough to avoid cracking; with the usual propergols, and in order to be able to compare several configurations, r_o can be assigned a minimal value r_n resulting from the empirical formula: (1)

$$(1)\quad r_n = \frac{0.06}{b};$$

ϵ is the radius of curvature at the points of G which are situated in the vicinity of an outer apex, and where the radius of curvature passes through a minimum; it is equal to zero if these apices are angular points; in the case of the moulded and adhered blocks, it has to be large enough to avoid cracking in the slow propergol before the moulding of the rapid propergol; as a rough assumption, it can have a value approximately equal to the value ϵ_n supplied by the empirical formula:

$$(2)\quad \epsilon_n = \frac{0.15}{\nu}$$

All things considered, a certain number of numerical characteristics are available which can be classed either as "parameters of construction" ($p, c, r_o, \epsilon, b, a, r_k$) or as "parameters of utilisation" ($r_o, \epsilon, a_M, \rho, \sigma$, and the noteworthy values of ϕ).

The calculation of a usable block of propergols is effected by starting from the conditions of production or of use. These conditions oblige various parameters to be situated within certain intervals. The problem consists, as from these data, in constructing blocks having the best properties.

One of the elements of the invention is a shape of the inner contour C_o, considered in itself independently of any dimensional indication, called "shape A", and constituted by a succession of circular arcs and of straight line portions which comprises in an elementary sector, such as that shown in FIG. 2:

a circular arc T'T, of radius r_o, having its centre merged with a main centre of curvature P situated on the main side, inside the segment OT';

a circular arc TT_1, connected tangentially to the arc T'T, having its centre at a point M_3'' situated on the second side of the sector and characterised by the angle $\widehat{OM_3''P} = \Omega''$;

a straight line portion T_1T_2, connected tangentially to this second circular arc, and of orientation characterised by the angle $\widehat{OM_3''T_1} = \lambda$;

a final circular arc T_2T'', of radius r'_o, connected tangentially to T_1T_2, having its centre at a point T_o situated on the second side inside the segment $T''M_3''$.

Such a shape can evolve according to the values of the radii of the circles and according to the length of its rectilinear portion.

More especially, r_o, r'_o, λ, and the length of the segment T_1T_2 can, separately or not, be nil.

If the angle at the centre Ω (equal to π/p) of the elementary sector is assumed known, the radius r_o of the arc of circle centred on P, and if the evolution of the function ϕ in the rapid phase can be imagined, it has been found that Ω'' and, consequently, the position of the point M_3'' can be determined by the relations,

$$(3)\quad \frac{\sin \Omega''}{\Omega + \Omega''} = \frac{\phi_r}{\phi M_3''} \cos \Omega_p,$$

with 8n

$$\Omega_p < \Omega'' < \frac{\pi}{2}$$

in which the angle Ω_p, called "angle of neutrality relative to Ω", is defined by the relations:

$$(4)\quad \tan \Omega p - \Omega p = \Omega, \quad O < \Omega p < \frac{\pi}{2},$$

and $\phi_{M_3''}$ is the value of ϕ relating to the flame front passing through M_3''.

The numerical parameters of a shape A can vary generally in a continuous manner within fairly large intervals, under some conditions of compatibility.

It is necessary that the line $T'\ TT_1\ T_2\ T''$ which has just been defined dows not leave the elementary sector. It can be seen immediately that, when the angle at P of the triangle $OM_3''P$ is acute

(that is to say if $\Omega + \Omega'' > \frac{\pi}{2}$),

the arc $T\ T_1$ runs the risk of intersecting the side OP if the radius r_o of the circular arc T' T is less than a certain minimum r_t, function of b, Ω, Ω''. Likewise, the angle $\lambda = \widehat{OM_3''T_1}$, where T_1 is the point of connection of $T\ T_1$ and of the segment $T_1\ T_2$, must not, all things equal moreover, exceed a certain minimum.

What may be referred to as a "normalised" inner contour, or contour of "shape N", an inner contour of shape A, is characterised in that:

r_o is equal to r_t when at the same time

$$\Omega + \Omega'' > \frac{\pi}{2}$$

and $r_n < r_t$; and it is equal to r_n, defined by the relation (1), in all the other cases;

λ is equal to the angle of neutrality Ω_p relative to Ω and defined by the relations (4).

Experiments have shown that such contours are those allowing the greatest coefficient of filling ρ to be obtained for any given values of p, b, r'_o and ϕ_d and taking (1) into account. And since the value chosen for r'_o has generally a negligible influence on the coefficient of filling ρ, it is used principally to adjust ϕ_o to the desired value.

There has also been found a certain number of relations to be respected between certain characteristics of the curves C_o and the characteristics of the curves G.

Referring now to FIG. 6 of the accompanying drawings, let an outer apex S of G situated inside, or on the sides of, an elementary sector bearing on its main side a main centre of curvature P; let P' be the apex-image adjacent to S, and O' the geometrical centre of the arc of circle or the ellipse belonging to G, in the vicinity of S, in the sector involved.

If G is of shape E, the distance $\epsilon' = P'P'O'$ is known as soon as n is assumed as well as a parameter of the arc of ellipse, for example the semi-small axis ϵ_b, and P′ is known to be on the segment PO′.

If G is of shape D, it has been found that with an approximation sufficient in practice, and by virtue of the fact that the radius ϵ of the round-off remains small, the point P′ can be placed on the segment PO′ at a distance $\epsilon' = P'O'$ from O′ such that, by laying down $q' = PO'$:

$$\epsilon' = \frac{\epsilon}{n-1 + \frac{\epsilon}{q'}}$$

The approximation made in this way has as a consequence that, if S is situated on the second side of the elementary sector, the apex-image relative to this apex is likened to two separate points according to whether the one or the other of the elementary sectors is considered as having this second side; however, this is not a drawback in practice, and moreover the two points in question are at the same distance from the centre O.

It has been found, on the one hand that, in order to minimise σ, it is desirable to place the outer apices of G on radii of the circle C_e cutting-up on this circle ν sectors of angles at the apices all substantially equal to

$$2\omega = \frac{2\pi}{\nu};$$

then a and b are given exactly or with an approximation sufficient in practice by the relations below, where ω_ν is called "angle of neutrality relative to ω":

$$(6)\ a = \frac{\sin \omega}{\sin \omega + \sin \omega'} \cdot \frac{\sin \omega'}{\omega + \omega'} = \phi_{ir} \cos \omega_\nu \ , O < \omega < \omega_\nu$$

$$(7)\ \tan \omega_\nu - \omega_\nu = \omega,\ O < \omega_\nu < \frac{\pi}{2},$$

$$(8)\ b = \frac{a}{n'} \cdot \frac{\phi_r}{\phi_{ir}} \cdot \frac{\sin \omega}{\omega} \cdot \frac{\Omega}{\sin \Omega} \cdot \frac{\cos \Omega_\nu}{\cos \omega_\nu}.$$

On the other hand, P'_i and O'_i are used to designate ($i = 1, 2, 3 \ldots j$) the j apex-images and the j geometrical centres of the ends of a separatrix, of shape D or, which are situated inside or on the sides of one and the same elementary sector; ϵ'_i refers to the distances $P'_i O'_i$, q_i the distances PP'_i of the apex-images at the main centre of curvature P of the sector, and a_i the distances of the same apex-images from the centre O; and it has been found:

that it is advantageous for the convenience of calculation to place, on j radii of C_e whose angles with a main side of the sector are worth either $(2J-1)\ \omega$ or $2J\ \omega$, with $J = 1, 2, \ldots$ etc., j, the geometrical centres O'_i when the separatrix is of shape D, or the apex-images P'_i when it is of shape E;

that in the two cases, the j lengths $q_i = PP'_i$ are the roots of a system of j equations expressing, on the one hand, the fact that a is the arithmetical mean of the ν distances to the centre O of the apex-images relating to the ν apices of the separatrix, on the other hand, the equality of value of the j different expressions which are obtained for the corrected distance of C_f when each of the j flame radii passing through the apices-images P'_i is followed, this equality being able to be expressed by the $j-1$ equations below: (9) $U_1 = U_2 = \ldots = U_j$, with $U_i = na_i + (n-1)\ '_i - q_i$. and $i = 1, 2 \ldots$ etc., j.

It will be apparent that the formulae given above and allowing the determination of the positions of the outer apices of the separatrix G are not absolutely mandatory, since they may have a certain number of reasonable approximations. Likewise, upon the effective realisation of the blocks of propergols in accordance with the invention, the theoretical positions of the outer apices of G, determined by the said mathematical formulae, will be able to be more or less respected as a result of the imperfections inherent in the physical processes used. It follows that blocks made in accordance with the invention cannot be limited to those for which the outer apices of G obey strictly the ideal positioning defined by the said formulae, but also extend to the other blocks of the same type in which the positionings of the outer apices of G are close to these theoretical positions.

Let $R_{i'}$ ($i' = 1, 2 \ldots j'$) be the inner apices of G situated inside or on the sides of the elementary sector and $R^*_{i'}$ the points of G situated in the vicinity of the $R_{i'}$ and such that their distances to the main centre of curvature P of the same sector pass through a relative minimum (the $R^*_{i'}$ can be wholly or partly merged with the $R_{i'}$). To determine the $R^*_{i'}$ comes back to determining the $R_{i'}$. The term $r_{ki'}$ denotes the length of the segment $PR^*_{i'}$, $\theta_{ki'}$ the acute angle of this segment and of the main side, and r_k the smallest of the lengths $r_{ki'}$; r_k is equal to the radius of the arcs of circle constituting C_k, and it has been found that its expression as a function of ϕ_k is given by the equations:

$$(10)\ \frac{r_k}{b} = \frac{\sin \Omega}{\sin \Omega'},\ \text{with}\ \frac{\sin \Omega'}{\Omega + \Omega'} = \frac{\phi_V}{\phi_k} \cos \Omega_\nu,\ O < \Omega' < \Omega_\nu$$

For $j' < 1$, the distances $r_{ki'}$ are chosen taking the following criteria into account:

first of all, they have to be spaced out so that ϕ evolves, just after the flame front C_k, in the sense desired; generally, it is desired that at the start of the mixed phase ϕ no longer increases, or at least begins very soon to decrease;

then, they must not be too small, so that the branches of G have a length sufficient to have a correct action on the flame fronts coming from the rapid propergol; if two sides of G having in common an inner apex R were too short, ϕ could rise too much before the flame fronts reach R; then, this point R being reached, ϕ would drop in a very pronounced manner, arriving at the end of the mixed phase at a value lower than that which would have been given by longer branches;

finally, they can satisfy various desirable convenient factors; for example, if they are all taken to be equal, the calculations of ϕ in the mixed phase will be facilitated; or, for the sake of regularity, they can be taken such that the distances to the centre O of all the inner apices are equal.

As for the angles $\theta_{ki'}$, they are determined so as to satisfy the imperatives of variation of ϕ and to facilitate calculation; they are chosen, preferably, such that the inner apices of G are not too remote from the bisectrices of the angles formed by the radii of C_e passing through two consecutive outer apices of G.

All the determinations below are made after the values of p, c, n and noteworthy values of ϕ have been assumed. These latter are supplied as desired by the users; p and c are chosen mainly as a function of the desired values of ρ and of σ, with the assistance of the tables of results of Example 1 below. As for n, it is determined by trial and error, by trying to construct the block with a certain value and by observing if the curve of evolution of ϕ which results therefrom is acceptable; a complete illustration of the method and of the influence of n on the function of shape is given in Example 3 below. However, it can be said in general that, if the usual conditions of maximum or of minimum are imposed on the variations of ϕ, there exists, for a wide range of values of p and of c, a certain interval of the possible values of n; it has been found that the mean point of this interval varies "grosso modo" in the converse sense of ν, and is slightly less than 2 for $\nu = 24$.

At a point situated on the axis of the block and such that the plane π normal to the axis at this point intersects the lateral surface of the central cavity, the "inherent locking" ("serrage propre") is by definition the quotient, of the area of the portion of surface of the central cavity which generates, at the start of the firing, gases directed towards the plane π divided by the area of the section of the central cavity according to π.

On the whole of the points above, the "inherent locking" ("serrage propre") has a maximum K_M which is a characteristic of the block. Now, it is necessary that K_M does not exceed a certain limit, as a function of the propergols used; and this circumstance is often troublesome, with blocks of somewhat elongated shape, when one uses fully the possibilities which are given by the invention of reducing the surface of the inner contour and of increasing, accordingly, the coefficient of filling.

A substantial improvement in this field is supplied by a very general family of bistellar blocks, characterised in that, if the sections of the axis cavity be considered to be made successively, through planes normal to the axis, always following the same flow direction of the gases, the first of these sections, situated at the origin of the flow, has a surface less than that of the final section, situated at the outlet of the block, and the surfaces of the intermediate sections never decrease; there can thus be obtained for the same value of K_M, a central cavity of volume less than that of which the sections would all have the same surface.

More especially, the invention allows the manufacture of bisteller blocks called "of the fifth group", belonging to the general family which has just been described, and characterised moreover in that:

they form part of the fourth group, that is to say, of that of the blocks of order p and of class c;

the sections of their cylindrical portion bear outlines of the flame front C_k all substantially identical;

the sections, through planes normal to the axis of the lateral surface of their central cavity, are all contours C_o belonging to the form A described above; for a given block, these contours are each constructed with the same values of Ω, b, r_o, r'_o; if these are considered one after the other by always following the same direction of flow of the gases, a finite number m of different contours are found, which are distinguished one from the other only by the values of $\lambda$and, accessorily, of Ω''.

Thus, the central cavity is composed of m successive portions, each containing sections having identical contours, and occupying on the axis of the block successive lengths marked $z_1, z_2 \ldots z_m$ in the direction of the flow of the gases.

The length of the portion, contained in an elementary sector, of an inner contour of shape A is marked s and called "elementary inner perimeter". On the m successive contours C_o of a block of the fifth group, considered in the same order as above, there is noted $s_1, s_2 \ldots s_m$ the elementary perimeters, $\rho_1, \rho_2 \ldots \rho_m$ the coefficients of filling, and $\lambda_1, \lambda_2 \ldots \lambda_m$ the values of λ; the coefficient of filling ρ and the elementary inner perimeter s of the block are by definition the quotients, of the sums $\rho_1 z_1 + \rho_2 z_2 + \ldots + \rho_m z_m$, and $s_1 z_1 + s_2 z_1 + \ldots + s_m z_m$ divided by the sum $z_1 + z_2 + \ldots + z_m$.

In order to determine a block of the fifth group, one can start from the outline, called "reference outline", of an inner contour C_o of shape A meeting the conditions imposed, corresponding to values marked Ω^*, Ω''^*, λ^*, b, r_o, $r'_o{}^*$ of the parameters of definition, and characterised itself by a coefficient of filling ρ^* and by an elementary inner perimeter s^*.

If a block of given length and with a central cavity were made whose sections normal to the axis are all identical to the reference outline, there would be obtained for the maximum "inherent locking" ("serrage propre") a value K^*_M which would by hypothesis be too great; the problem is therefore to define a block of the fifth group, of same outer dimensions, which is constructed with values of Ω, b, r_o, r'_p identical to those of the reference outline, which retains the values ρ^* and s^* of ρ and of s, and for which the maximal "inherent locking" ("serrage propre") has a value less than K^*_M and as low as possible.

It has been found, in practising the invention, that the block sought is theoretically characterised in that:

m has to be of the largest possible value and s_m of the smallest possible value;

since there exists, with the conditions laid down, a linear relationship between the ρ_i and the s_i ($i = 1, 2 \ldots m$) which can be written $s_o - s_i = B(1-\rho_i)$, s_o and B being positive constants, the elementary inner perimeters s_i have to form, with the constant s_o, a decreasing geometrical progression of $m + 1$ successive terms s_o, $s_1, s_2 \ldots s_m$;

the z_i ($i = 1, 2 \ldots m$) have to all be equal one to another.

However, in practice:

m cannot be very large, without excessive complications in practice; and, moreover, its influence on the value of K_M is rather low;

s_m cannot be less than the value of s for which λ is nil;

s_1 cannot exceed a certain maximum corresponding to a line T' T T_1 T_2 T'' of FIG. 2 which would be, of course, situated inside the elementary motif and of which, furthermore, the shortest distance to the main side of the same sector would have a minimum value not nil ζ, still compatible with a good circulation of the combustion gases along the inner surface of the central cavity.

It has been found that, for m fixed and for s_1 chosen the greatest possible, the optimal block is that for which the numbers $s_1, s_2 \ldots s_m$ form a geometrical progression and the numbers $z_1, z_2 \ldots z_m$ are all equal. It is therefore sufficient to settle s_1 as a function of the value tolerated for ζ; there can be deduced therefrom by calculation, since s^* and m are known, the terms of the sequence $s_2, s_3 \ldots s_m$.

However, it can happen that the final term or some of the final terms of this sequence are less than the value of s corresponding to $\lambda = 0$. It has been observed in this case that without noticeable disadvantage, for the corresponding contours C_o, the desired values of the perimeter can be made by taking λ nil and by causing Ω'' to vary. The conditions of the optimum are no longer then exactly observed and, on the other hand, the coefficient of filling of the block becomes very slightly less than ρ^*; however, the differences are negligible in practice.

The choice of a relatively high value of r'_o on the reference outline can be justified by the benefit of this parameter as means of regulating of ϕ_o and by its low influence on the value of ρ. It has been found, in practising the invention, that the maintenance of such a value of r'_o on a block of the fifth group is not, generally, specially disadvantageous as regards the coefficient of filling; in fact, if on a contour of shape A, r'_o is caused to increase as from zero by at the same time causing λ to vary so that ζ remains constant, it has been noted that ρ does not begin to decrease, as one would have thought, but that it increases slightly before reaching a maximum.

The non-restrictive examples below of blocks in accordance with the invention relate, except for the last one, to blocks where the central cavity has everywhere the same section normally to the axis.

EXAMPLE 1

It will be seen that, if the value of n, is settled, general information on the wishes of the users is enough to be able to calculate several essential characteristics of the block.

In fact, the more the quantity ρ is kept to a high value, the more it is necessary to keep p relatively small ($p \leq 8$, generally if it is desired that $\rho \geq 0.94$); the more the quantity σ is desired to be kept low, the more ν has to be made large ($\nu \geq 20$, generally if it is desired that $\sigma \leq 0.02$); however, the class c is, generally, at the most equal to 4, and for $c > 2$ the calculations become complicated. All that allows a couple of values of p and of c to be chosen for test purposes.

Thus, it is sufficient to settle on a value of ϕ_w in accordance with the desired aspect of the variation of ϕ in order to obtain a by the relations (6) and (7).

The "limit residual rate" σ_o, equal by definition to the quotient of the surface between C_f and C_e divided by the surface interior to C_e, and worth consequently $\sigma\rho$, is then determined. The benefit of ρ_o is, on the one hand, that it can be calculated, with an approximation which is broadly sufficient, by assuming C_f to be formed from circular arcs tangential to C_e and all having for radius $1-a$, this calculation then necessitating only the knowledge of ν and of a, on the other hand, that since ρ is always rather close to unity, the obtaining of σ_o already furnishes reasonably precise information on ρ. Then, b is obtained as from the first minimum ϕ_r by the relation (8).

This allows ρ to be calculated by choosing a shape of inner contour, for example a shape N with $r'_o=0$ which will be called "shape N_o". This shape N_o, for which $\phi_o = \phi_d$, is entirely defined by the numbers p, b, ϕ_o; and once ρ is obtained, ρ is immediately available through the relation $\sigma=\rho\sigma$.

Finally, if n is known, it is sufficient to settle on ϕ_o, ϕ_r, ϕ_w in order to obtain a (which gives an idea of the value of the minimal shaping (Retreint)a_M then the coefficient of filling ρ and the rate of residual σ.

In all the cases, the couples of values of ρ and of σ obtained with the bistellar blocks for values of n acceptable in practice are, all things equal moreover, much better than with monostellar blocks.

For these latter, it is known that the evolution of ϕ is that of the rapid phase shown in FIG. 3 (with $\phi_k \equiv \phi_f = 1$).

The tables (11) to (16) below give a comparison between some monostellar and bistellar blocks all having an inner contour of shape N_o (r_o there is equal to r_n, unless there is an indication to the contrary).

The monostellar blocks in question have for characteristic values of ϕ only ϕ_o and ϕ_r. There is attributed to them, in a purely symbolical manner, the class zero. They are defined entirely by the three numbers p, ϕ_o and ϕ_r. In fact, it is sufficient to determine their inner contour, for which the distance OP = b is supplied by the relations (6) and (7) by replacing there ν by, ω by Ω, and a by b, whilst Ω'' is supplied by the relation (3).

The whole of the Tables (11) to (16) shows that at the same time a_m, ρ and σ are improved when the possibilities of variation of ϕ are increased by acting on the minima ϕ_r and ϕ_w.

TABLE (11)

p	c	n	a	b	ρ	σ
		$\phi_o = 1$			$\phi_r = \phi_w = 0.96$	
16	0	—	—	0.715 1	0.600 3	0.054 1
20	0	—	—	0.742 6	0.536 7	0.044 6
24	0	—	—	0.763 2	0.489 8	0.038 0
16	1	2.55	0.715 1	0.280 4	0.929 4	0.034 9
20	1	2.2	0.742 6	0.337 5	0.897 7	0.026 7
24	1	1.95	0.763 2	0.391 4	0.860 9	0.021 6

TABLE (12)

p	c	n	a	b	ρ	σ
		$\phi_o = 1$			$\phi_r = \phi_w = 0.93$	
16	0	—	—	0.686 6	0.639 7	0.044 1
20	0	—	—	0.713 6	0.579 2	0.035 5
24	0	—	—	0.733 8	0.534 0	0.029 6
16	1	2.55	0.686 6	0.269 3	0.935 2	0.030 1
20	1	2.2	0.713 6	0.324 4	0.906 3	0.022 7
24	1	2	0.733 8	0.366 9	0.873 4	0.018 0
24	1	1.95	0.733 8	0.376 3	0.872 5	0.018 1

TABLE (13)

p	c	n	a	b	ρ	σ
		$\phi_o = 1$			$\phi_r = \phi_w = 0.9$	
6(*)	0	—	—	0.523 6	0.977 0	0.097 3
8	0	—	—	0.566 2	0.888 1	0.073 4
12	0	—	—	0.623 6	0.756 9	0.049 6
16	0	—	—	0.660 8	0.674 0	0.037 1
20	0	—	—	0.687 2	0.616 4	0.029 3
24	0	—	—	0.707 0	0.573 7	0.024 1
16	1	2.6	0.660 8	0.254 2	0.942 0	0.026 5
8	2	2.6	0.660 8	0.218 0	0.960 6	0.026 0
20	1	2.3	0.687 2	0.298 8	0.920 3	0.019 6
10	2	2.3	0.687 2	0.259 7	0.948 5	0.019 1

TABLE (13)-continued

p	c	n	a	b	ρ	σ
		$\phi_o = 1$		$\phi_r = \phi_w = 0.9$		
24	1	2	0.707 0	0.353 5	0.888 1	0.015 5
12	2	2	0.707 0	0.310 8	0.927 4	0.014 5
8	3	2	0.707 0	0.282 7	0.952 0	0.014 5
6	4	2	0.707 0	0.262 9	0.968 1	0.014 3

(*)For this block, r_o is equal to r_f, since $r_n < r_f$.

TABLE (14)

p	c	n	a	b	ρ	σ
		$\phi_o = 0.9$		$\phi_r = \phi_w = 0.9$		
6	0	—	—	0.523 6	0.936 3	0.101 6
8	0	—	—	0.566 2	0.844 1	0.077 3
12	0	—	—	0.623 6	0.722 5	0.051 9
16	0	—	—	0.660 8	0.645 6	0.038 7
20	0	—	—	0.687 2	0.592 1	0.030 5
24	0	—	—	0.707 0	0.552 4	0.025 0
16	1	2.6	0.660 8	C.254 2	0.938 8	0.026 6
8	2	2.6	0.660 8	0.218 0	0.956 4	0.026 1
20	1	2.3	0.687 2	0.298 8	0.916 4	0.019 7
10	2	2.3	0.687 2	0.259 7	0.942 9	0.019 2
24	1	2	0.707 0	0.353 5	0.883 2	0.015 6
12	2	2	0.707 0	0.310 8	0.920 1	0.015 0
8	3	2	0.707 0	0.282 7	0.943 0	0.014 6
6	4	2	0.707 0	0.262 9	0.958 1	0.014 4

TABLE (15)

p	c	n	a	b	ρ	σ
	$\phi_o = 0.9$			$\phi_r = \phi_w = 0.85$		
6	0	—	—	0.490 8	0.963 9	0.086 5
8	0	—	—	0.530 9	0.881 0	0.064 0
12	0	—	—	0.585 2	0.770 5	0.041 3
16	0	—	—	0.620 6	0.700 1	0.029 9
18	0	—	—	0.634 1	0.673 5	0.026 0
20	0	—	—	0.645 8	0.650 8	0.023 0
24	0	—	—	0.664 8	0.614 1	0.018 4
16	1	2.6	0.620 6	0.238 7	0.946 3	0.022 1
8	2	2.6	0.629 6	0.204 7	0.960 6	0.021 8
18	1	2.45	0.634 1	0.258 8	0.937 7	0.018 7
9	2	2.45	0.634 1	0.223 6	0.956 2	0.018 3
20	1	2.3	0.645 8	0.280 8	0.927 2	0.016 1
10	2	2.3	0.645 8	0.244 1	0.950 0	0.015 7
24	1	2	0.664 8	0.332 4	0.898 4	0.012 6
12	2	2	0.664 8	0.292 2	0.931 2	0.012 2
8	3	2	0.664 8	0.265 8	0.951 0	0.011 9
6	4	2	0.664 8	0.247 2	0.963 5	0.011 7

TABLE (16)

p	c	n	a	b	ρ	σ
			$\phi_o = 0.9$; $\phi_r = 0.81$; $\phi_w = 0.85$			
6(*)	0	—	—	0.465 9	0.981 8	0.076 9
8	0	—	—	0.504 0	0.905 7	0.055 9
12	0	—	—	0.555 9	0.803 5	0.035 1
16	0	—	—	0.589 7	0.738 0	0.024 9
18	0	—	—	0.602 7	0.713 3	0.021 5
20	0	—	—	0.613 9	0.692 1	0.018 8
24	0	—	—	0.632 2	0.657 8	0.014 9
16	1	2.6	0.620 6	0.227 5	0.951 2	0.022 0
8	2	2.6	0.620 6	0.195 1	0.963 1	0.021 7
18	1	2.45	0.634 1	0.246 7	0.943 7	0.018 6
9	2	2.45	0.634 1	0.213 0	0.959 7	0.018 3
20	1	2.3	0.645 8	0.267 6	0.934 3	0.016 0
10	2	2.3	0.645 8	0.232 6	0.954 7	0.015 7
24	1	2	0.664 8	0.316 8	0.908 7	0.012 5
12	2	2	0.664 8	0.278 5	0.938 5	0.012 1
8	3	2	0.664 8	0.253 3	0.956 2	0.011 8
40	4	2	0.664 8	0.235 6	0.966 8	0.011 7

(*)For this r_o is equal to r_f because $r_n < r_f$.

EXAMPLE 2

Here is the complete description of a plan of block of class 2, with separatrix E of second class and consequently regular, for propergols of ratio of speeds $n = 2$.

It is desired that $a_M < 0.73$, $\rho > 0.9$, $\sigma < 0.02$, and ϕ shall be between 0.9 and 1 during the entire firing, with $\phi_o = 0.9$. The value p is chosen to be 12, hence $\nu = 24$.

The flame front C_k arriving at G comprises 12 circular arcs which each give rise to four rectilinear segments after having traversed four hyperbolic arcs forming part of G. In this way, the motif of FIG. 7 is obtained: P is a main centre of curvature of C_o, situated on the main side OB' of the elementary sector B'OB'', of angle at the centre $\Omega = 15°$, of C_e. In this sector, G comprises two hyperbolic arcs HJ and NQ possessing a focus at P, turning their convexity towards P, having eccentricities equal to n, connected by an elliptical arc JSN whose main focus P' is on the bisectrix OA of the angle B'OB'' (S is an outer apex of G). The flame front portions in the slow propergol situated in the vicinity of P' are substantially rectilinear; the ellipse transforms them substantially into circular fronts centred on P'.

Thus, in the sector in question, C_f is composed substantially of the two circular arcs AD' and AD'' centred on P'. On the other hand, in order to satisfy the condition relative to a_M, one adopts for OP' the value $a = 0.71$. That determines completely C_f, whose perimeter is then found equal to 1.010 6 l_e, l_e being the perimeter of C_e equal itself to 2π.

In order to determine P, referring to FIG. 8 of the accompanying drawings, a circular arc M'M'' centred on P is considered, having its end M' on O B' and its end M'' on OB''. By laying down $\widehat{CM''P} = \alpha$, it is found that, if M'' is displaced on OB'', the arc M'M'' has a length which passes through a minimum for α equal to $\Omega_p = 47°29'$; by writing that this minimum length is worth 0.9 times the quotient of the length of the arc

D'AD'', divided by n for the distance OP is found the value $b = 0.310\,9$. On the other hand, there exist two values of α, viz: $\alpha_1 = 25°25'$ and $\alpha_3 = 76°39'$, such that the corresponding arcs M'M'' have for length the quotient of the length of the arc D'AD'' divided by n; let M''_1 and M''_3 be the corresponding positions of M''. The contour C_o adopted is composed, in the sector B'OB '', of two circular arcs T'T, TT'' (FIG. 7) having in common a point T of the segment PM''_3 and admitting for respective centres P and M''_3 (therefore here $\lambda = 0$). The point T is determined by the condition that the length of the contour T'TT'' is 0.9 times the quotient of the length of the arc D'AD'' divided by n. The calculation gives $PT = r_o = 0.032\,2$, which determines entirely C_o, as well as ρ. The inner contour is therefore of shape A, with the length of the segment T_1T_2 (see FIG. 2) equal to zero, and with $M''_3T = r'_o = 0.066\,6$. It is found that ρ=0.908 5.

The outline of G can, for example, be made in such a way that the hyperbolic arcs HJ and NQ and the elliptical arc JSN have, at their points of connection J and N, tangents which are practically merged. For this, by iteration first of all the ellipse to which the arc JSN belongs is defined. This ellipse has as eccentricity ($1/n$, as major axis the straight line PP', and as main focus P'; it is determined if the distance $2\,\epsilon_b$ of the apices J' and N' of its minor axis is known. The distance $2\,\epsilon_b$ must be fairly large so that the round-off at S has a sufficient radius of curvature; and it has to be small enough so that ϕ does not vary too much in the vicinity of S. A reasonable value is $2\,\epsilon_b = 0.015$. The ellipse being thus defined, first of all the ends J and N of the hyperbolic arcs are placed at the apices J' and N' of the minor axis of the ellipse; the hyperbolas are located under these conditions and the directions of the tangents to these curves at J' and at N' are noted; then for points J and N the points of the ellipse are taken where the tangents to this latter have the directions which have just been obtained; and so forth.

As regards the points H and Q, one of them at least has to be between O and the arc M'_1 and M''_1 so that the corresponding hyperbola intersects this arc and that thus ϕ does not run the risk of increasing further beyond M'_1 and M''_1. For example, Q is placed between O and M''_1, at a distance from this point sufficient so that, if the machining or the forming of the intermediate core leaves a slight round-off at Q, the point M''_1 is still in the region of the slow propergol. Thus, QM''_1 is assumed to be 0.007 5, which determines Q; the calculation gives OQ = 0.462 3. The points Q and N' are therefore known, which allows the hyperbolic arc N'Q to be completely determined, and in particular the angle ψ'' of the straight line PP' and of the tangent at N' to this arc. The value ψ'' is found to be 14°27'. Point N is now adopted as the point of the ellipse where the tangent to this latter forms precisely the angle ψ'' with PP'. The point N is thus determined; at rectangular coordinates of origin O and of axis of the abscissae OB', its coordinates are $x_N = 0.708\,9$ and $y_N = 0.101\,2$. The determination of the hyperbola is recommenced by this time taking Q and N as the ends of the arc instead of Q and N'; it is then found that the tangent at N to the hyperbola forms with PP' the angle $\psi_1'' = 14°22'$, very little different from ψ''. The difference between ψ'' and ψ_1'' is negligible, and it is not necessary to make another iteration.

The flame front C_h passing through N is constituted, between the point N and the straight line OB'', by a segment of straight line NN_o whose angle β'' with OB'' is 44°55', and which has as its length 0.121.4.

It is now a question of defining the hyperbola HJ, by replacing first of all J by the adjoining apex J' of the minor axis of the ellipse. Now, the flame front C_h has to have a corrected perimeter l_h between l_f and 0.9 l_f, and rather close to l_f so that the first flame front in the slow phase C_g may have a corrected perimeter still greater than 0.9 l_f. To obtain a value approximating to l_h, it can be allowed that, in the sector B'OB'', the flame front C_h is comparable to the whole of the segment NN_o, of the segment N'J' (equal to $2\,\epsilon_b$) and of a certain segment $J'J'_o$ having its end J'_o on OB' and forming with OB' a certain angle β'. Calculation shows then that the condition $0.9\, l_f < l_h < l_f$ is equivalent to the condition $84°27' > \beta' > 49°42'$.

For β' is chosen the value $\beta_\nu = 90° - \omega_\nu$, where $\omega_\nu = 39°9'$ is the angle of neutrality relative to ω=7°30', and which is convenient for calculation. With the values thus adopted for β' and for β'', the minimum ϕ_{w} of ϕ, in the phase of the combustion going from C_g to C_f is found equal to 0.903 4, which is acceptable.

As for C_f, it comprises the circular arcs AD' and AD''. The calculation of the residual gives the value $\sigma = 0.015\,4$, which is suitable.

It remains now to construct an arc of hyperbola HJ' whose end H, situated on OB', is still not determined, but of which is known the end J', a focus P, the eccentricity which is equal to n, and finally the direction of the transverse axis since this latter forms with OB' an angle obviously equal to β'. Such data allows the curve to be completely defined. In this way, it is found that the tangent to the arc HJ' at J' forms with PP' the angle ψ'=7°59'. Therefore, as point J is taken the point of the ellipse where the tangent to this curve forms the same angle ψ' with PP'; for the coordinates of J are found the values $x_J = 0.711\,2$, $y_J = 0.086\,8$. The determination of the hyperbola is recommenced in the same conditions as above, by simply replacing J' by J. This time it is found that the tangent to the hyperbola at J forms with PP' an angle differing from ψ' by less then one minute of arc; a new iteration would be useless. The point H is thus determined; it is at a distance PH = 0.184 4 from P, whilst PQ is equal to 0.180 9. It is reached by the combustion after the point Q, which belongs therefore to the first flame front C_k of the mixed phase.

It would be advisable finally to examine the variations of the corrected perimeter in the zone where the two propergols burn together. In the sector B'OB'', a flame front comprises, between C_k and C_h, a circular arc centred on P and situated in the rapid propergol, and one or two rectilinear segments situated in the slow propergol; its corrected perimeter is expressed easily as a function of the radius r of the circular arc. Therefore, by successive points the curve giving ϕ as a function of r is constructed, in order to verify if, in this zone, ϕ remains, or not, between 0.9 and 1. It is observed in this way that ϕ is equal to $0.988\,4 = \phi_k$, for $r = 0.180\,9 = r_k$ (point Q), that it decreases as far as minimum ϕ_μ worth about 0.955 for r close to 0.24, that it then increases in order to reach the value 0.989 1, close to ϕ_h, for $r = 0.4$ (in the vicinity of J and N), and that it decreases almost immediately afterwards, somewhat suddenly, to arrive at the value $0.943\,6 = \phi_g$ shortly after the point S.

The graph of FIG. 9 of the accompanying drawings sums up the variations of ϕ during the entire combustion. It is found that $1 - y_f = 0.041$.

TABLE (21)

n	r_k	ϕ_k	ϕ_μ	ϕ_h	ϕ_g	f
1.9	0.093 5	0.909 7	0.896 6	0.939 8	0.9	0.003 4
1.91	0.098 3	0.915 5	0.903 7	0.940 3	0.9	0.003 6
1.916 2	0.101 3	0.919 4	0.908 1	0.940 7	0.9	0.003 6
2	0.137 3	0.983 6	0.945 0	0.945 0	0.9	0.004 2
2.13	0.137 5	1	0.990 4	0.998 3	0.945 5	0.003 2
2.14	0.136 9	1	0.991 9	1.003 0	0.949 6	0.003 1
1.905 = n_1	0.096	0.913	0.9	0.940	0.9	0.003 5
2.134 = n_2	0.137	1	0.991	1	0.947	0.003

Therefore the block can be constructed for any value of n such that $n_1 \leqslant n \leqslant n_2$; to each value of n satisfying this condition there corresponds an interval (r'_k, r''_k) of the possible values of r_k. The Table (22) below gives some of these intervals, obtained by interpolation.

TABLE (22)

n	r'_k	r''_k
1.905 = n_1	$r'_k = r''_k$	0.096
1.916 2	0.093	0.101
2	0.087	0.137
2.1	0.123	0.139 5
2.134 = n_2	$r'_k = r''_k$	0.137

EXAMPLE 4

This relates to a block differing from that of Example 3 only by its order, equal to 20, and only by ϵ_b, equal to 0.009. In this case $a = 0.687\ 2$.

Table 23 below gives its characteristics calculated for several values of n and of r_k.

The second value of n (2.125 1) is that allowing the construction of a similar block with $\epsilon_b = 0$, and $\phi \equiv 1$ in the rapid phase and in the slow phase as far as C_f; the last two values of n (2.104 and 2.336) are the limits n_1 and n_2, obtained by interpolation.

TABLE (23)

n	r_k	ϕ_k	ϕ_μ	ϕ_h	ϕ_g	f
2.1	0.095 0	0.911 8	0.897 7	0.949 6	0.9	0.003 2
2.125 1	0.104 7	0.926 1	0.914 2	0.950 9	0.9	0.003 5
2.3	0.136 0	1	0.982 8	0.985 3	0.924 7	0.003 4
2.32	0.134 8	1	0.986 7	0.993 4	0.931 6	0.003 2
2.34	0.133 7	1	0.989 5	1.001 5	0.938 6	0.003 1
2.104 = n_1	0.096	0.914	0.9	0.950	0.9	0.003 3
2.336 = n_2	0.134	1	0.989	1	0.937	0.003 1

EXAMPLE 5

This relates to a block differing from those of Examples 3 and 4 only by the values of p and of ϵ_b, equal respectively to 16 and to 0.011; the parameter a is then worth 0.660 8.

The Table (24) below relates to two values of n straddling

TABLE (24)

n	r_k	ϕ_k	ϕ_μ	ϕ_h	ϕ_g	f
2.6	0.130 5	1	0.975 8	0.976 9	0.903 5	0.003 5
2.7	0.125 6	1	0.988 6	1.009 6	0.930 4	0.002 9

Thus the value $n = 2.6$ is acceptable. The heavy drop of ϕ between ϕ_h and ϕ_g is observed, due essentially to the relatively high values of n and of ϵ_b. These values result from that of ν, in accordance with what has been said above on the subject of the choices of n and of ϵ.

The value $n = 2.7$ is not acceptable, since ϕ_h is greater than 1. However, it becomes acceptable if one takes $\epsilon_b = 0.007\ 5$; then, with $r_k = 0.118$ for example, there is obtained $\phi_k = 0.982$, $\phi_\mu = 0.966$, $\phi_h = 0.983$, $\phi_g = 0.929$; thus, the difference $\phi_h - \phi_g$ is much less strong than with $\epsilon_b = 0.011$.

EXAMPLE 6

This relates to a block differing from that of Example 3 only by its separatrix which is of the first sort (FIG. 12); thus, G is still of shape E, C_o is of shape N_o, and $p = 24$, $c = 1$, $\epsilon_b = 0.007\ 5$, $\phi_o = 1$, $\phi_v = \phi_w = 0.9$, $0.9 \leqslant \phi \leqslant 1$, $a = 0.707\ 0$.

The aspect of the variations of ϕ in the mixed phase, as a function of n and of r_k, is substantially the same as for Example 3. The Table (25) below gives the results of the calculations of characteristics for several values of n, more especially for those serving for obtaining lower and upper limits n_1 and n_2.

TABLE (25)

n	r_k	ϕ_k	ϕ_μ	ϕ_h	ϕ_g	f
1.85	0.127 2	0.950 1	0.896 9	0.938 6	0.9	0.023 7
1.86	0.130 4	0.956 1	0.902 9	0.939 1	0.9	0.022 8
1.9	0.143 2	0.981 3	0.924 4	0.941 2	0.9	0.020 0
2	0.173 6	1.050 1	0.946 4	0.946 4	0.9	0.015 6
2.05	0.142 9	1	0.962 2	0.998 8	0.948 8	0.014 3
2.06	0.142 2	1	0.963 7	1.003 4	0.952 8	0.014 0

FIG. **10** of the accompanying drawings gives a precise outline of a portion of the straight section of the block, with some flame fronts (of which the doubled dashes relate to the portions located in the rapid propergol). The hyperbolic arcs of the separatrix have their "relative maximal arrows (sagittae), f", that is to say the quotients of the maximal distance of a point of the arc to its chord, worth 0.004 3 for the arcs leaving from a main side and 0.021 2 for the arcs leaving from a second side.

EXAMPLE 3

This refers (FIG. **11** of the accompanying drawings) to a block of order **24** and of class m having an inner contour N_o, with a separatrix E of second sort and obviously regular; the value ϕ_o is taken to be 1, $\phi_v = \phi_w =$ 0.9 and ϕ has to remain between 0.9 and 1. It is proposed to show on this Example the influence of n.

Since the values of n possible for this block remain close to 2, there is taken, for the semi-minor axis of the ellipses containing the outer apices of the separatrix, the fixed value $\epsilon_b = 0.007$ 5 giving to ϵ values close to 0.006 5. Whatever n may be, the distance a is equal to 0.707 0. FIGS. **13**, **14** and **15** each show the family of the graphs of ϕ in the mixed phase, from ϕ_k to ϕ_h, for a given value of n and for various values of ϕ_k; in the abscissae are the distances r between the point P and the portions of flame front situated in the rapid propergol; the flame front C_h is that passing through the apices of the minor axis of the ellipses.

The first value of n (graph of FIG. **13** of the accompanying drawings) has been chosen in the following manner: if a block is considered which would differ from that studied only in that $\epsilon_b = 0$ (the outer apices of G being then all angular, which is obviously theoretical), and that ϕ is identically equal to 1 in the entire rapid phase and in the slow phase as far as C_f, then it is observed that the interval of the possible values of n is reduced to the single value $n = 1.916$ 2, leading to $\rho =$ 0.823 0, $\sigma = 0.035$ 0, and $\phi_\mu = 0.962$ 7; and it has appeared interesting to try by way of comparison this same value of n on the block in question. The other two values of n are 2 (FIG. **14**) and 2.1 (FIG. **15**).

In these Figures can clearly be seen the respective influences of ϕ_k and of n; thus, when n is fixed:

ϕ_h varies in the opposite direction to ϕ_k;

the minimum ϕ_μ (marked approximately by a circle surrounded by a square on the curves) can, for certain values of ϕ_k, go to ϕ_h;

the amplitude of the variation of ϕ between ϕ_k and ϕ_h is variable, but sometimes remarkably small; thus, for $n = 2$ and $\phi_k = 0.96$, ϕ varies between 0.96 and 0.948 in the main portion of the mixed phase, from C_k to C_h;

the curves all approximate to one another in a certain "gathering zone" situated in the second half of the variation of ϕ.

It is noted that the gathering zone rises or falls on the graph according to whether n increases or decreases.

For reasons of clarity, in the drawings, the values of ϕ_g are not given in the graphs; they figure in the Tables (18) to (20) below, where there are at the same time the other characteristics of the corresponding blocks, and more especially:

the shortest distance r_k of a main centre of curvature to the separatrix G, a distance equal here to the length PR of FIG. **11**;

the relative maximum arrow f of the hyperbolic arcs of G, defined as for Example 2.

TABLE (18)

ϕk	r_k	ϕ_μ	ϕ_h	ϕ_g	f
		$n = 1.916$ 2 ; $\rho = 0.885$ 9 ; $\sigma = 0.015$ 6			
0.91	0.093 0	0.898 1	0.947 2	0.906 3	0.003 3
0.92	0.101 8	0.908 5	0.940 3	0.899 6	0.003 6
0.93	0.109 3	0.916 9	0.934 1	0.893 7	0.003 9
0.94	0.116 2	0.922 8	0.928 4	0.888 1	0.004 2
0.95	0.122 8	0.922 8	0.922 8	0.882 6	0.004 5
0.96	0.129 1	0.917 4	0.917 4	0.877 3	0.004 8
0.97	0.135 2	0.911 9	0.911 9	0.872 0	0.005 1
0.98	0.141 2	0.906 6	0.906 6	0.866 8	0.005 5
0.99	0.147 1	0.901 2	0.901 2	0.861 5	0.005 8
1	0.152 9	0.895 8	0.895 8	0.856 2	0.006 1

TABLE (19)

ϕ_k	r_k	ϕ_μ	ϕ_h	ϕ_g	f
		$n = 2$; $\rho = 0.88$ 1 ; $\sigma = 0.015$ 4			
0.91	0.089 1	0.902 5	0.986 4	0.940 3	0.002 6
0.92	0.097 5	0.914 0	0.979 6	0.933 7	0.002 8
0.93	0.104 7	0.924 2	0.973 7	0.928 0	0.003 1
0.94	0.111 3	0.933 3	0.968 1	0.922 5	0.003 3
0.95	0.117 6	0.941 5	0.962 7	0.917 2	0.003 5
0.96	0.123 7	0.948 0	0.957 3	0.912 1	0.003 7
0.97	0.129 5	0.951 2	0.952 1	0.906 9	0.003 9
0.98	0.135 3	0.946 8	0.946 8	0.901 8	0.004 2
0.99	0.140 9	0.941 6	0.941 6	0.896 7	0.004 4
1	0.146 5	0.936 3	0.936 3	0.891 6	0.004 6

TABLE (20)

r_k	ϕ_k	ϕ_μ	ϕ_h	ϕ_g	f
		$n = 2.1$; $\rho = 0.905$ 0 ; $\sigma = 0.015$ 3			
0.084 5	0.91	0.905 4	1.032 5	0.980 2	0.002 0
0.092 8	0.92	0.917 2	1.025 9	0.973 8	0.002 2
0.099 7	0.93	0.927 8	1.020 2	0.968 2	0.002 4
0.106 0	0.94	0.938 0	1.014 8	0.963 0	0.002 5
0.112 0	0.95	0.947 7	1.009 5	0.957 9	0.002 7
0.117 8	0.96	0.957 1	1.004 4	0.952 9	0.002 8
0.123 4	0.97	0.965 9	0.999 3	0.948 0	0.003 0
0.128 8	0.98	0.973 8	0.994 3	0.943 0	0.003 2
0.134 2	0.99	0.980 3	0.989 2	0.938 1	0.003 3
0.139 5	1	0.983 4	0.984 1	0.933 2	0.003 5

The smallest possible value, n_1 of n is that for which there exists a value of ϕ_k giving at one and the same time $\phi_\mu = \phi_g = \phi_w = 0.9$. The flame fronts in the mixed phase are formed, in the slow propergol, from segments of a straight line all forming, for a given block, the same angle β with the main side where they end; and if $\phi_g = \phi_w$, this angle β is the complementary β_ν of the angle of neutrality ω_ν. On the other hand, for a given value of n, the knowledge of β allows to be defined all the elements of the separatrix. Consequently, to have here n_1, it is sufficient to seek, by interpolation, the value of n which, with $\beta = \beta_\nu$, gives $\phi_\mu = 0.9$.

The greatest possible value, n_2, of n is that which, for $\phi_k = 1$, gives $\phi_h = 1$; it is obtained in the same manner by interpolation.

The Table (21) below gives the characteristics ϕ_μ, ϕ_h, ϕ_g; f for values of n and r_k giving either $\phi_g = 0.9$ or ϕ_h close to 1, then the approximate values of n_1 and of n_2 as well as the corresponding characteristics obtained by interpolation.

TABLE (25)-continued

n	r_k	ϕ_k	ϕ_μ	ϕ_h	ϕ_g	f
$1.855 = n_1$	0.129	0.953	0.9	0.939	0.9	0.023
$2.053 = n_2$	0.142 5	1	0.963	1	0.937	0.014

It is noted that, in relation to the block of Example 3 having a separatrix of the second sort, the interval of the possible values of n is displaced downwards, the minimum ϕ_μ of ϕ in the rapid phase is more pronounced, and the relative maximal arrow f of the separatrix is clearly greater whilst still remaining low in absolute value.

In short, the action of the slow propergol is more energetic. This is due to the fact that the acute angle of the tangents to the separatrix and to a flame front incident in the rapid propergol, at a point common to these two curves, is distinctly less, all things being equal moreover, than for a separatrix of the second sort.

EXAMPLE 7

The most interesting blocks of class 2 are those having a separatrix of the second sort (FIGS. **16** and **17**) and ipso facto regular.

In the elementary motifs of these blocks are marked: r_{k1} and r_{k2}, respectively, the distances to the centre of main curvature P of the portions of G adjoining the inner apices R_1 and R_2 situated on the main side and on the second side; C_{k1} and C_{k2}, respectively, the flame fronts of which $r = r_{k1}$ and $r = r_{k2}$; ϕ_{k1} and ϕ_{k2}, respectively, the corresponding values of ϕ.

r_k and $r_{k'}$, respectively, the smallest and the largest of the two lengths r_{k1} and r_{k2}; C_k and $C_{k'}$, respectively, the flame fronts for which $r = r_k$ and $r = r_{k'}$; ϕ_k and $\phi_{k'}$, respectively, the values of ϕ relative to these two flame fronts.

The graph of ϕ as a function of r still comprises an angular point for the point of coordinates $r_{k'}$, $\phi_{k'}$. This point can have its upper ordinate or not at ϕ_k.

And the minimum ϕ_{μ}' of ϕ between ϕ_k and $\phi_{k'}$ can have a value distinct from ϕ_k and from $\phi_{k'}$, obtained for values of r between r_k and $r_{k'}$. In practice, the value of $\phi_{\mu'}$ is never very different from ϕ_k and is therefore not troublesome; on the other hand, $\phi_{k'}$ can be distinctly greater than ϕ_k.

The choice of the inner apices can be effected in many ways, and this as follows:

p, a, b, ϵ_b being already given, values of n and of $'_k$ are settled by taking account of what is desired for $'_g$, according to a method similar to that of Example 3; therefrom r_k is deduced; and one of the apices R_1 or R_2 is chosen as being that whose distance to P is r_k. This allows the arc of separatrix leaving from this apex to be completely defined. Then it is examined how ϕ varies at the start of the mixed phase. If ϕ_y is decreasing there, $r_{k'}$ is chosen, by simply paying attention to the values which stem therefrom for ϕ_μ and for ϕ_h; if ϕ is increasing, attention is given in the first instance to taking $r_{k'}$ such that $\phi_{k'}$ is not too great. It is then observed that there exists a gap (n_1, n_2) of values of n for which this construction is possible, as a function of the conditions imposed on ϕ.

The determination of n_1 and of n_2 is more difficult than in the case of class 1; in general, for $n = n_1$, the construction of G is possible only with a single couple (r_{k1}, r_{k2}) giving to ϕ_μ and to ϕ_g the minimum values permitted; and for $n = n_2$ it is possible only with a single couple (r_{k1}, r_{k2}) giving to ϕ_k (or to $\phi_{k'}$) and to ϕ_h the maximum values permitted.

Thus, for the block of Example **2**, numerous couples (r_{k1}, r_{k2}) allowing one to respect the condition $0.9 \leq \phi \leq 1$ are possible.

The Table (26) below gives several thereof, with their results on ϕ in the mixed phase. The first couple indicated is that of Example 2; the second is characterized by the fact that R_1 and R_2 are at an equal distance from the centre O of the block, and thus give a particular regular aspect to G; the others are examples fairly close to extreme cases (extreme values of r_{k1} for $n = 2$, extreme values of n).

TABLE (26)

n	2	2	1.85	2	2	2.05
r_{k1}	0.184 4	0.174 1	0.17	0.15	0.23	0.212
r_{k2}	0.180 9	(*)0.201 4	0.17	0.22	0.16	0.175 5
ϕ_k	0.988 4	0.976 9	0.949 8	0.938 9	0.954 0	0.986 8
$\phi_{\mu'}$	0.986 3	0.976 9	—	0.938 9	0.947	0.984
ϕ_k'	0.986 3	0.995 3	—	0.995 8	0.994 4	0.998 2
ϕ_μ	0.955	0.960 2	0.904 6	0.964 2	0.977	0.983
ϕ_h	0.989 1	0.983 5	0.943 2	0.983 3	0.979 5	0.997 9
ϕ_g	0.943 6	0.936 3	0.909 3	0.935 9	0.930 3	0.947 9

(*)these values of r_{k1} and of r_{k2} entail $OR_1 = OR_2 = 0.485$

EXAMPLE 8

This refers (FIG. **16** of the accompanying drawings) to a block of the order 12 and of class 2, having an inner contour N_o, having a separatrix D of the second sort, with $\epsilon = 0.0065$, $\phi_o = 0.9$, $\phi_{v} = \phi_{w} = 0.85$, $0.85 \leq \phi \leq 1$, and $n = 2$, which has the special feature of having a separatrix of simple shape, since there is chosen for this curve a monostellar regular polygon whose inner and outer apices are all replaced by circular round-offs of the same radius ϵ. Its main parameters, which already figure in the Table (15) above, are $a = 0.664\ 8$, $b = 0.292\ 2$, $\rho = 0.932\ 2$, $\sigma = 0.012\ 2$; hence $a_M = 0.671\ 3$.

By choosing $\phi_k = 0.925$, the following are found: $r_{k1} = 0.165\ 0 = r_k$; $r_{k2} = 0.190\ 1 = r_{k'}$.

The noteworthy values of ϕ are, in these conditions:

TABLE (27)

ϕ_μ	$\phi_{k'}$	ϕ_μ	ϕ_h	ϕ_g
0.920	0.934 4	0.916 2	0.922 8	0.874 2

FIG. 16 gives the outlines on the scale of C_o and of G for the numerical values defined above.

It can be verified on this block that, in the slow propergol, the flame fronts have very extended arcs and can, in practice, be taken to be straight lines.

For example, towards the end of the mixed phase, the flame front C_h in the elementary sector of FIG. 16 of the accompanying drawings comprises two arcs in the slow propergol, each having one end on the separatrix and the other end on the principal side as regards the first and on the second side as regards the second one. For each of these arcs, the angle can be calculated, the sides of which each make tangents to the ends and it is clear that the relative maximal arrow f is at the most equal to half the value in radians of this angle. There is thus obtained:

for the first arc: $f < 0.002\ 9$;

for the second arc: $f < 0.016\ 3$.

By way of demonstrating more specifically the design of the propergol of this Example, the embodiment will be described according to the segment shown in FIG. **16.** It will be understood that the basic structure of FIG. **16** is repeated to provide the finished propellant, that is, the sector shown is 1/24 of the entire structure.

For convenience in the following description, all decimal numbers are separated into three-digit groups, except for the last group.

The order p was chosen as **12,** and class c equal to 2. Consequently, the inner contour C_o is in star form with $p = 12$ arms, and the separatrix G is in star form with $\nu = pc = 24$ arms. These were chosen because such a value of ν promotes a low value of the residual rate σ (as is stated above; value $p = 12$ leads to a coefficient of filling which is already sufficiently high; and the value $c = 2$ leads to a block relatively easy to calculate and of very symmetrical design.

The inner contour chosen is of the standardized type N_o. This type of contour is a particular case of FIG. **2.** It will be noted that the shape N_o is shape N with $r'_o = 0$, and that the shape N is itself a particular case of the shape A.

The shape N_o is characterized on the one hand in that the function of the shape ϕ remains constant between ϕ_o and ϕ_d (see FIG. **3**) at the beginning of combustion, while the value of the radius of curvature r_o in star form leads to rates of mechanical stresses in the mass of the propergol (due to the shrinking after casting) of the same order of magnitude for all the contours. It is therefore a shape which makes it possible to compare, on realistic bases, various designs of blocks, and which at the same time is perfectly valid in practice.

The block has a separatrix of shape **D** and of the second sort. Separatrix D is composed solely of straight-line segments connected by arcs of circle. It is of the second sort, since its summits are not located on radii which pass through the summits of the inner contour, and consequently its design is very symmetrical (a summit of C_o is at an equal distance from two consecutive summits of G, and this has the useful consequence that the residual rate σ is in particular not very sensitive to an error committed by the manufacturer of propergol on the value of the ratio n).

Since $\phi_o = 0.9$, $\phi\nu = \phi\omega = 0.85$; $0.85 \leqslant \phi \leqslant 1$, and $n = 2$, the curve of variations of ϕ has an appearance similar to FIG. **3, 4,** and **5.**

The value of n was chosen as a function of what has been described at length in Examples 3, 4, 5, 6, 7 on the subject of the possible values of this parameter and of its influence on the characteristics of the block, in application of the general rule given above.

The values $a = 0.664\ 8$, $b = 0.292\ 2$, $p = 0.932\ 2$, $\sigma = 0.012\ 2$, and $a_M = 0.6713$ given in the text were obtained by calculation from the preceding data (p, c, and the values of ϕ).

The length a is, in FIG. **16,** the distance OP', furnished by formulae (6) and (7). The calculations described here were all made with a scientific office calculator of the Olivetti Programma 101 type, working to 10 decimals. The following were successively obtained:

1. Calculation of $\omega = \pi/\nu$ with $\nu = p.c = 24$: $\omega = 0,130\ 899\ 6939$*
2. Calculation of ω_ν such that $\tan \omega_\nu / -\omega_\nu = \omega$, ω being between o and $\pi/2$ (Formula 7) shows $\omega_\nu = 0,683\ 399\ 1255$,
3. Calculation of ω' such that

$$\frac{\sin \omega'}{\omega + \omega'} = \phi_w \cos \omega_\nu$$

(Formula **6**) with ω' between o and ω_ν and $\phi_w = 0.85$ provides $\omega' = 0.251\ 849\ 2623$.

4. Calculation of a such that

$$a = \frac{\sin \omega'}{\sin \omega + \sin \omega'}$$

(Formula **6**) provides $\sin \omega = 0.\ 130\ 526\ 1922$ and $\sin \omega' = 0.258\ 867\ 2199$, so that $a = 0.664\ 796\ 0954$.

5. b is the distance OP of FIG. **2** relative to the inner contour calculated using formula (8).

$$b = \frac{a}{n'} \cdot \frac{\phi\nu}{\phi\omega} \cdot \frac{\sin \omega}{\omega} \cdot \frac{\Omega}{\sin \Omega} \cdot \frac{\cos \Omega p}{\cos \omega_\nu},$$

with $n' = n$ line 23) (or $n = 2$), $\phi_\nu = 0.85$, and $\phi_w = 0.85$. Ω_p is calculated by formulae (4) $\tan \Omega p - \Omega p = \Omega$, $o < \Omega p < \pi/2$, with $\Omega = \pi/p = \pi/12$ to give $\Omega = 0.261\ 799\ 3878$, $\Omega p = 0.828\ 629\ 9487$, and $\cos \Omega p = 0.675\ 886\ 1307$. Knowing that $\cos \omega_\nu = 0.775\ 430\ 8844$, b is $0.292\ 227\ 3405$.

6. The coefficient of filling ρ may be defined, in FIG. **2,** as the difference between unity and the quotient of the surface located inside the contour $OT'TT_1T_2T''O$ by the surface of the sector of angle at the center

$$\Omega = \frac{\pi}{p}.$$

This quotient is obtained by an elementary geometrical calculation when the characteristics of the contour are known. And this is indeed the case, since an inner contour of shape N_o is known as soon as p and b are known. In fact, if reference is made to FIG. **2,** $r_o = PT = PT'$ is such that

$$r_a = r_n = \frac{0.06}{bp}$$

(from formula **1**)

so that r_o is $0.017\ 109\ 9664$. Ω'' is angle $OM_3''P$ so that

$$\frac{\sin \Omega''}{\Omega + \Omega''} = \frac{\phi\nu}{\phi M_3''} \cos\Omega p,$$

where $\Omega p < \Omega'' < \pi/2$, per formula (3). λ = angle $OM_3''T$, is equal to the angle of neutrality Ωp, so that ϕ_{M3}'' is equal to 100_o. $\phi_o = 0.9$ and $\phi_\nu = 0.85$, so that Ω'' = 1.196 055 3090. Since $r'_o = T_oT_2 = T_oT'' = 0$, line $T'TT_1T_2T''$ is accordingly entirely determined so that ρ is calculated as 0.931 153 5505.

The residual rate, σ, the quotient of surface between C_f and C_e/ by the surface between C_o and C_e is calculated utilizing the value obtained for p. C_f is composed of 24 arcs of circles of radius $1-a$ internally tangential to unit circle C_e at 24 points forming a regular polygon, so that calculation of the surface between C_f and C_e is a matter of geometry and $\sigma = 0.012\ 156\ 2340$.

The value a_M is the distance OS in FIG. **16** and $a_M = OD' + \epsilon$, the latter being chosen as 0.0065 by reference to empirical formula (2), $\epsilon_n = 0.15/\nu$. Taking P'O' as ϵ' and PO' as q and utilizing formula 5 to furnish a relationship between quantities ϵ' and q', that is, between the points P, P', and O', angle POO' is ω, OP' is a, and OP is b. OO' is accordingly calculated as 0.671 051 3215 and a_M is 0.677 551 3215.

The value 0.925 is selected for ϕ_k because at this last instant of combustion in the rapid phase there is considerable propergol to be burned and the empty space for circulation of combustion gases inside the propellent is relatively limited. Too high a value for ϕ_k could lead to excessive gas pressures.

Then $r_k = r_{kl}$ is the distance PR_1 in FIG. **16** given by ratios **10**

$$r_k = b\ \frac{\sin \Omega}{\sin \Omega'},$$

where

$$\frac{\sin \Omega'}{\Omega + \Omega'} = \frac{\phi v}{\phi k}$$

cos Ωp and Ω'

lies between O and Ωp. From the known values of Ωp, ϕv, ϕ_k, and b, the values $\Omega' = 0.976\ 030\ 9665$ and $r_k = 0.165\ 047\ 8395$ are derived.

This value or r_k determines the position of point R_1, inner summit of the separatrix. As this latter is a regular polygon whose summits are all replaced by roundoffs of same radii, the positions of all the other inner summits are known, and the separatrix is completely determined.

The calculation of r_{k2}, distance from point P to the round-off R_2A_2, is a matter of pure geometry. The following is found:

$r_{k2} = 0.190\ 133\ 2988$

The numerical calculation of ϕ, point by point, is not absolutely necessary in the rapid phase and in the slow phase. In fact, ϕ is proportional to a certain sum of lengths of axes of curve.

These curves are arcs of circles and straight-line segments very strictly in the rapid phase and approximately in the slow phase. By elementary geometry, the evolution of this sum may be thus easily provided.

However, in the mixed phase combustion, the flame fronts are arcs of circles of the 4th degree. Their lengths, at various instants, are numerically calculated to be able to deduce threfrom a graph point by point of the graph of ϕ and thus insure that the variations of the shape function in this period remain in the limits imposed, that is, between 0.85 and 1. In particular, it is important to determine the minimum ϕ_μ of ϕ in the mixed phase. The following values of ϕ have been found, as a function of the reduced thickness y. (It is noted that y is proportional to the combustion time).

The values computed for ϕ are shown in the following tabulation by phase:

Y	ϕ
RAPID PHASE	
$Y_o = 0$	$\phi_o = 0.9$
$Y_d = 0.061\ 049$	$\phi_d = 0.9$
$Y_v = 0.081\ 361$	$\phi_v = 0.85$
$Y_k = 0.190\ 755$	$\phi_k = 0.925$
MIXED PHASE	
$Y_{k'} = 0.164\ 622$	$\phi_{k'} = 0.934\ 672$
0.19	0.917 337
0.20	0.916 657
0.21	0.916 316
0.22	0.916 229
0.23	0.916 322
0.25	0.916 914
0.27	0.917 848
0.30	0.919 961
0.33	0.921 603
$Y_{g'} = 0.342\ 793$	$\phi g. = 0.922\ 839$
SLOW PHASE	
$Y_g = 0.356\ 524$	$\phi g = 0.874\ 151$
$Y_w = 0.591\ 693$	
$Y_f = 0.968\ 049$	$\phi_f = 1$
$Y_e = 1$	$\phi_e = 0$

It will be seen from the foregoing that the minimum value of ϕ_μ is about 0.9162. The relatively long duration of the slow phase (from $y_g = 0.356$ to $y = 1$) is shown, as is the very short time for combustion of the residue (from $y_f = 0.968$ to $y = 1$).

To put the foregoing results into physical terms, the rapid propergol has a combustion velocity of 15 mm/sec; the slow, 7.5 min/sec. C_o is a 12-branched (or 12-pointed) star having sharp internal summits or corners and rounded outer summits. G is a 24-branched star having circular arc inner and outer summits.

The physical structure shown in FIG. **16** for a 1-meter diameter solid propellant is accordingly readily derived. Since the radius was taken as unity and in the actual propellant the radius is 500 mm (half of 1 meter), it is only necessary to multiply the ratios based on the radius by 500 mm to obtain the actual dimensions of C_o and G. The radius of C_e is of course 500 mm, less any small inhibiting layer which might be utilized.

Taking first the dimension b, the distance OP in FIG. **16**, it is 0.292 227 × 500 mm, or 146.1 mm. The radius r_o of the circular arc constituting the outer apex of Co and having its center at P is 8.55 mm (0.017 110 × 500 mm).

The half-angle made by r'_o, the sharp inner apex of "the angle of 47° Co, is 90° less angle λ (between T'' M_3'' and the perpendicular at T_1; see FIG. **2** for a diagram of all these terms), since angle $T_2T_1M_3'$ is a right angle and T'' coincides with T_2. λ is equal to the angle of neutrality Ωp and Ωp was calculated above as 0.828 629 9487. Conversion from the radians used in the computation (multiplication by $180/\pi$) gives an angle of 4° 476 996. Subtraction of this value from 90° gives 42° 523 004 as the angle of the inner apex with OR_2. Since the total apex is twice as large by virtue of the replication of the image of the sector shown, the total inner apex angle is twice 42° 523 004 or 85° 046 008. This value can be rounded off to 85°3'.

The radii of curvature ϵ of the inner and outer apices of G are 3.25 mm (0.0065 × 500 mm). The distance a_M (shown as OS in FIG. **16**) to the center of curvature of the outer apices of G is 333.8 mm (0.677 551 × 500 mm). The distance r_k from P to R, is 0.165 068 and the distance b (that is OP in FIG. 16) is 0.292 227, the total of these being 0.457 295. The distance from O to the inner apex of G is accordingly 228.6 mm (0.457 295 × 500 mm).

Such a solid propellant has a weight of about 1300 kg

The noteworthy values of ϕ in the mixed zone are ϕ_k, $\phi_{k'}$, $\phi_{g'}$, and ϕ_g, corresponding respectively to the flame fronts passing through R_2, through R_1, tangential to G in the vicinity of S_1, in the vicinity of S_2.

The Table (28) below gives the noteworthy values of ϕ and, below each of them, the corresponding values of the reduced thickness.

TABLE (28)

ϕ_r	ϕ_k	$\phi_{k'}$	ϕ_μ	ϕ_h	$\phi_{g'}$	ϕ_g	ϕ_w
0.9	0.989 0	0.989 1	0.955 9	0.983 4	0.969 4	0.943 1	0.900 8
0.095 3	0.168 8	0.174 2	0.250 0	0.378 3	0.384 5	0.415 6	0.682 3

per linear meter, i.e., per meter of length along the axis of the cylinder of revolution.

EXAMPLE 9

A block of order 8 and of class 3, having an inner contour N_o, having a separatrix E of the first sort (FIG. 18), has been studied, with a view to first improvement of the block of Example 2, on the following data:

$$n = 2;\ a = 0.705\ 15;\ 0.9 \leq \phi \leq 1;\ \epsilon_b = 0.007\ 5.$$

The value b is chosen as 0.282 8, that is to say a value very close to that relative to the block of order 8 and of class 3 of Table (13) above; in this way, there is obtained:

$$\rho = 0.952\ 0;\ \sigma = 0.014.4;\ r_o = 0.026\ 5.$$

The calculations of a_1 and of a_2, effected by taking $\widehat{POP_2'} = 2\ \omega = \pi/12$, gives: $a_1 = OP_1' = 0.697\ 3$; $a_2 = OP_2' = 0.713\ 0$.

The inner apices R_1 and R_2 of the separatrix have been obtained in the following manner:

To find R_2, the point M_1' (FIG. 19 of the accompanying drawings) is determined on the second side where there would pass a flame front subsequent to C_r and corresponding to $\phi = 1$, if the rapid propergol occupied the entire space a between C_o and C_e; then, the point R_2 has been placed on the segment OM''_1 in such a way that $R_2M_1'' = 0.007\ 5$. The distances of P to the points M_1'' and R_2 have then been found to be equal to $r_1'' = PM_1'' = 0.216\ 6$; $r_{k2} = PR_2 = 0.210\ 2$.

As regards the apex R_1, its distance r_{k1} to P has been taken greater than r_{k2} but very slightly less than r''_1; the value $r_{k1} = PR_1 = r''_1 - 0.001 = 0.215\ 6$.

To obtain the angle $\theta_{k1} = \widehat{P_1'\ PR_1}$, firstly an outline of separatrix is determined between S_1 and S_2 which may be constituted, as from the apices B_1 and B_2 of the ellipses forming the roundoffs in the vicinity of the outer apices S_1 and S_2, of two hyperbolic arcs $B_1R'_1$, R'_1B_2 (FIG. 19) each possessing the following property: in the slow propergol, the rectilinear portions of flame front which they engender form with the bisectrix Δ of the angle $\widehat{P_1'\ OP'_2}$ angles both equal to the complement β_ν of the angle of neutrality ω_ν relative to ω. The numerical determination of the hyperbolic branches of which these arcs form a part is a known problem. Therefore, let R_{11} and R_{12} be the points, adjoining R'_1, where the hyperbolic branches bearing the arcs R'_1B_1 and R'_1B_2, respectively, intersect the circle of centre P and of radius r_{k1}; it is noted that the angles $\theta_{k11} = \widehat{P_1'PR_{11}}$ and $\theta_{k12} =$ angle $\widehat{P_1'PR_{12}}$ are very close, and that $\theta_{k11} > \theta_{k12}$. The relative position of the points R_1', R_{11}, R_{12} is therefore that indicated by FIG. 19 of the accompaning drawings. For θ_{k1} is adopted a value equal approximately to the average of the two angles θ_{k1} and $\boldsymbol{\theta}_2$, in other words $\theta_{k1} = 0.322\ 9$. The separatrix is thus determined.

FIG. 18 of the accompanying drawings gives an outline on the scale of C_o and of G corresponding to the numerical values defined above.

EXAMPLE 10

This relates to a block of order 6 and of class 4, with an inner contour N_o, with a separatrix E of the second sort, with $\phi_o = 1$, $\phi_r = \phi_w = 0.9$, $0.9 \leq \phi \leq 1$, $\epsilon_b = 0.007\ 5$, $n = 2$ and, accordingly, $a = 0.706\ 55$, $b = 0.262\ 9$.

The elementary motif is that of FIG. 20, and there is obtained:

$$\rho = 0.968\ 1,\ \sigma = 0.014\ 2,\ r_o = 0.038\ 0.$$

The calculation of a_1 and of a_2, effected by taking, to begin with, $\widehat{POP'} = \omega = \pi/24$, $\widehat{POP_2'} = 3\ \omega$, gives $a_1 = OP_1' = 0.693\ 1$, $a_2 = OP_2' = 0.720\ 0$, which determines the apex-images P_1' and P_2', as well as the points of connection B_1, B_1', B_2, B_2' of the elliptical arcs to the hyperbolic arcs of the separatrix (as above, these points are the apices of the minor axes of the ellipses).

The definition of the inner apices R_1, R_2, R_3 of the separatrix is effected by a method similar to that used for the previous example.

For the apex R_3, situated on the second side of the elementary motif, firstly there is defined on this side the point M_1' (FIG. 21 of the accompanying drawings) which would be reached, if the rapid propergol occupied the entire space between C_o and C_e, by a flame front subsequent to C_r, and for which ϕ would be 1; then the point R_3 is taken on the segment OM_1'', at the distance 0.007 5 from the point M_1''. This entails $PR_3 = r_{k3} = 0.233\ 0$.

To define R_2, hyperbolic arcs $R_2'\ B_1'$ and $R_2'\ B_2$ are constructed (FIG. 21 of the accompanying drawings) which, if they were elements of separatrix, would engender in the slow propergol portions of rectilinear flame fronts forming with the bisectrix Δ' of the angle $B_1'\ PB_2$ angles equal one and the other to the complement β_ν of the angle of neutrality ω_ν relative to ω; the hyperbolic branches which bear these arcs intersect the circle of centre P and of radius r_{k3} respectively at two points R_{21}, R_{22} adjoining R'_2, and it is found that $\theta_{k21} = \widehat{R_1\ PR_{21}} > \theta_{k22} = \widehat{R_1PR_{22}}$. Then, the apex R_2 is placed in the centre of the circular arc $\widehat{R_{21}\ R_{22}}$ of centre P; in this way: $PR_2 = r_{k2} = r_{k3} = 0.233\ 0$; $\widehat{R_1PR_2} = \theta_{k2} = 0.377\ 2$.

On the apex R_1 is imposed the condition that the arc of separatrix $R_1\ B_1$ engenders in the slow propergol rectilinear portions of flame front forming with the main side an angle equal to β_ν. In this way it was found that $PR_1 = r_{k1} = 0.222\ 6$.

The separatrix is completely determined.

The Table (29) below gives several values of ϕ and of r in the mixed zone, with the corresponding noteworthy flame fronts.

TABLE (29)

flame front	r	ϕ
C_k (point R_1)	0.222 6	0.972 6
C_k' (points R_2 and R_3)	0.233 0	0.983 5
	0.25	0.967 3
	0.275	0.959 3
	0.30	0.958 9
	0.35	0.968 6
	0.40	0.980 7
C_h (points B_1 and B'_1)	0.438 2	0.993 5

The minimum ϕ_μ is worth approximately 0.958 9.

FIG. 20 of the accompanying drawings gives an outline on the scale of C_o and of G, for the numerical values defined above.

EXAMPLE 11

One can construct a block distinguished from the previous one only by its separatrix, this latter being of shape D and of the second sort; the data are thus: $p = 6$, $c = 4$, $n = 2$, $\phi_o = 1$, $\phi_v = \phi_w = 0.9$, $0.9 \leqslant \phi \leqslant 1$, $\epsilon = 0.006\ 5$.

The elementary motif is that of FIG. 22 of the accompanying drawings. According to Table (13) above $\rho = 0.968\ 1$, $\sigma = 0.014\ 3$, $a = 0.707\ 0$, $b = 0.262\ 9$, $r_o = 0.038\ 0$.

By taking $\widehat{O_1'OP} = \omega = \pi/24$, $\widehat{O_2'OP} = 3\ \omega$, the calculation of a_1 and of a_2 leads to the values $a_1 = OP_1' = 0.693\ 7$, $a_2 = OP_2' = 0.720\ 3$; and it is found that $d_1 = OO_1' = 0.700\ 0$, $d_2 = OO_2' = 0.726\ 5$, $\sigma_1 = \widehat{P_1'OO_1'} = 0.000\ 7$, $\sigma_2 = \widehat{P_2'OO_2'} = 0.001\ 8$.

To simplify matters, the distances r_{k1}, r_{k2}, r_{k3} to the main centre of curvature P of the three inner apices R_1, R_2, R_3 of G of one and the same elementary motif are taken equal one to another. Therefore, when ϕ_k is given, r_k is deduced therefrom immediately and the position of the inner apices R_1 and R_3 becomes known.

At the same time the inner apex R_2 is determined by imposing on the rectilinear segments of the separatrix A_2B_1' and A_2' B_2 the condition of being equally inclined to the straight line Δ_2 which joins P to the centre of the small circular arc A_2A_2'. Such a condition is in no way imperative, but it greatly facilitates the calculation of ϕ in the mixed phase and in the slow phase. Thus, the outline of the block is completely defined as from ϕ_k.

The Table (30) below gives, for $\epsilon = 0.006\ 5$, the values of r_k, of the sine of the angle θ_2 of the straight line Δ_2 and of the main side, and finally of ϕ_μ and of ϕ_h for various values of ϕ_k; ϕ_h is here the value of ϕ for the flame front passing through the point B_1.

TABLE (30)

ϕ_k	r_k	sin θ_2	ϕ_μ	ϕ_h
1	0.239 3	0.384 5	0.972 5	0.986 9
0.99	0.233 3	0.384 9	0.964 2	0.990 8
0.98	0.227 2	0.385 4	0.955 2	0.944 8
0.97	0.221 0	0.385 8	0.945 9	0.998 8

The curves of variation of ϕ in the mixed phase, from ϕ_k to ϕ_h, and still with $\epsilon = 0.006\ 5$, are given for $\phi_k = 1$ and for $\phi_k = 0.99$ in FIG. 23 of the accompanying drawings where they are united, for greater clarity, by hatchings. It is observed that the influence of ϕ_k is similar to that observed in Example 3.

If ϵ is caused to vary without changing the other parameters of base a, b, n, the position of the centres O'_1 and O'_2 of the round-offs of the exterior apices varies only very slightly, but the aspect of the variations of ϕ in the mixed phase is completely modified. This is what is shown in FIG. 23 by the curves representative of ϕ as a function of r from ϕ_k to ϕ_k, for ϵ successively equal to 0, to 0.003, to 0.006 5 and to 0.008, whilst ϕ_k always remains equal to 1. An increase of ϵ has thus an effect similar to an increase of n.

The Table (31) below gives for these new values of ϵ the lengths r_k, a_1, a_2 and the sine of the angle θ_2 defining the separatrix.

TABLE (31)

ϵ	r_k	sin θ_2 (for $\phi_k = 1$)	a_1	a_2
0	0.239 3	0.383 1	0.693 6	0.720 5
0.003	0.239 3	0.383 7	0.696 6	0.723 3
0.008	0.239 3	0.384 8	0.701 4	0.727 9

FIG. 22 of the accompanying drawings gives the outline to scale of C_o and of G for the numerical values given at the start of this Example.

EXAMPLE 12

This relates to a block of the fifth group constructed on the basis of a modification of the block of Example 11, of which the inner contour is such that $\Omega = \pi/6$, $b = 0.262\ 9$, $r_o = r_n = 0.038\ 0$, $\Omega'' = 1.479\ 7$, $\lambda = \Omega_p = 0.985\ 6$, $r'_o = 0$, $\rho = 0.968\ 1$, and which is assumed to have a maximal arrow which is too high.

A reference outline is therefore envisaged in which r_o is already slightly increased; r_o is chosen to be 0.045 and, on the other hand, for constructional reasons, $m = 4$ and $\zeta = 0.005$.

With these values of r_o and of ζ, the Table (32) below shows how ρ_1 varies as a function of r'_o.

TABLE (32)

r'_o	0	0.01	0.03	0.05	0.07	0.08
ρ_1	0.971 7	0.972 6	0.973 5	0.973 7	0.972 8	0.972 3

The value r'_o is chosen to be 0.03; the reference outline is finally characterised by $\Omega^* = \pi/6$, $b = 0.262\ 9$, $r_o = 0.045$, $\Omega''^* = 1.479\ 7$, $\lambda_o = \Omega_p = 0.985\ 6$, $r'_o = 0.03$, $\rho^* = 0.960\ 1$, $s = 0.248\ 8$, $\phi_o = 0.940\ 6$.

For $\zeta = 0.005$ and $r'_o = 0.03$, $\lambda_1 = 1.228\ 2$, $s_1 = 0.308\ 7$, $\rho_4 = 0.973\ 5$.

Therefore, the three numbers s_2, s_3, s_4 can be determined, which numbers have to form with s_1 (taken as the first term) a geometrical progression of which the four terms have s as arithmetical mean. Then:

$s_2 = 0.264\ 7$, $s_2 = 0.227\ 0$, $s_4 = 0.194\ 6$.

However, there does not exist a value of λ giving, with the fixed values of Ω, b, r_o, Ω'', an elementary perimeter equal to s_4; therefore, the value of Ω'' must be calculated which, for the fixed values of Ω, b, r_o and for λ=0, allows s to have the value s_4. On the other hand, with the fixed values of Ω, b, r_o, Ω'' the values of λ_2 and λ_3 of λ are calculated to make $s = s_2$ and $s = s_3$. Finally, four inner contours of shape A having the following characteristics are obtained:

$s_1 = 0.308\ 7$; $\Omega'' = \Omega_p = 1.479\ 7$; $\lambda = \lambda_1 = 1.228\ 2$; $\rho_1 = 0.973\ 5$
$s_2 = 0.264\ 7$; $\Omega'' = \Omega_p = 1.479\ 7$; $\lambda = \lambda_2 = 1.083\ 5$; $\rho_2 = 0.963\ 6$
$s_3 = 0.227\ 0$; $\Omega'' = \Omega_p = 1.479\ 7$; $\lambda = \lambda_3 = 0.698\ 9$; $\rho_3 = 0.955\ 3$
$s_4 = 0.194\ 6$; $\Omega'' = \Omega''_4 = 1.223\ 2$; $\lambda = \lambda_4 = 0$; $\rho_4 = 0.945\ 1$

The final coefficient of filling, equal to the arithmetical means of ρ_1, ρ_2, ρ_3, ρ_4, is worth 0.959 4.

In relation to the reference outline, the coefficient of filling has therefore decreased by 0.000 7, which is negligible; the initial value ϕ_o of ϕ has not varied; however, it is found that the maximal "inherent locking" K^*_M has decreased by 23.3% comparatively to a block having a central cavity of constant section and in accordance with the reference outline.

What is claimed is:

1. A solid propellant for use as a gas generator, particularly for the reaction propulsion of vehicles in space or in a gaseous or liquid medium, said propellant having a lateral outer surface which is a cyclinder of revolution and which does not participate in combustion and the propellant having a lateral inner surface defining a central cavity of a shape elongated in the direction of the axis of the cylinder, the inner surface being ignitable at the initial instant of firing, and wherein said solid propellant is formed from two propergols having different speeds of combustion, the two propergols contacting one another without interruption along a continuous separation surface or separatrix surrounding the inner surface, the more rapidly combustible propergol occupying the space between the inner surface and the separatrix and the more slowly combustible propergol occupying the remaining volume of the solid propellant between the separatrix and the outer surface and wherein the solid propellant through a plane perpendicular to the axis of the cylinder satisfies the following two conditions:

1. the inner surface is closed, continuous, and is substantially star-shaped having p branches with p at least equal to 3, the shape of the star being such that the rapid phase of the combustion in which only the rapidly combustible propergol burns, ends at the instant when the section of the surface of combustion is a line formed only of p consecutive arcs having their concavity towards a point and forming a curve parallel to a portion of the end of a different branch of the star, said arcs being composed substantially of circular arcs whose centers of curvature are situated on p radii of said inner surface, each radius intersection said inner contour at a point of the end of a different star branch; and
2. the separatrix does not touch the outer surface or the inner surface and has substantially the shape of a star having ν branches, with ν at least equal to p, the separatrix star shape being such that toward the end of the slow phase of the combustion in which only the more slowly combustible propergol burns, the cross section of the surface of combustion which reaches said outer surface is a line formed solely of ν consecutive arcs having their concavity towards said point, being substantially tangential to said outer surface, and being placed opposite the end of one of said ν branches of said separatrix, said ν arcs being themselves substantially composed of circular arcs whose centers are all situated in the vicinities of the ends of said ν star branches.

2. A solid propellant as claimed in claim 1 wherein the inner surface at the end opposite that from which the gases formed upon combustion exit has a surface less than the surface at the end from which the gases exit and the surface increases from the opposite end to the end from which the gases exit.

3. A solid propellant as claimed in claim 1, wherein said outer surface has p' circular sectors, with p' at least equal to p, so that the portions of the surface of combustion contained in said sectors are all superimposable one on the other and wherein all the portions of said separatrix contained in the same sectors are similarly superimposable.

4. A solid propellant as claimed in claim 1, wherein the portions of the combustion surface and the separatrix contained in any main sector of said outer surface limited by two consecutive main sides each has, as an axis of symmetry, the inner bisectrix of the said main sector; and wherein the sides of said main sectors each intersect the separatrix at a point of the end of one of the ν star branches.

5. A solid propellant as claimed in claim 3, wherein the portions of said surface of combustion and said separatrix contained in any main sector each have the inner bisectrix of said sector as an axis of symmetry.

6. A solid propellant as claimed in claim 5, wherein the portion of said inner surface situated in any main sector and subtending an angle at the center of $2\ \Omega = 2\ \pi/p$, comprises substantially the successive elements, connecting tangentially one to the other in order from a side to the bisector of $2\ \Omega$ a first circular arc having a radius r_o, centered on the side of the sector of curvature P, which is on said side of said sector, inside at a distance b from the axis a second circular arc having its center at a point situated on the bisectrix of the sector and characterized by the angle Ω''; from the point on the bisector to the axis and from the point on the bisector to the main center of the curvature a straight line section of length ζ, and of orientation characterized by the angle λ from the point on the bisector to the axis and from the point on the bisector to the beginning of the straight line section and a third circular arc having its end on the bisector between the axis and the point on the bisector and having its center at a point on the segment from the end of sector to the point on the bisector.

7. A solid propellant as claimed in claim 6, wherein the following values substantially apply:

r_o equal to

$$r_n \quad \frac{0.06}{bp};$$

λ equal to the angle Ωp defined by the relation: tan Ωp - Ωp = Ω,

$$O < \Omega p < \frac{\pi}{2}.$$

8. A solid propellant as claimed in claim 6, wherein that at least one of the values r_o, r'_o, ζ, λ is substantially nil.

9. A solid propellant as claimed in claim 6 wherein

$$\frac{\sin \Omega''}{\Omega + \Omega''} = \frac{\phi\nu}{\phi_{M3}''} \cos \Omega p,$$

-continued

$$\tan \Omega_p - \Omega_p = \Omega, \text{ with}$$

$$O < \Omega_p < \Omega'' < \frac{\pi}{2}$$

and

$$b = \frac{a}{h} \cdot \frac{\phi v}{\phi w} \quad \frac{\sin \omega}{\omega}$$

$$\frac{\Omega}{\sin \Omega} \quad \frac{\cos \Omega p}{\cos \omega_\nu}$$

where ϕ is the shape function (equal to the corrected perimeter of $C(t)$ the surface of combustion at time t divided by the corrected perimeter of C_f, the surface of combustion at first contact with C_e, the outer surface of the cylinder), ϕ_v is the shape function of minimum value in the more rapidly burning propergol, ϕ_{M_3}'' is the maximum shape function in the more rapidly burning propergol, a is the arithmetic means of the distances from the axis to the inner surface, n' is equal to the rate of combustion in the more rapidly burning propergol divided by the rate of combustion in the more slowly burning propergol, ϕ_w is the final minimum value in the more slowly burning propergol, and $\tan \omega_\nu - \omega_\nu = \omega$.

10. A solid propellant as claimed in claim **7**, wherein at least one of the values r_o, r'_o, ζ, λ is substantially nil.

11. A solid propellant as claimed in claim **1**, wherein the sides of the ν star branches are comprised by portions of straight lines or hyperbolas and the ends of the branches are portions of a circle, a Descartes oval, or an ellipse.

12. A solid propellant for reaction propulsion having a substantially cylindrical outer surface and which comprises two separate substantially homogeneous propergols, the first of which has a combustion speed which is higher than the combustion speed of the second propergol, the first propergol defining a central axial cavity in the cylinder and extending inwardly from said cavity to the second propergol and the second propergol occupying the volume between the first propergol and the cylindrical outer surface, the cross-section of the surface of the first propergol at the interface with the cavity perpendicular to the axis of the cylinder being in substantially the shape of a star with p branches, p, being at least three, the outer surface of the first propergol defining a separatrix which is coextensive with the inner surface of the second propergol the cross-section of the surface of the separatrix perpendicular to the axis of the cylinder having the form of a star having ν branches wherein ν is greater than p, the maximum distance between the separatrix and the axis of the cylinder being not more than about 0.75 times the radius of the cylinder.

13. A reaction propulsion device comprising a propellant according to claim **12** capable of producing gases upon combustion and a generally cylindrical chamber enclosing the propellant, the chamber having at least one aperture to permit the efflux of gases from the chamber.

* * * * *

UNITED STATES PATENT OFFICE

CERTIFICATE OF CORRECTION

PATENT NO. : 3,918,365

DATED : November 11, 1975

INVENTOR(S) : PAUL ARRIBAT

It is certified that error appears in the above-identified patent and that said Letters Patent are hereby corrected as shown below:

Column 1, line 5, after "1971" insert a comma

Column 1, line 32, "with" should read --without--

Column 2, line 17, "c_e" should read --C_e--; line 45, correct the spelling of "different"

Column 4, line 41, "apexim-" should read --apex-im--

Column 4, line 64, "R_i" should read --B_i--

Column 5, line 49, correct the spelling of "conform"

Column 8, line 61, after "a_M" insert a comma

Column 10, line 7, cancel "8n"

Column 10, line 26, correct the spelling of "does"

Column 11, line 1, "P'P'O'" should read --P'O'--

Column 11, line 13, ahead of the equation insert --(5)--

Column 11, line 39, "$O > w: > w_V$" should read --$O > w' > w_V$--

Column 12, line 21, "2..." should read --2....,--

UNITED STATES PATENT OFFICE

CERTIFICATE OF CORRECTION

PATENT NO. : 3,918,365

DATED : November 11, 1975

INVENTOR(S) : PAUL ARRIBAT

It is certified that error appears in the above-identified patent and that said Letters Patent are hereby corrected as shown below:

Column 12, line 35, "$\underset{b}{(10)}\ r_k = \frac{\sin \Omega}{\sin \Omega'}$" should read --$(10)\ r_k = b\,\frac{\sin \Omega}{\sin \Omega'}$--

Column 14, line 16, "b" should read --b*-- and "r_o" should read --r_o*--

Column 15, line 46, "P_o" should read --σ_o--

Column 15, line 54, "p" should read --σ--

Column 16, line 5, "p" should read --σ--

Column 17, line 20, "C.254 2" should read --0.254 2--

Column 18, line 44, "$\widehat{CM''P}$" should read --$\widehat{OM''P}$--

Column 19, line 24, delete "("

Column 24, line 16, after "straddling" insert --2°--

Column 25, line 61, "'k" should read --ϕ_k--

Column 25, line 62, "'g" should read --ϕ_g--

Column 28, line 14, after "tan w_v" cancel the solidus (/)

CERTIFICATE OF CORRECTION

PATENT NO. : 3,918,365

DATED : November 11, 1975

INVENTOR(S) : PAUL ARRIBAT

It is certified that error appears in the above-identified patent and that said Letters Patent are hereby corrected as shown below:

Column 29, line 3, "100_o" should read --ϕ_o--

Column 29, line 65, correct the spelling of "therefrom--

Column 30, line 53, cancel " "the angle of 47°"

Column 30, line 55, "$T_2T_1M_3'$" should read --$T_2T_1M_3''$--

Column 30, line 60, "4° 476 996" should read --47°. 476 996--

Column 30, line 61, "42° 523 004" should read --42°. 523 004--

Column 30, line 64, "42° 523 004" and "85° 046 008" should read,respectively --42°. 523 004-- and --85°. 046 008--

Column 31, line 65, correct the spelling of "accompanying"

Column 31, line 67 "θ_2" should read --θ_{k2}--

Column 32, line 42, "M_1'" should read --M_1''--

Column 34, line 53, "b" should read --b*-- and "r_o" should read --r^*_o--

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INVENTOR(S) : PAUL ARRIBAT

It is certified that error appears in the above–identified patent and that said Letters Patent are hereby corrected as shown below:

Column 34, line 54 "r'_o" should read --r'^*_o--

Column 34, line 55, "s" should read --s*--

Column 34, line 60, "s" should read --s*--

Column 35, line 10, "means" should read --mean--

Column 35, line 52, "intersection" should read --intersecting--

Column 37, lines 10-19, the mathematical equation should all be in one line

Signed and Sealed this

Thirteenth Day of July 1976

[SEAL]

Attest:

RUTH C. MASON
Attesting Officer

C. MARSHALL DANN
Commissioner of Patents and Trademarks