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Coupland

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[54] **ELECTRICAL COILS FOR GENERATING MAGNETIC FIELDS**

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[52] **U.S. Cl.** **335/213,** 335/216

[51] **Int. Cl.** **H01f 5/00**

[58] **Field of Search** 335/210, 213, 216

[56] **References Cited**

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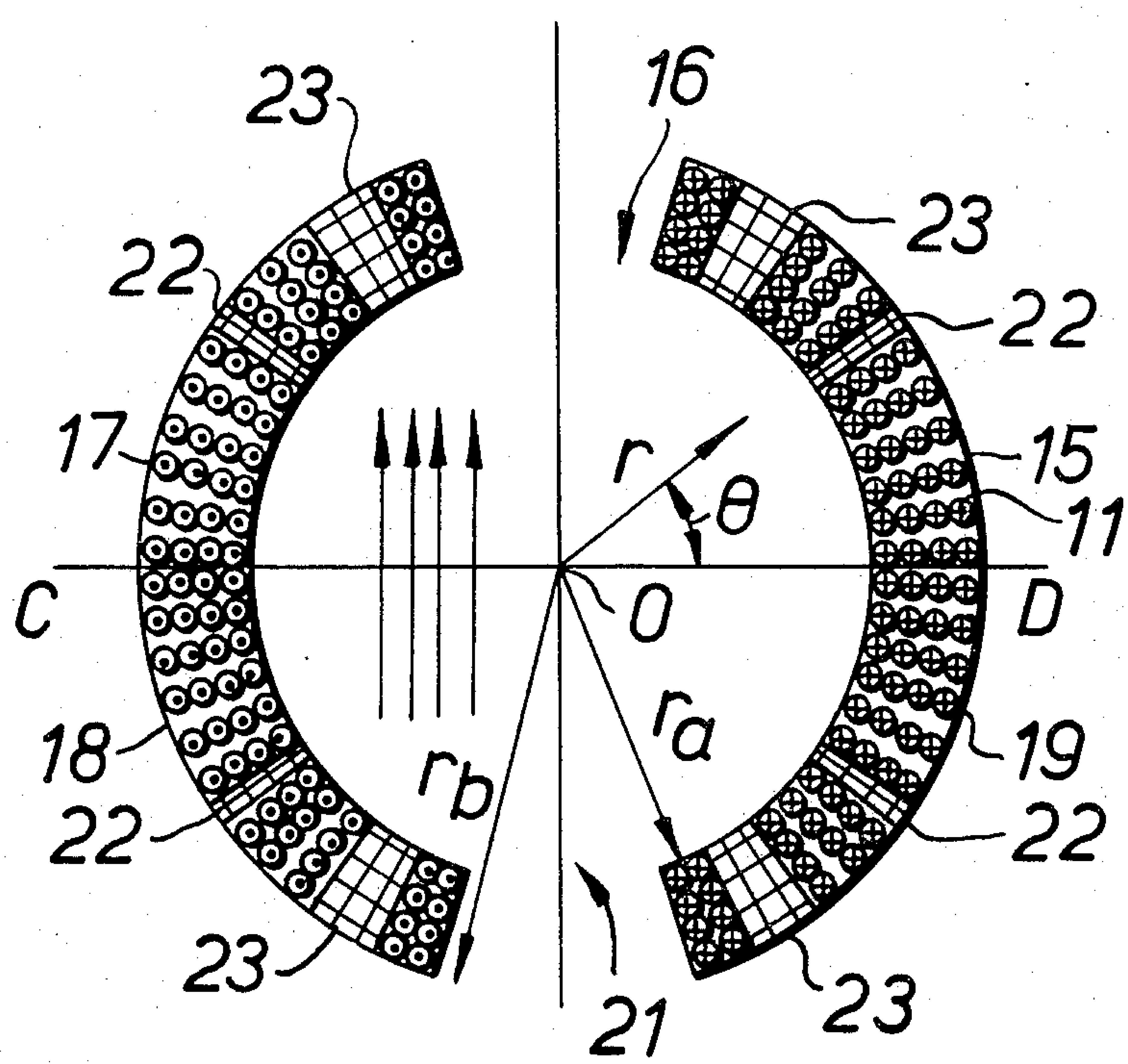
Primary Examiner—George Harris
Attorney—Larson, Taylor and Hinds

[57] **ABSTRACT**

A novel approach to the mathematical analysis of the magnetic field produced by the coil windings of accelerator beam bending/focussing magnets has led to the proposal for inserting accurately located and dimensioned azimuthal spaces in the windings. The spaces provide a simple means for improving the uniformity of the magnetic field. A mathematically specified pattern of end winding reduces their disturbing influence.

10 Claims, 6 Drawing Figures



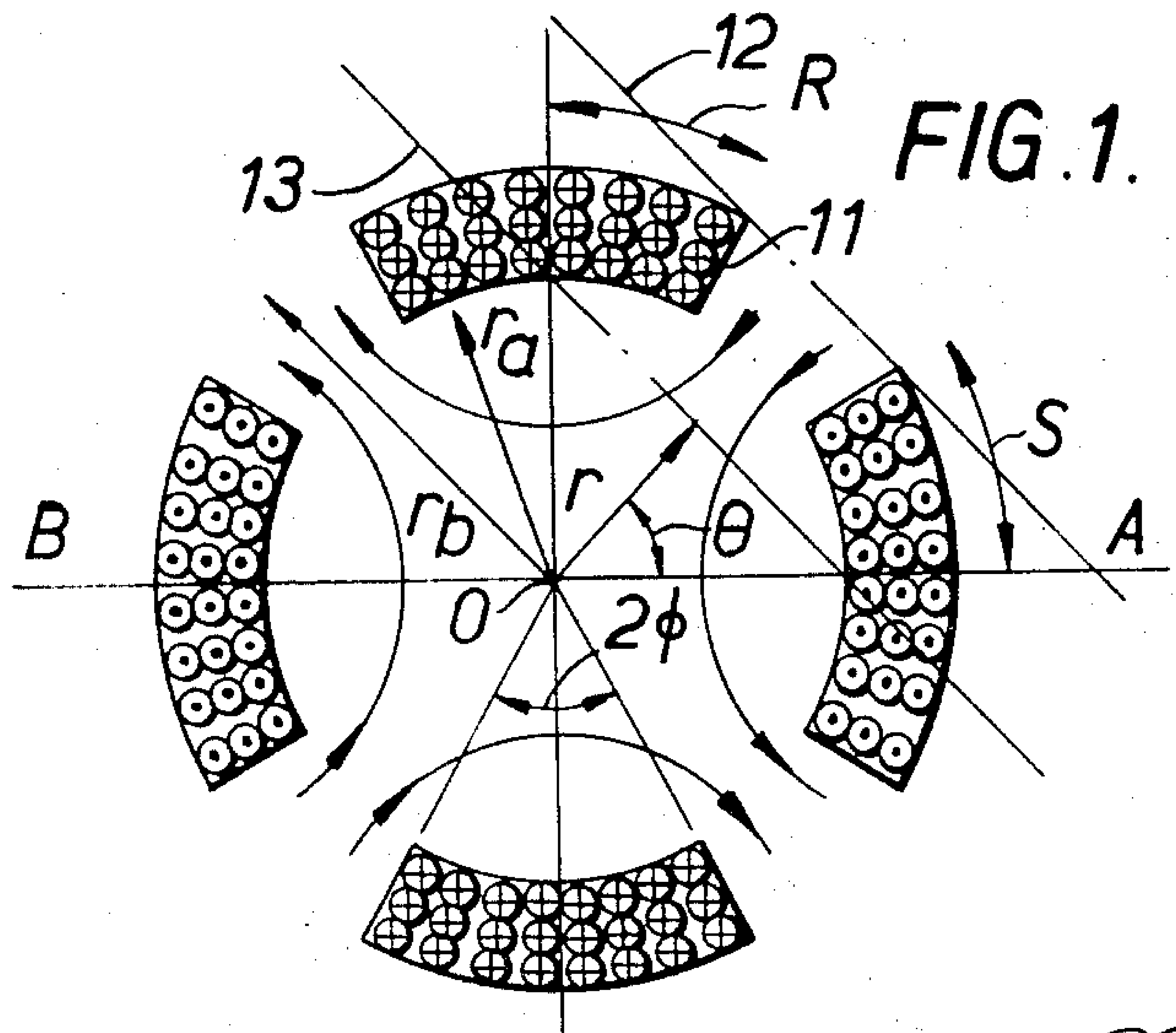


FIG.1.

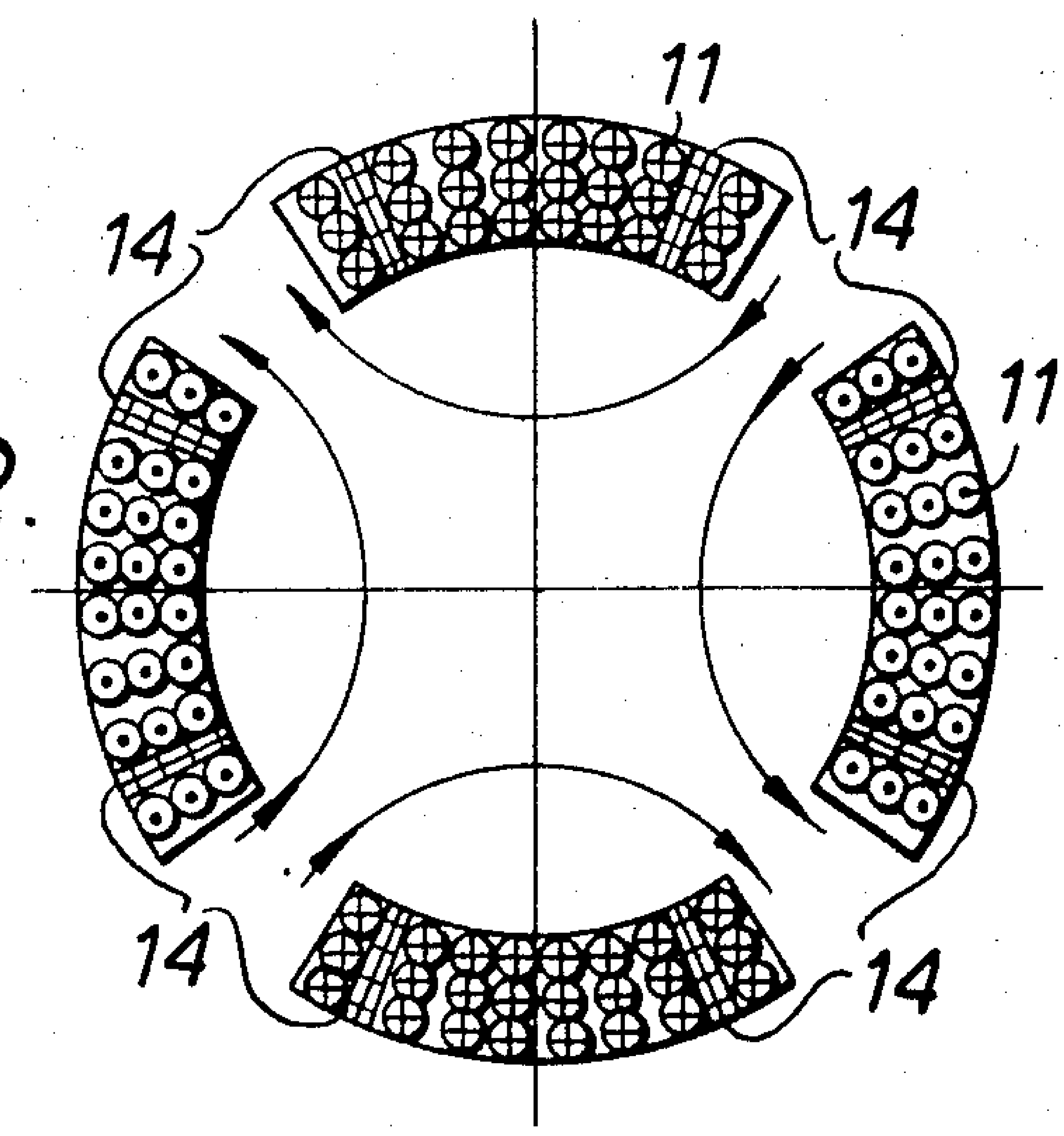


FIG.2.

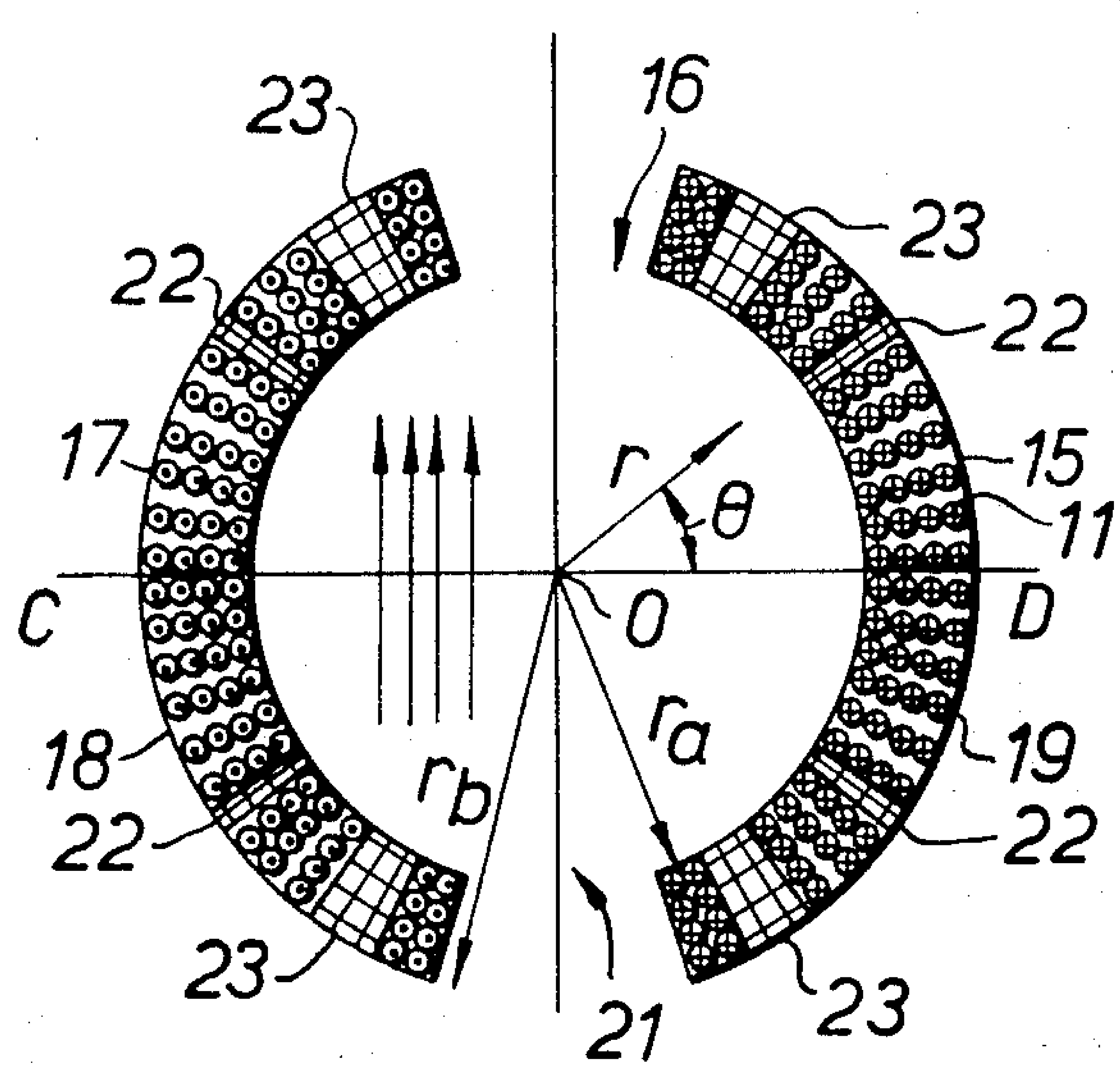


FIG.3.

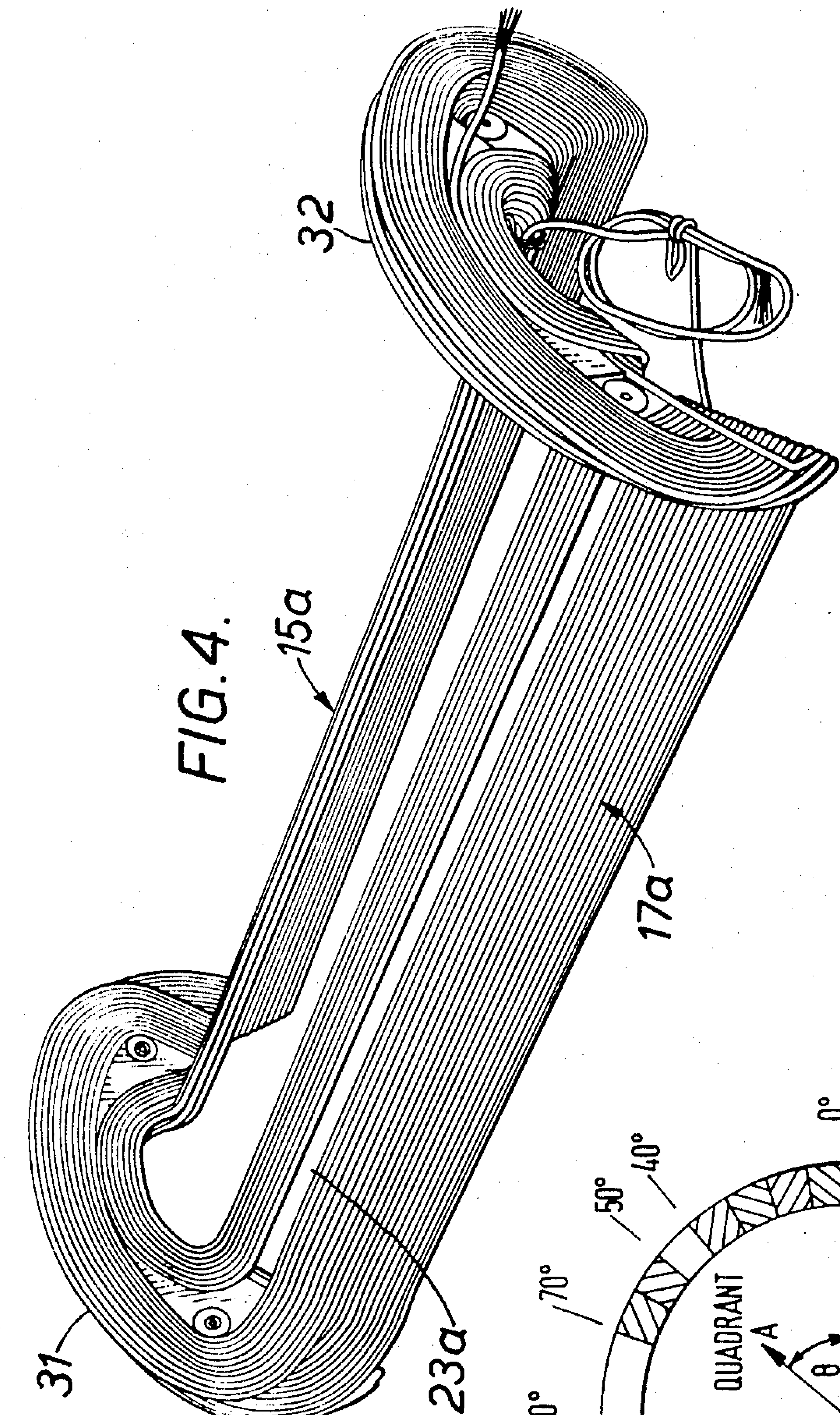


FIG. 4.

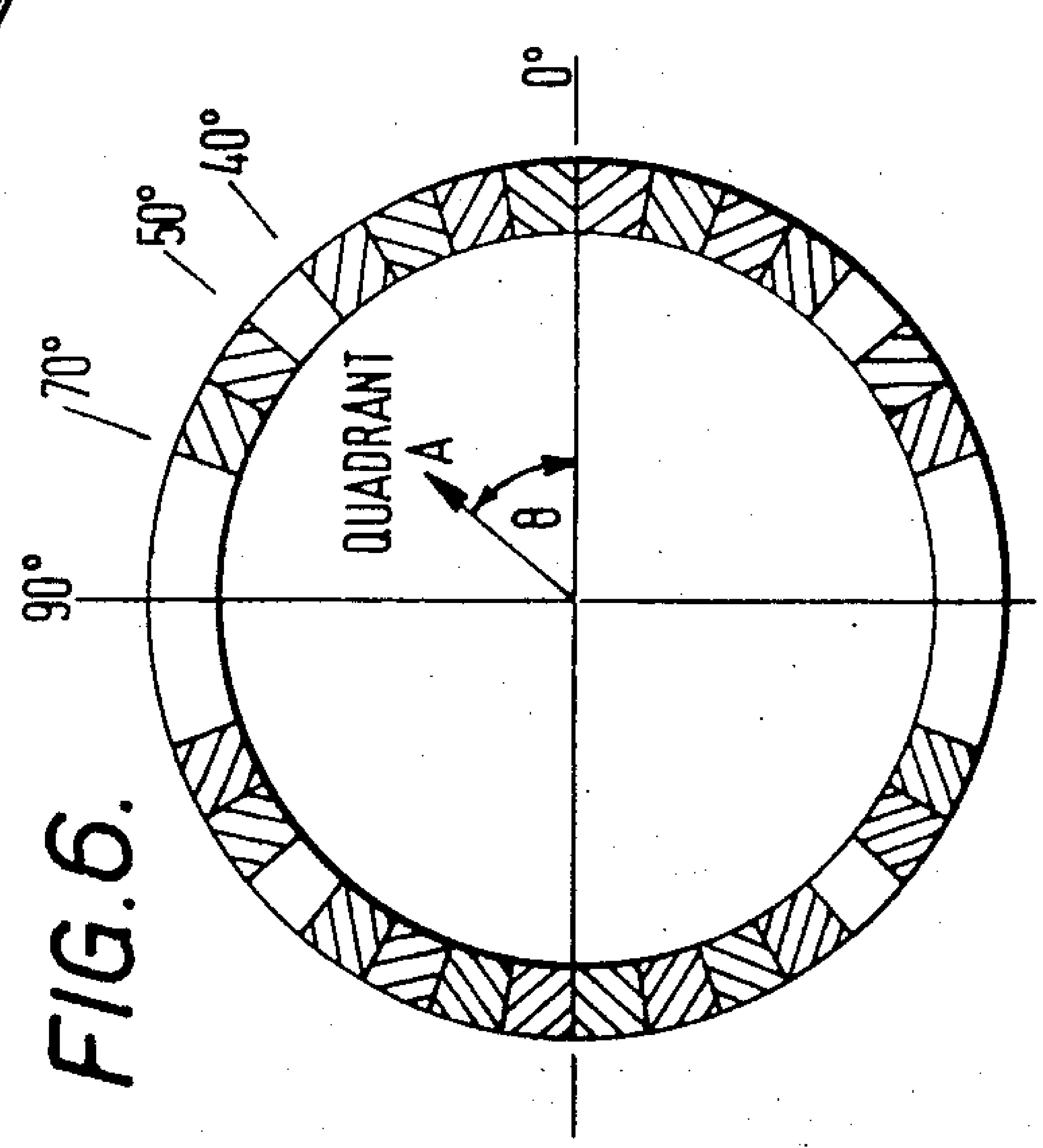


FIG. 6.

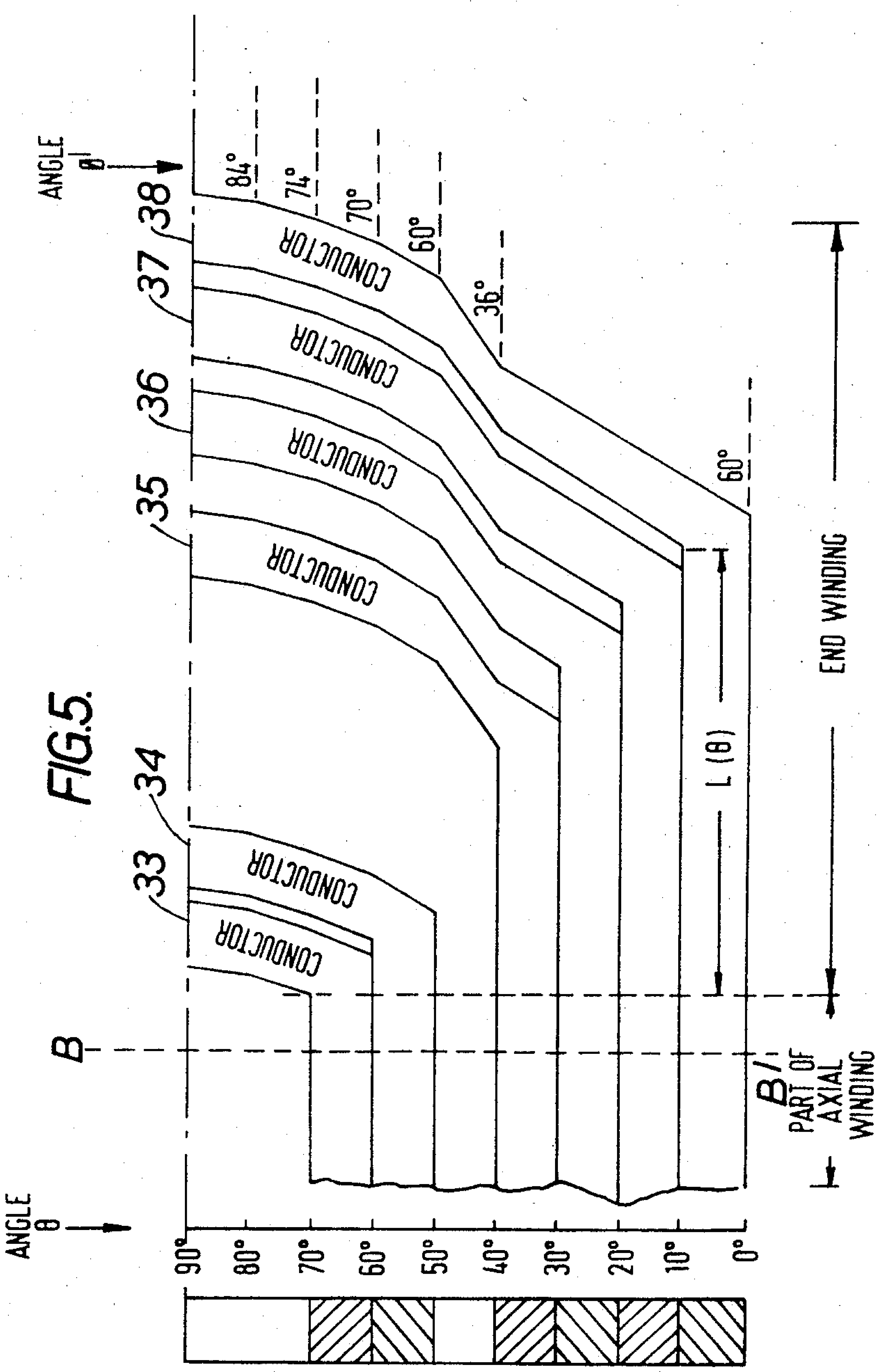

ANGLE
ø'
38
37
36
35
34
33
84°
74°
70°
60°
36°
60°
CONDUCTOR
END WINDING
L (θ)
B
B'
PART OF AXIAL WINDING
ANGLE
θ
90°
80°
70°
60°
50°
40°
30°
20°
10°
0°

FIG.5.

ELECTRICAL COILS FOR GENERATING MAGNETIC FIELDS

BACKGROUND OF THE INVENTION

The invention relates to electrical coils for generating magnetic fields and more particularly to such coils for generating the magnetic fields for beam bending or beam focusing in charged particle accelerators.

If one considers long electrical conductors arranged around the surface of an infinitely long cylinder, with the lengths of the conductors parallel to the axis of the cylinder, then with axial current flow in the conductors and a cos θ distribution of current density around the surface of the cylinder, a uniform magnetic dipole field within the cylinder will be generated. A cos 2θ distribution of current density will generate a pure magnetic quadrupole field within the cylinder.

Electrical coils for generating magnetic fields for beam bending or beam focusing in charged particle accelerators are constructed so as to approximate as closely as manufacturing difficulties will allow to the ideal situation described above. It will be appreciated that the cylinder cannot be infinitely long and, in practice, the windings are in the form of a coil, a winding running up the length of one side of the cylinder crossing over at an end to run back down the other side.

Although it is not necessary in the calculation to limit the winding to a thin shell, the winding thickness in practice is likely to be relatively small if a high field superconductor is used. A high current density winding is particularly important for a multipole magnet in order to limit the volume of conductor required.

One proposal for approximating the sinusoidal current density distribution is made by W. B. Sampson in Proceedings of the International Conference on Magnet Technology, Oxford, 1967, page 574. In this proposal, the circumference of the cylinder is divided into 10° equi-depths slots in which are wound the appropriate average number of conductors for a stepwise approximation to the sine wave.

Another, constructionally simpler, proposal is to use a constant current density winding, a constant radial or slot depth, but choose the azimuthal length of the windings so as to reduce field errors in the beam space. Such a proposal is adopted by Asner and Iselin in Proceedings of the International Conference on Magnet Technology, Oxford, 1967, page 32, who propose further refinement by the addition of further coils wound radially outside the basic coil.

The present invention is based upon a novel approach in the mathematical analysis of the magnetic field generated by electric current in the windings from which it has been appreciated that considerable improvement in the field uniformity (or field purity in the case of magnetic quadrupole fields or fields of higher orders) may be achieved by the insertion of accurately located and dimensioned spacers within the coil windings.

SUMMARY OF THE INVENTION

The invention provides, in one of its aspects, an electrical coil for generating a magnetic field, of order N which coil is of the form in which the windings lie in a bundle between two parallel planes spaced apart, the windings lying in two side runs and two end runs where the windings cross from one side run to the other, the general direction of the lengths of the windings in the two side runs being parallel to the said two planes, the side runs being substantially longer than the end runs so that the magnetic field is principally defined by the side runs, the windings being arranged so that there is at least one region, but not more than 4N regions, each region being defined in cross-section by an area within each perimeter respectively of the bundle of windings as seen in cross-section in the two side runs, from which region the windings are absent, the number of windings per unit cross-sectional area in the side runs being substantially constant throughout the regions where windings are present, and the location and extent of the region or regions where the windings are absent being selected to enhance the uniformity or purity of the magnetic field generated by the coil.

It is an important feature of the invention that the windings in the said two side runs are straight and parallel with one another.

The invention provides, in another of its aspects, an electrical coil for generating a magnetic field wherein the windings of the coil, as seen in cross-section, are distributed within boundaries defined by concentric circles and generally radially extending lines at the azimuthal limits of the coil windings, and wherein the windings are so arranged that there is at least one region defined by an area within the said boundaries as seen in cross-section, from which region the windings of the coil are absent, the location and extent of the region being selected to enhance the uniformity or purity of the magnetic field generated by the coil.

Preferably said area defining the region comprises a sector of the outer circular boundary truncated by the inner circular boundary.

Preferably the coil windings are divided by azimuthal spacers inserted in the windings, the spacers occupying the said regions as defined above.

For generating a uniform dipole magnetic field, the arrangement may comprise coil windings which, as seen in cross-section, are within boundaries defined by concentric circles, the azimuthal limits of the coil windings being at 67.40° in each quadrant measured from a reference axis, and spacers in each quadrant, the azimuthal extent of the spacers being from 43.50° to 52.60° measured from the reference axis, the coil windings being otherwise uniformly distributed within the boundaries.

For generating a quadrupole field of high purity, the azimuthal limits of the coil windings, of which limits there will be two in each quadrant defining the four magnetic pole regions, and the azimuthal extents of the spacers, of which there will be two in each quadrant, may be derived by dividing by two the aforementioned angles for the dipole field coil. Similarly, the corresponding angles for a sextupole field of high purity may be derived by dividing by three the aforementioned angles for the dipole field coil, and so on for higher order fields.

While the fields generated by coils with limits so defined will be of improved uniformity or purity as compared with the fields produced by similar coils without spacers, the dipole field may be still further improved if the azimuthal limits of the coil windings are at 71.81° in each quadrant measured from the reference axis with spacers from 33.38° to 37.17° and from 53.16° to 63.36°.

Corresponding improvement in higher order fields may be obtained by including additional spacers, but the higher the number of spacers, the more stringent become the manufacturing tolerances on dimensions and locations of the spacers which have to be met. In practice, one spacer per pole for quadrupole fields and fields of higher orders, and two spacers per pole for dipole fields, is a likely application of the technique of insertion of spacers in windings of magnets for accelerators.

BRIEF DESCRIPTION OF THE DRAWINGS

Specific constructions of electrical coil embodying the invention will now be described by way of example and with reference to the accompanying drawings in which:

FIGS. 1 to 3 are cross-sectional views of three forms of coil,

FIG. 4 is a perspective view of a winding for a form of coil,

FIG. 5 is a development showing a pattern of end winding of a coil, and

FIG. 6 is a section on line B—B of FIG. 5.

DESCRIPTION OF PREFERRED EMBODIMENTS

FIG. 1 is included for illustrating the principles of the mathematical analysis on which the present invention is based and shows windings of the form adopted by Asner and Iselin as mentioned above for generating a quadrupole field.

In the FIGS. 1 to 3 the general form of the magnetic field is indicated by arrowed lines.

Referring to FIG. 1, there are four coil windings, one in each quadrant. The reference R and associated arrows shows the extent of the bundle of windings 11 in one run of one of the coils. At the ends, the windings 11 cross over into the other run, the extent of which is indicated by the reference S. The perimeter or boundary of the bundle of windings in the run R, as seen in cross-section, is defined by an outer circle of radius r_b, an inner circle of radius r_a, and radial lines at azimuthal locations defined by angle θ equal to 60° and 90°. The run S has its boundaries defined by the same circles, but the azimuthal limits are at $\theta = 0°$ and $\theta = 30°$. The other three coils are symmetrically arranged and the azimuthal extent of the coils is defined by the angle indicated as 2ϕ, where, in this case, $\phi = 30°$. The four poles of the magnetic quadrupole field generated by passing electrical currents through the windings 11 in the senses indicated by the dots and crosses in the windings are located in the four pole gaps encompassed by the windings in each quadrant. Measured from the reference axis BOA, these pole gaps extend from 30° to 60° in each quadrant.

It will be seen that the windings of the coil in the first quadrant in FIG. 1 lie in a bundle between two parallel planes spaced apart. The planes extend perpendicularly to the plane of the paper and contain the two lines respectively marked 12 and 13. The FIG. 1 shows in cross-section the side runs R and S which extend perpendicularly to the plane of the paper a distance which is substantially greater than the length of the end runs where the windings cross over from side run S to side run R. In a typical example for a magnet for an accelerator the ratio of the length of the side runs to the length of the end runs may be between 100:1 and 10:1.

For the mathematical analysis on which the present invention is based, at any radius r_1 the current density j in a shell of thickness dr_1, is Fourier analyzed into its angular components $j_n \cos n\theta$, from which the vector potentials A_n at a point (r, θ) may be calculated for points inside and outside the current shell using the formulas:

$$A_n = \frac{\mu_o j_n r_1 dr_1}{2n}\left(\frac{r}{r_1}\right)^n \cos n\theta, \text{ for } r<r_1$$

and

$$A_n = \frac{\mu_o j_n r_1 dr_1}{2n}\left(\frac{r_1}{r}\right)^n \cos n\theta, \text{ for } r>r_1$$

The origin of θ is chosen to exclude terms $\sin n\theta$, and in the case of two-fold symmetry as is required for a quadrupole, n will take only the values 2, 6, 10, 14, . . . or $(4p + 2)$ where p is an integer. If the Fourier coefficients j_n are independent of radius as in the case of sectored coils as shown in FIG. 1, then simple integration of these radial expressions gives by superposition the combined effect for a thick coil of inner and outer radii r_a and r_b. At points in the winding, A is given by the following expressions, whence by the usual curl relations the components of field within the coil are obtained. These are necessary in considering the mechanical forces on the conductors and in the case of superconductors also in checking that the critical field is not exceeded for the current density in question.

Hence for

$$r_a \le r \le r_b,\ A = \sum_{n\neq 2} A_n + A_2$$

where

$$A_n = \frac{\mu_o j_n}{2n}\left[r^2\left(\frac{2n}{n^2-4}\right) - \frac{1}{n+2} r_a^2 \left(\frac{r_a}{r}\right)^n - \frac{1}{n-2} r_b \left(\frac{r}{r_b}\right)^n\right] \cos n\theta \quad (1)$$

and

$$A_2 = \frac{\mu_o j_2}{4}\left[\frac{r^4 - r_a^4}{4r^2} + r^2 \ln r_b/r\right] \cos 2\theta \quad (2)$$

which is a special case. Each of these expressions must of course satisfy the "Poisson like" equation for the region r_a to r_b, that is

$$\frac{1}{r}\frac{\delta}{\delta r}\left(r\frac{\delta A_n}{\delta r}\right) + \frac{1}{r^2}\left(\frac{\delta^2 A_n}{\delta \theta^2}\right) = -\mu_o j_n \cos n\theta$$

where μ_o is the permittivity of free space.

For equal and equispaced coils each of angular length 20, as in FIG. 1, the Fourier coefficients of the current density are obtained as

$$j_n = \frac{8}{\pi}\int_0^{\pi/4} j(\theta) \cos n\theta d\theta$$

which becomes

$= 8/\pi\ 1/n\ j \sin n\theta$, if the current density is constant, $j(\theta) = j$ for $0 < \theta < \phi$

and zero, $j(\theta) = o$ for $\phi < \theta < \pi/4$

Continuity conditions at the inner boundary, $r = r_a$ then give the relative field components in the aperture in this sectored case as

$$C_n=\left|\frac{\frac{\delta A_n}{\delta r}}{\frac{\delta A_2}{\delta r}}\right|_{\substack{r=r\\ \theta=0^\circ}}=\left|\frac{nA_n}{2A_2}\right|_{\substack{r=r\\ \theta=0^\circ}}=\frac{\frac{j_n}{n-2}\left(1-\left(\frac{r_a}{r_b}\right)^{n-2}\right)}{\frac{j_2}{2}\ln\frac{r_b}{r_a}}$$

where the total azimuthal field within the aperture will be given by

$$B_\theta=\frac{\mu_o j_2 r \ln n\frac{r_b}{r_a}}{2}\left[\cos 2\theta+C_6\left(\frac{r}{r_a}\right)^4\cos 6\theta+C_{10}\left(\frac{r}{r_a}\right)^8\cos 10\theta\cdots+C_n\left(\frac{r}{r_a}\right)^{n-2}\cos n\theta\cdots\right]$$

By performing the analysis this way the amplitudes of the octupole and higher harmonics are immediately to hand in convenient form for trajectory calculations.

By way of illustration component values are given in Table I for coils of configuration as shown in FIG. **1** in which the azimuthal half-length of the coils is $\phi = 30°$. The values of C_n are estimated for coils with $r_b = r_a\sqrt{2}$, and will be a little lower for thicker coils.

TABLE I

Fundamental	Harmonics				
$j_2/j=$	$j_6/j_2=$	$j_{10}/j_2=$	$j_{14}/j_2=$	$j_{18}/j_2=$	$j_{22}/j_2=$ etc.
1.10	0	−0.22	0.14	0	−0.091
$C_2=$	$C_6=$	$C_{10}=$	$C_{14}=$	$C_{18}=$	$C_{22}=$ etc.
1.00	0	−0.072	0.032	0	−0.013

In practice the maximum usable aperture will probably be only 1.8 to 1.9 r_a due to an inner wall thickness for the windings, or even less if there has to be an annular space for a temperature transition 300° K to 4.2° K. The maximum harmonic error at 0.8 r_a of 1.2 percent due to C_{10} may be acceptable for a number of applications.

However, an order of magnitude improvement may be obtained by increasing the length of each coil and introducing a simple azimuthal space in the winding of each coil, in effect replacing each single coil with two coils.

This is illustrated in FIG. **2**, where the spaces in the windings are formed by spacers **14**. These spacers **14** extend along the length of the runs of each coil and it can be seen that their general location is defined, in cross-section, by an area within each perimeter respectively of the bundle of windings **11** as seen in cross-section in the two side runs of each coil.

The parameters computed for the coil are an overall azimuthal half-length of $\phi = 33.70°$ and an azimuthal space from 21.75° to 26.30°. The component values, corresponding to those given in Table I, for this coil, again for $r_b = r_a\sqrt{2}$, are given in the following II:

TABLE II

$j_2/j=$	$j_6/j_2=$	$j_{10}/j_2=$	$j_{14}/j_2=$	$j_{18}/j_2=$	$j_{22}/j_2=$
1.04	0.0020	−0.0018	−0.0065	−0.18	0.22
$C_2=$	$C_6=$	$C_{10}=$	$C_{14}=$	$C_{18}=$	$C_{22}=$
1.00	0.0010	−0.0060	−0.0015	−0.030	0.032

Within an aperture of 1.6 r_a the field error from any of the harmonics is now less than 0.1 percent. On the other hand, the manufacturing tolerance on the angular boundaries is correspondingly tighter. For example, a change of less than 0.05° in any of the boundaries is enough to reduce j_6 to 0.

In principle, better fields result from introducing further carefully chosen spaces in the windings, but the windings have to be more accurately defined, with their boundaries accurate to 0.01°.

Apart from the different radial factors affecting the effectiveness within the useful aperture of the different current harmonics and hence the preferred compromise, the same coil proportions will be best for a uniform dipole field or for a sextupole magnet after scaling the angles by 2 or ⅔ respectively. Whilst the quality of the sextupole field will be improved and is likely to be more than adequate, the quality of the dipole field will be correspondingly poorer due to the lower order attenuation of the harmonic terms.

For generating a dipole field of high uniformity, it is considered to be worthwhile including two spacers in the coil winding for each pole — i.e. effectively using three coils per pole. Such an arrangement is shown in FIG. **3**. One side run **15** of the coil windings for forming pole **16** has an azimuthal extent from $\theta = 0$ to $\theta = 71.84°$. The other side run **17** is symmetrically arranged in the second quadrant. The other coil winding having runs **18** and **19** for forming pole **21** is similarly symmetrically arranged in the third and fourth quadrants.

Each run **15, 17, 18, 19** has an azimuthal spacer **22** from 33.33° to 37.12° as measured from the reference axis COD. Each run **15, 17, 18, 19** also has an azimuthal spacer **23** from 53.14° to 63.38° measured from the reference axis COD.

The following table III shows the harmonic components for the coil arranGement of FIG. **3**:

TABLE III

$\frac{j_1}{j}=1.022$	$\frac{j_3}{j_1}=0$	$\frac{j_5}{j_1}=+0.00013$	$\frac{j_7}{j_1}=-0.00016$	$\frac{j_9}{j_1}=-0.000082$
$C_1=1.00$	$C_3\simeq0$	$C_5\simeq0$	$C_7\simeq0$	$C_9\simeq0$
$\frac{j_{11}}{j_1}=-0.00043$	$\frac{j_{13}}{j_1}=-0.18$	$\frac{j_{15}}{j_1}=0.23$	$\frac{j_{17}}{j_1}=0.082$	etc.
$C_{11}=-0.0001$	$C_{13}=-0.016$	$C_{15}=0.017$	$C_{17}=0.0054$	

The values of C are based on a coil where $r_b = 2r_a$. All the lower order components, C_3 to C_{11}, are approximately zero and in principle may be made exactly zero by defining the five angular boundaries to a greater precision. The most significant errors are due to the components C_{13} and C_{15} where the impure dipole field as characterized by the azimuthal component is given by:

$$B_\theta=\frac{\mu_0}{2}j(r_b-r_a)\left[\cos\theta+C_3\left(\frac{r}{r_a}\right)^2\cos 3\theta+C_5\left(\frac{r}{r_a}\right)^4\cos 5\theta\ldots+C_n\left(\frac{r}{r_a}\right)^{n-1}\cos n\theta\ldots\right]$$

where

$$C_n=\frac{j_n}{j_1}\frac{\left(1-\left(\frac{r_a}{r_b}\right)^{2-n}\right)}{(n-2)\left(\frac{r_a}{r_b}-1\right)}$$

At 70 percent of full aperture, the contribution from C_{13} will be 0.025 percent and that from C_{15} will be 0.013 percent. The components C_{13} and C_{15} could also be made zero by the introduction of a further azimuthal space in the windings, but in practice it is believed that the field quality would be limited bY failure in construction to realize the strict symmetry implicit in the analysis. Further, if a superconductor is used, diamagnetic effects in the windings will spoil the field quality if a wire conductor is not used.

FIG. 4 is a perspective view of one layer of windings for one half of a coil for a dipole field. Side runs corresponding to 15 and 17 of FIG. 3 can be seen, as indicated by references 15*a* and 17*a* in FIG. 4. An azimuthal space at 23*a* can also be seen. The figure is principally included, however, to illustrate the general form of the end windings 31 and 32.

For magnets for high energy accelerators it is important, in making the windings, to approach, as closely as possible, to constancy in the integral:

$$\int_{-\infty}^{+\infty} B_y dz$$

the integral being taken along the beam paths through the magnet and parallel to the z axis.

With the arrangement as shown in FIG. 4, the end windings 31, 32 make no contribution to the above integral. This arrangement is therefore a satisfactory solution to the end winding problem where space permits the radial extension at the ends.

However, mathematical analysis has shown that a satisfactory approach to the above condition may be achieved, without the large radial extension, by following an end winding pattern illustrated in FIGS. 5 and 6.

The cross-sectional view of FIG. 6 shows windings (the sectioned areas) similar to FIG. 3 for a dipole field, but with only one azimuthal space in each quadrant. FIG. 5 is a development for showing in plan a typical improved end winding pattern, the figure showing only the windings in the quadrant marked A of FIG. 6. For the development, the windings are uncurled so that their curvature from $\theta = 0$ to $\theta = 90$ apparent in FIG. 6 is shown planar in FIG. 5.

As indicated on the drawing, FIG. 5 shows part of the axial winding, B—B marking the section line of FIG. 6, and one quadrant of end winding. The plan shows a scale of θ from 0° to 90° for a quadrant of a dipole magnet (the scale would be θ from 0° to 45° for a quadrupole magnet, etc.) and the other axis corresponds with the axial distance z. The conductors 33, 34, 35, 36, 37, 38 are arranged parallel to each other and having an inclination ϕ' to the z axis.

To satisfy the requirement, mentioned above, that the integral

$$\int_{-\infty}^{+\infty} B_y dz$$

is constant, the axial component of the nett current element in the end winding configuration has to show a similar Fourier analysis with angle θ as for the side windings. To satisfy this condition for an ideal pure $\cos\theta$ system, a solution is:

$$A\,\theta \cot \phi' + L(\theta) = B \cos \theta$$

where A and B are constants and $L(\theta)$ is the axial length as shown (see FIG. 5) for conductor 38 only.

Suitable approximate values of ϕ' are indicated in FIG. 5 for the various ranges of θ. However, ϕ' does not need to be constant, or even the same at each θ for all of the inclined conductors. If ϕ' is not the same at each θ for all the conductors, the pattern will necessarily be more complex.

The concentric type of construction described with reference to the accompanying drawings lends itself to the making of a compact combined function magnet, for example where an inner section comprises a quadrupole winding with two or possibly three coils per pole and an outer section comprises a dipole winding having three coils per pole. In this way, a beam of charged particles, for example, may be acted upon simultaneously by a dipole and a quadrupole field.

The self-magnetic field in the outer windings may be derived by differentiating the solutions, equation (1) above, and adding the various harmonic components. To this must be added terms of the type $r^{n-1} \cos n\theta$ due to the decaying quadrupole field of the inner section. For the inner windings, the net field will be given in a similar way from equation (2) above plus the uniform field from the dipole winding. A separate treatment has to be applied to the end sections of the coil where field increases can also occur.

The choice of the dipole windings section as the outer section is helpful in improving the quality of the dipole field in the useful aperture and at the same time minimizing the dimensions of the quadrupole, so saving in winding material, especially as the field gradient $B_\theta + r$ varies slowly as $1n\ (r_b/r_a)$. With the dipole winding, the magnetic field depends directly upon the thickness of the winding $(r_b - r_a)$ and consequently there is less incentive to minimize the mean radius. A further advantage of the concentric arrangement is that it is easier to ensure that the two magnets are self-aligned to a common axis.

The technique described above for improving the uniformity or purity of the magnetic field is also advantageous where superconducting windings are employed because one avoids unnecessary magnetic field rise in the superconducting coils themselves.

The invention is not restricted to the details of the foregoing examples. For instance, the side runs of the coil windings need not necessarily be confined within concentric circular boundaries as seen in cross-section. That is, the side runs of the windings need not conform to the surface of a cylinder. The arrangement may, for example, be such that, as seen in cross-section, the side runs of the windings are contained within rectangular boundaries. In this case, the field uniformity or purity can similarly be enhanced by spacers of predetermined dimensions and locations. However, the mathematical calculation of the parameters of the spacers is considerably more complex than that for the above-described concentric arrangements.

Further, the windings may be formed from wide, thin tapes of superconductor, which may be braided.

I claim:

1. An electrical coil for generating a magnetic field of order N, which coil is of the form in which the windings lie in a bundle between two parallel planes spaced apart, the windings lying in two side runs and two end runs where the windings cross from one side run to the other, the general direction of the lengths of the windings in the two side runs being parallel to the said two planes, the side runs being substantially longer

than the end runs so that the magnetic field is principally defined by the side runs, the windings being arranged so that there is at least one region, but not more than 4N regions, each region being defined in cross-section by an area within each perimeter respectively of the bundle of windings as seen in cross-section in the two side runs, from which region the windings are absent, the number of windings per unit cross-sectional area in the side runs being substantially constant throughout the regions where the windings are present, and the location and extent of the region or regions where the windings are absent being selected to enhance the uniformity or purity of the magnetic field generated by the coil.

2. An electrical coil as claimed in claim **1**, wherein the windings in the said two side runs are straight and parallel with one another.

3. An electrical coil for generating a magnetic field as claimed in claim **2** wherein the windings of the coil, as seen in cross-section, are distributed within boundaries defined by concentric circles and generally radially extending lines at the azimuthal limits of the coil windings, and wherein the said area defining the region from which the windings are absent is an area within the said boundaries as seen in cross-section.

4. An electrical coil as claimed in claim **3**, wherein the said area defining the region comprises a sector of the outer circular boundary truncated by the inner circular boundary.

5. An electrical coil as claimed in claim **3**, wherein the coil windings are divided by azimuthal spacers inserted in the windings, the spacers occupying the said regions as defined above.

6. An electrical coil as claimed in claim **5**, wherein, for generating a uniform dipole magnetic field, the arrangement comprises coil windings which, as seen in cross-section, are within boundaries defined by concentric circles, the azimuthal limits of the coil windings being at 67.40° in each quadrant measured from a reference axis, and spacers in each quadrant, the azimuthal extent of the spacers being from 43.50° to 52.60° measured from the reference axis, the coil windings being otherwise uniformly distributed within the boundaries.

7. An electrical coil as claimed in claim **5**, wherein, for generating a quadrupole field of high purity, the azimuthal limits of the coil windings, of which there are two in each quadrant defining four magnetic pole regions, are at 33.70° in each quadrant measured from mutually perpendicular reference axes, and the azimuthal extents of the spacers, of which there are two in each quadrant, are from 21.75° to 26.30° measured from the reference axes.

8. An electrical coil as claimed in claim **6**, wherein, for generating a magnetic field of order N, the azimuthal limits of the coil windings, of which limits there will be $N_{/2}$ in each quadrant defining the N magnetic pole regions, and the azimuthal extents of the spacers, of which there are $N_{/2}$ in each quadrant, are derived by dividing by $N_{/2}$ the angles mentioned in claim **6** for the dipole field coil.

9. An electrical coil as claimed in claim **5**, wherein, for generating a uniform dipole magnetic field, the arrangement comprises coil windings which, as seen in cross-section, are within boundaries defined by concentric circles, the azimuthal limits of the coil windings being at 71.84° in each quadrant measured from a reference axis, and two spacers in each quadrant, the azimuthal extent of the spacers being from 33.33° to 37.12° and from 53.14° to 63.38°.

10. An electrical coil for generating a magnetic field, which coil is of the form in which the windings lie in a bundle between two parallel planes spaced apart, the windings lying in two side runs and two end runs where the windings cross from one side run to the other, the general direction of the lengths of the windings in the two side runs being parallel to the said two planes, the side runs being substantially longer than the end runs so that the magnetic field is principally defined by the side runs, the windings in the end runs being arranged so that for any length of winding of azimuthal location θ and inclination ϕ' to a z axis defined parallel to the said general direction of the lengths of the windings in the side runs, the following condition is satisfied:

$$A\,\theta \cot \phi' + L(\theta) = B \cos \theta$$

where A and B are constants and $L(\theta)$ is the length of the winding continuing in the direction of the side run beyond an end plane, the said end plane being defined as containing the point of diversion from side run to end run of the winding for which $L(\theta)$ is zero.

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