Aug. 27, 1963 G. RAISBECK 3,102,161

PREVENTION OF CORONA DISCHARGE

Filed April 11, 1960 2 Sheets-Sheet 1

FIG. 1

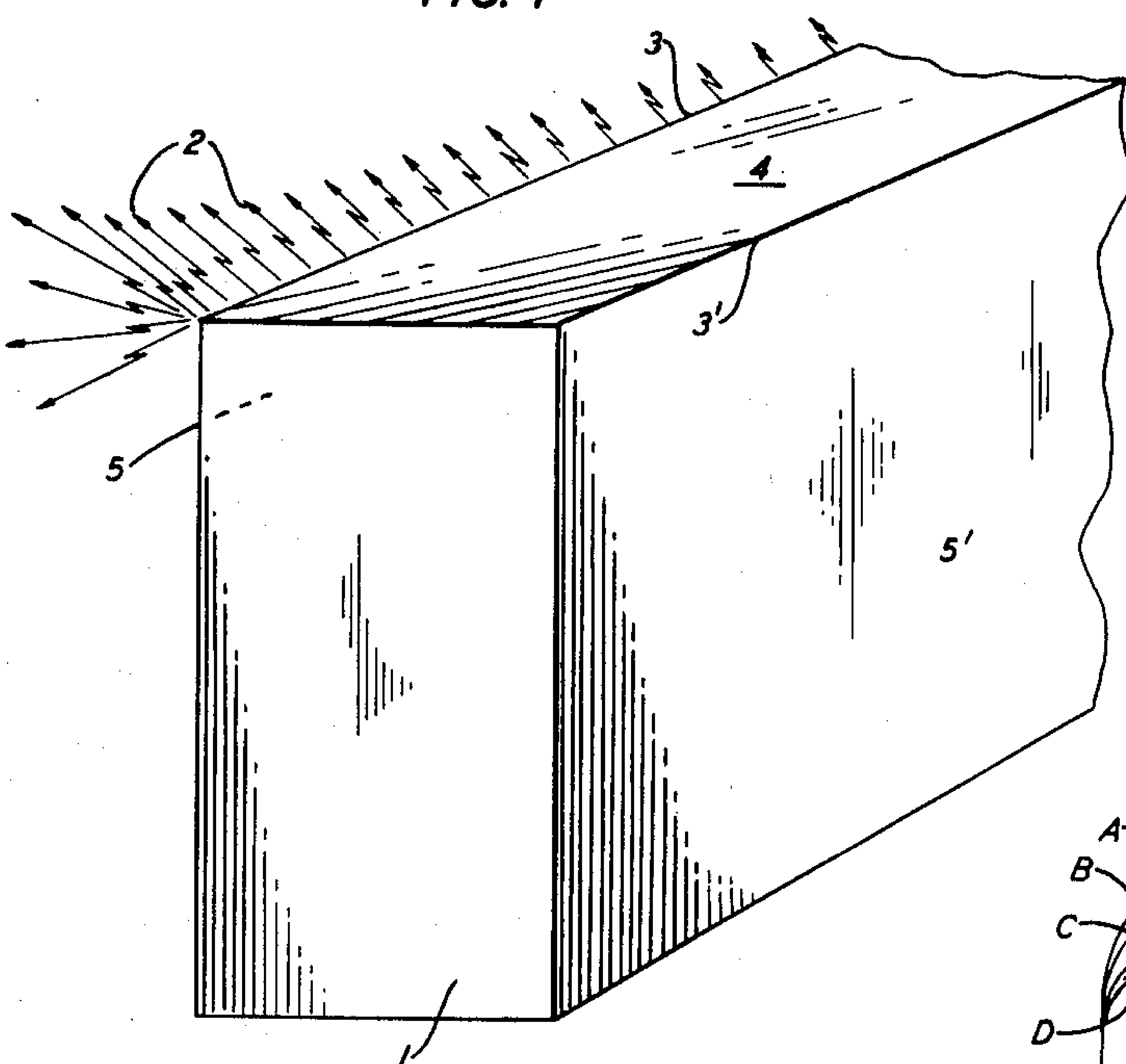


FIG. 2

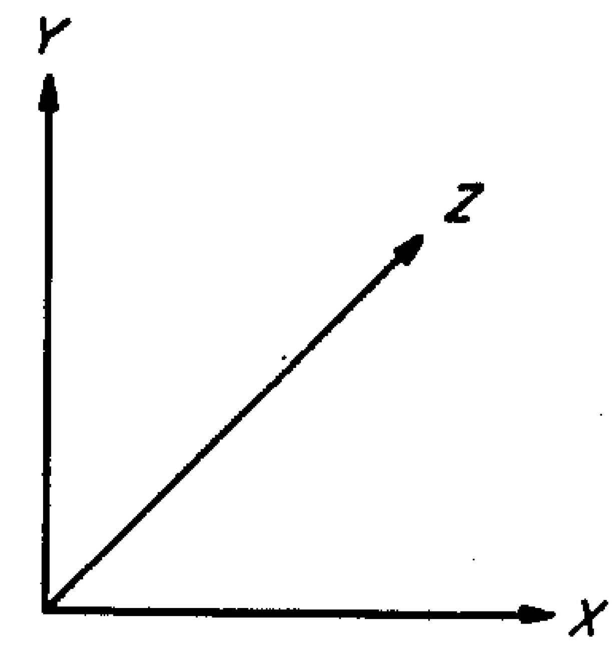


FIG. 5

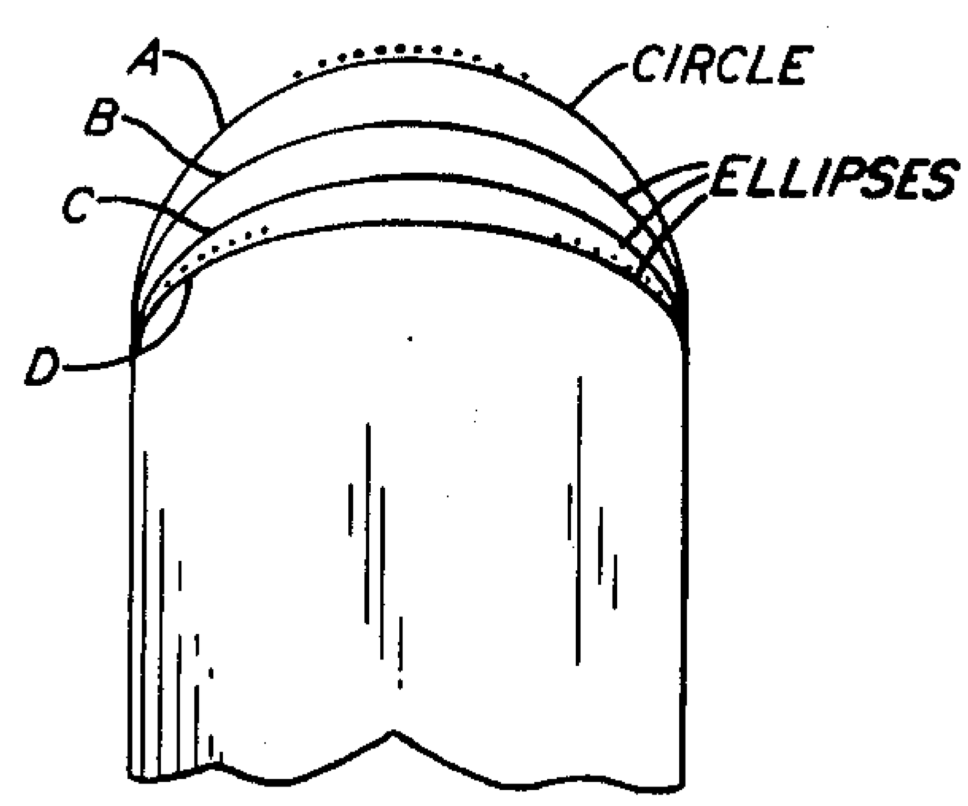


FIG. 3

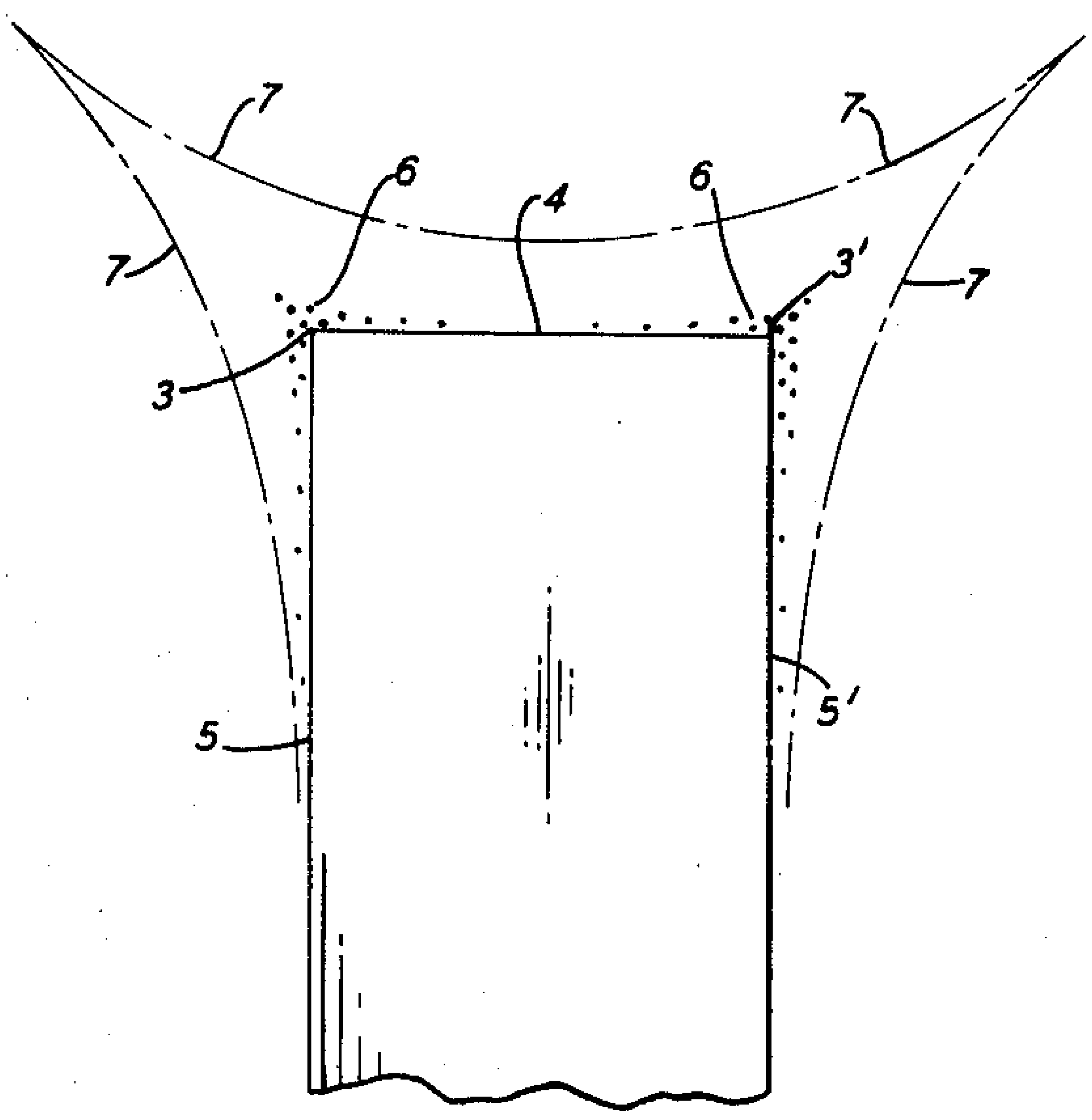
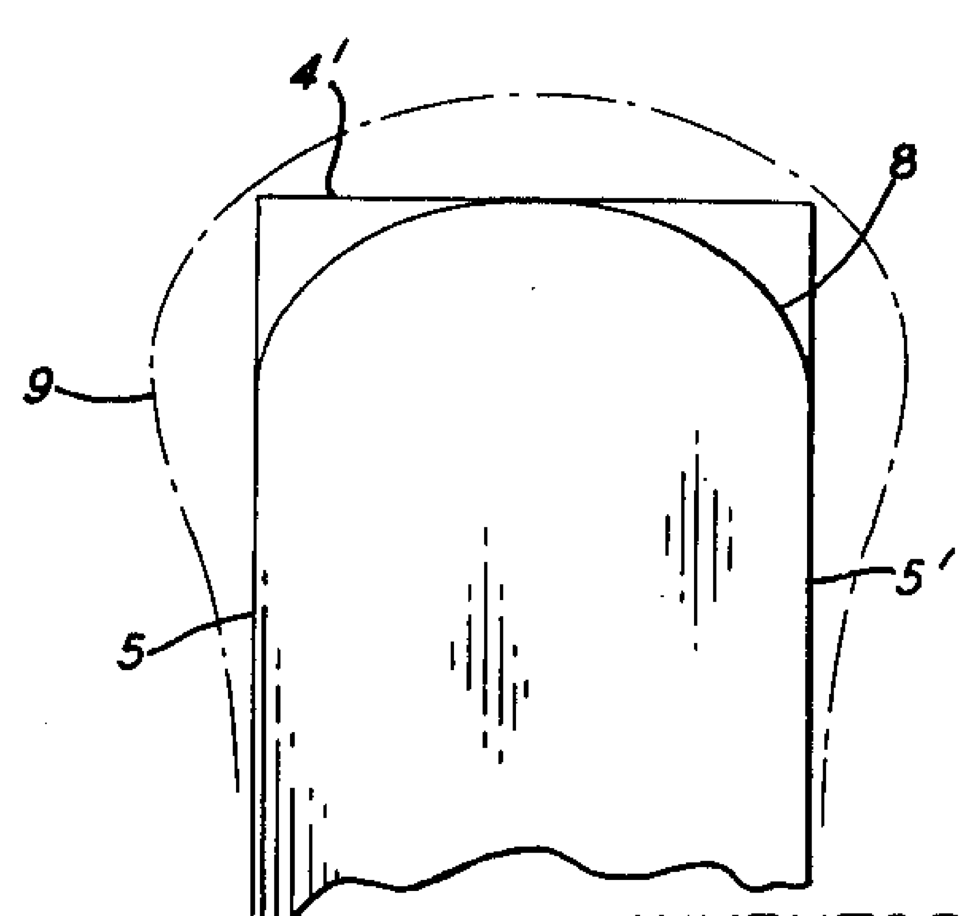


FIG. 4



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Filed April 11, 1960 2 Sheets-Sheet 2

FIG. 6

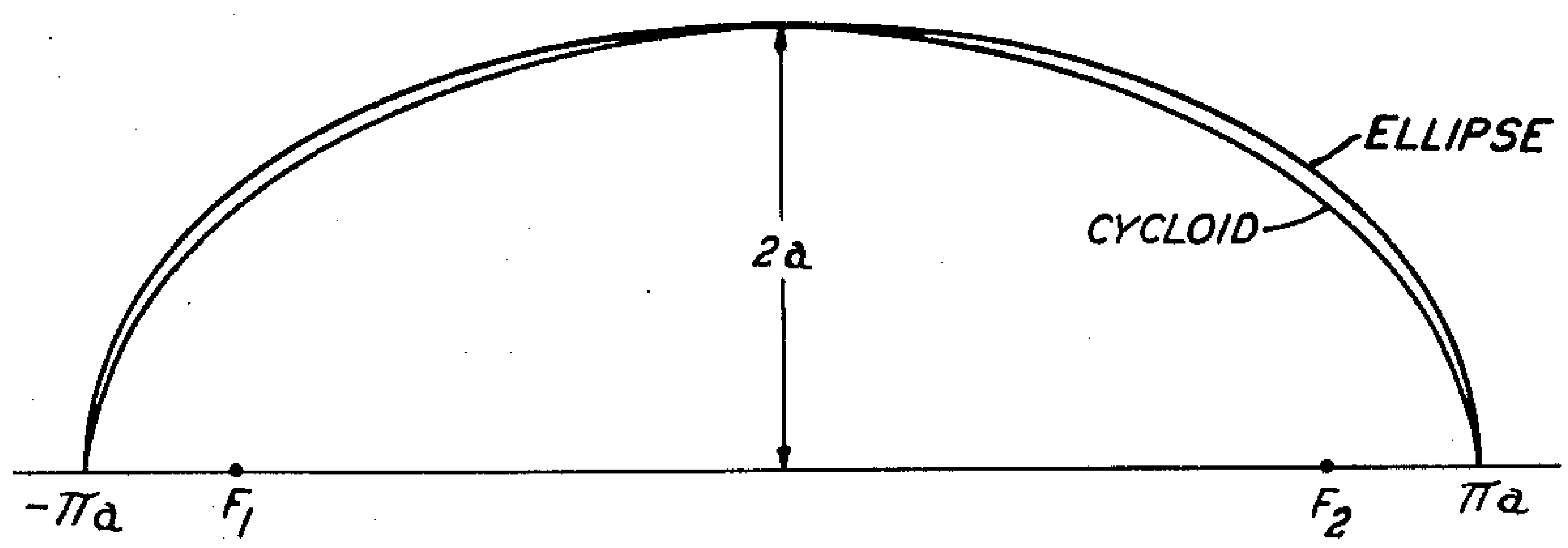


FIG. 7

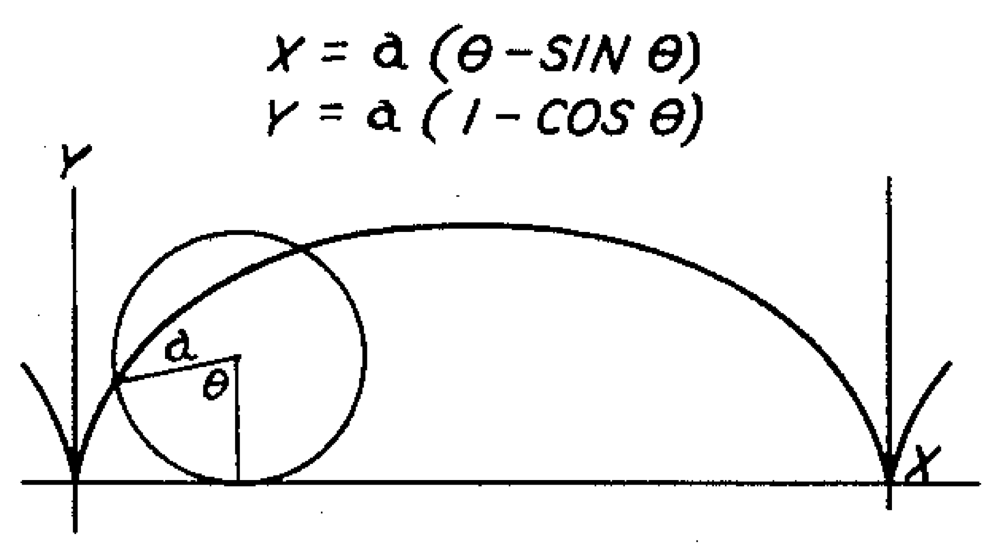


FIG. 8

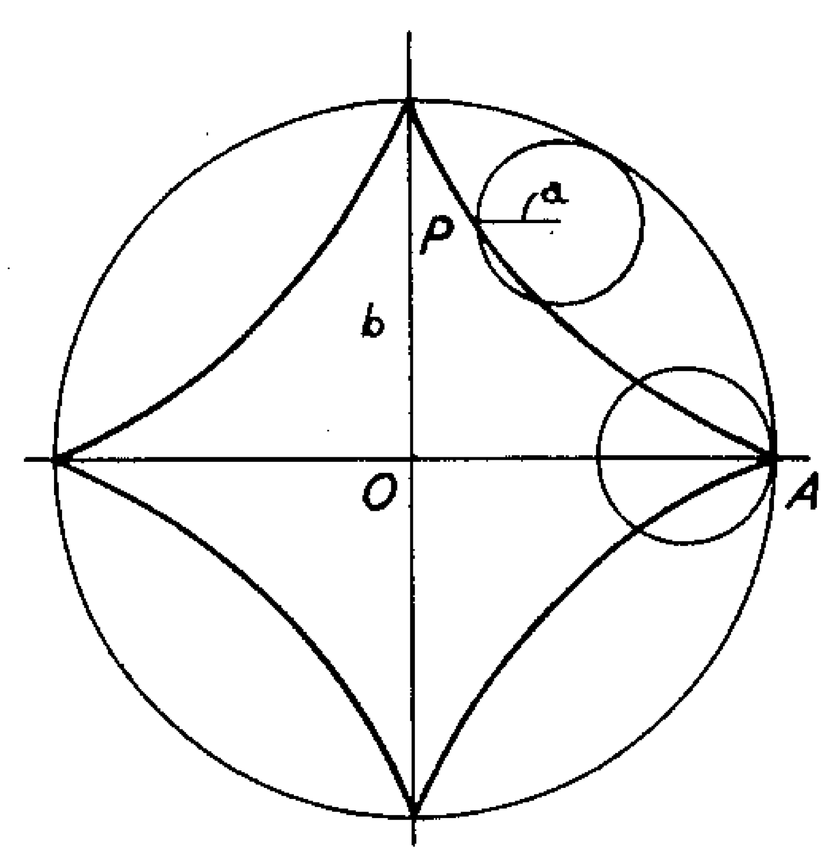


FIG. 10

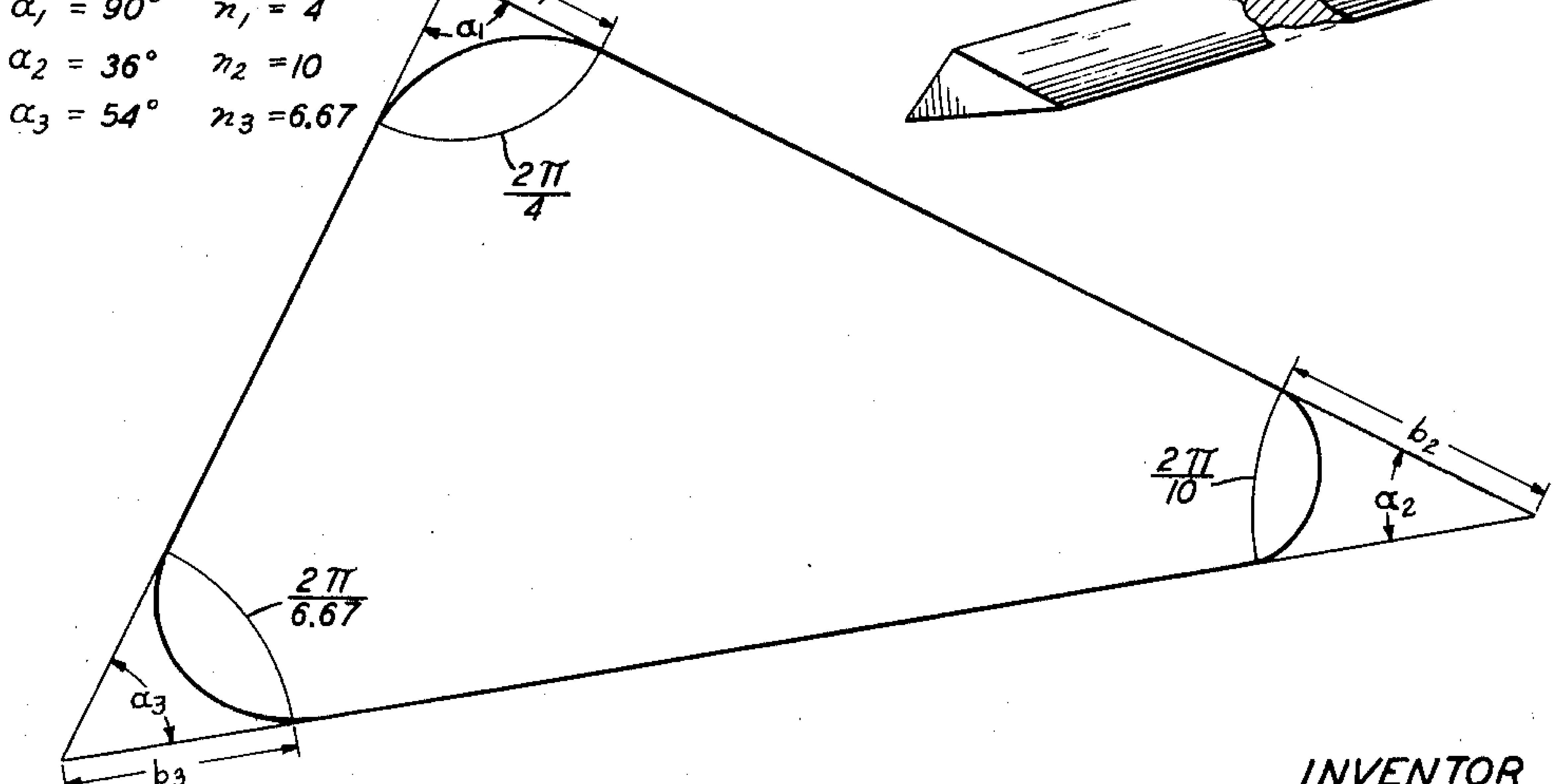
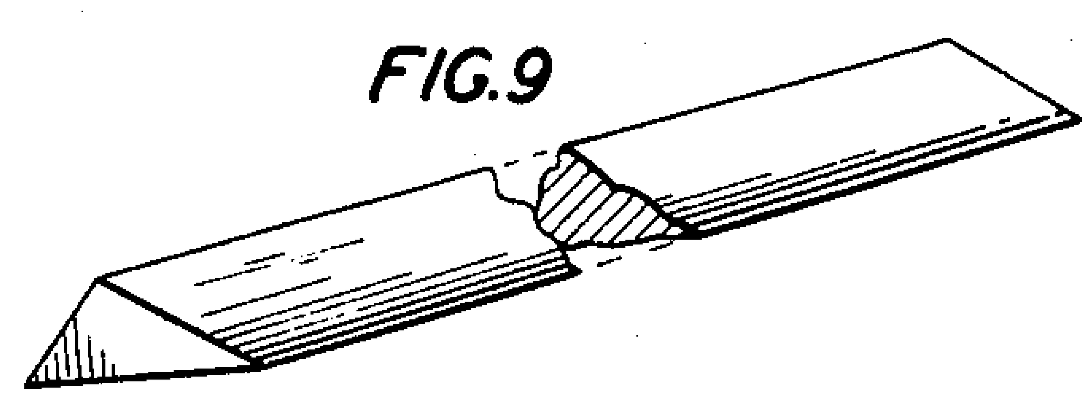


FIG. 9



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1

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PREVENTION OF CORONA DISCHARGE

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2 Claims. (Cl. 174—127)

This invention deals with electric conductors and especially with the prevention of corona discharge from such conductors.

When an electrical conductor having one or more sharp edges or corners is supplied with a sufficiently high electric potential, a discharge of electricity, known as a corona discharge, takes place from the edge to the surrounding atmosphere. The phenomenon stems from the fact that, for any potential of the conductor as a whole, the intensity of the electric field is greatest at the sharpest edge, and may reach a magnitude at which the ambient air breaks down while the intensity at other parts of the conductor is still far below this magnitude. The corona discharge thus limits the potential to which the conductor can be raised. The difficulty is espectially serious in the case of an elongated conductor for carrying power at high voltage; i.e., a bus-bar or strip of rectangular cross section, of which the major sides are substantially flat, and usually parallel. While it is known that the corona discharge can be reduced by rounding the sharp corners and edges of the conductor, there has not been available to the designer any general principle to guide him in his choice of a contour.

The invention stems from the discoveries that, given two plane conducting surfaces, it is possible to find a curved conducting surface to interconnect them over which the electric field intensity is uniform, and hence no greater at any one point than it is at any other point; and that the contour of the curved surface can be generated, in every case, by the movement of a point fixed to the circumference of a generating circle of appropriate radius as it rolls along an auxiliary track. In the most important, though special, case in which the two plane conductors to be interconnected are parallel and spaced apart a distance D, the track is a straight line, and the radius a of the generating circle is given by

$$2\pi a = D$$

When the generating point of the rolling circle touches the track at one end of the travel, and touches the track again for the first time at the other end, it traces out a single full arch of a cycloid curve between adjacent cusps, making exactly one full revolution in doing so. The contour of the interconnecting surface should conform with this cycloid curve.

In the general case in which the two plane surfaces are not parallel, the track along which the generating circle rolls becomes an arc of a larger circle of which the center lies at the point at which the plane surfaces would meet if extended, so that it intersects each of the plane surfaces at right angles. The generating circle rolls on this circular track, from coincidence with one of the intersection points to coincidence with the other, rolling on the inside when the angle between the planes is less than 180 degrees and on the outside when the angle between the planes is more than 180 degrees. In each case, its radius is such that, having done so, a point of its circumference that initially coincided with one of the plane surfaces now coincides with the other plane surface. The path traced out by this generating point as the smaller circle rolls on the larger one is a hypocycloid curve or an epicycloid curve, accordingly as the smaller circle rolls on the inside of the larger one or on the outside of it. The contour of the surface interconnecting

2

the two non-parallel plane surfaces should conform to the resulting hypocycloid, or epicycloid, curve.

Significant confirmation of the discovery is found in the fact that, as the angle between the plane surfaces decreases without limit, the circular track in the general case reduces to the straight track in the special case and the hypocycloidal connecting surface of the former becomes the cycloidal connecting surface of the latter.

The invention will be fully apprehended from the following description of illustrative embodiments thereof taken in connection with the appended drawings in which:

FIG. 1 is a perspective view of a portion of an elongated conductor having a rectangular cross section;

FIG. 2 is a diagram of a coordinate system of axes referred to in the specification;

FIG. 3 is an end view of the conductor of FIG. 1 showing distribution of its surface charges and of the electrostatic field surrounding it;

FIG. 4 is an end view of a conductor otherwise like that of FIG. 3 and in which the straight short side and the edges bounding it have been replaced by a cycloidal contour;

FIG. 5 is an end view of a conductor of rectangular cross section in which the narrower side has been rounded to elliptical contours of various eccentricities;

FIG. 6 is a diagram showing a single arch of a cycloid curve and a semiellipse having the same base line and the same height;

FIG. 7 is a diagram illustrating the generation of a cycloid;

FIG. 8 is a diagram illustrating the generation of a hypocycloid curve;

FIG. 9 is a perspective view of an elongated conductor having a triangular cross section; and

FIG. 10 is a diagram showing the cross section of a conductor bounded by three straight sides interconnected by three hypocycloidal contours.

Referring now to the drawings, FIG. 1 is a perspective view of a bus-bar: a heavy strip **1** of a highly conductive metal such as copper, intended to carry electric currents from one point to another, at high voltage. As a rule, the dimensions of such a strip stand in high numerical ratios to each other: the length (Z-axis, FIG. 2) is much greater than the width (Y-axis) which, in turn, is much greater than the thickness (X-axis). Unless the sharp corners of such a strip be somewhat rounded, corona discharge **2** may take place from any or all of them, when the strip is raised to a high potential. This discharge may be excessive, and may be the cause of a serious loss of power.

FIG. 3 shows the upper part of the cross section, in the X—Y plane, of the strip **1** of FIG. 1 when the edges **3**, **3'** at which its plane sides **4**, **5**, **5'** intersect are infinitely sharp. As is well known, when such a strip is raised to a high potential, positive or negative, the mutual repulsions among the electric charges on its surface tend to drive them onto the corners. This is indicated in FIG. 3 by the concentration of charges **6** which increases from the approximate center of each plane surface toward both of the bounding edges. Inasmuch as the electric field intensity depends on the charge density, it, too, increases from the center of each plane face toward both of the bounding edges. With infinitely sharp edges, the field intensity, represented by the broken line **7**, reaches infinite magnitude at each of those edges. Such a very large, localized, field intensity makes for prohibitive corona discharge.

FIG. 4 shows the sharp edges of FIG. 3 to have been rounded, by removal of the material of the strip **1**, included between the rounded outline and the rectangular one, to provide the narrowest bounding face **4** of the strip **1** with a contour **8** defined by a cycloid curve. The electric field intensity **9** increases, as before, from a low magnitude at the centers of the wider faces to a moderate magnitude at their points of tangency with the cycloidal contour **8**. Now, however, the field intensity **9** is uniform at this moderate magnitude over the entire cycloidal contour, and the high intensity conditions that prevailed at the sharp edges of FIG. 3 have been eliminated.

While any rounding of the sharp edges of FIG. 3 is advantageous, the cycloidal contour of FIG. 4 is optimal. (This may be established through complex domain analysis, including the transformation, by the technique of conformal mapping, of one complex plane onto another. Indeed, it was through analysis of this sort that the contours of the invention were discovered. The intricacies of the analysis render it unsuitable for inclusion in this specification.) The optimal character of the cycloidal contour may be inferred from the following elementary considerations.

From the fact that the electric field intensity in the neighborhood of a conductor may be expressed as the gradient of a potential, and from the large number of widely different electrostatic problems for which analytic solutions have been obtained, one may surmise that the optimal contour is an analytic curve, expressible in terms of the dimensions of the conductor to be rounded. To avoid the introduction of spurious edges, its ends must coincide with and be tangent to the plane faces of the strip. Because the strip, before rounding, is symmetrical, the contour should evidently be symmetrical, so that it rounds the two edges that bound the narrow face of the strip in the same way, and to the same extent. Thus it should be parallel at its midpoint to the original (unrounded) narrow face, falling monotonically from this midpoint to its point of tangency with the wider faces.

A semicircle, curve A of FIG. 5, comes at once to mind as satisfying all of these requirements. But with a semicircular contour, mutual repulsions among the electric charges cause them to reach concentrations at the midpoints of the contours that exceed the concentrations elsewhere, so that the electric field intensity cannot be uniform over the contour having, rather, a maximum magnitude at the midpoint. This is indicated, in FIG. 5, by the central concentration of dots on curve A. Consequently, corona discharge commences at this midpoint.

To prevent it, the semicircular contour must be somehow flattened. The natural way to "flatten" a semicircle is to treat it as a semiellipse of zero eccentricity; i.e., an ellipse whose major and minor axes are alike in magnitude, and then gradually to alter the eccentricity of the ellipse until the requisite amount of flattening has been obtained. Semiellipses of successively greater eccentricities are shown in curves B, C, D of FIG. 5. When the semiellipse of curve D has been arrived at for which the ratio of the length of the major axis to the length of the minor axis is equal to $\pi/2$, it turns out that the semicircle has been too much flattened. The contour now contains two points, one on each shoulder of the semiellipse, at which the field intensity has a magnitude that exceeds its magnitude at all other points. This is indicated by concentrations of dots on both shoulders of curve D.

FIG. 6 shows, to the same scale for comparison, a single arch of a cycloid and the semiellipse of curve D of FIG. 5 for which the half major axis has the magnitude πa while the half minor axis has the magnitude $2a$. The ratio of its major axis to its minor axis is thus $\pi/2$; i.e., equal to the ratio of the length of the base of the cycloid to its height above the base at its midpoint. It will be observed that except at its two end points and at its midpoint, the cycloid lies everywhere inside of the semiellipse, its shoulders drooping somewhat more than those of the semiellipse. It can be shown by detailed calculation that the greater droop of the shoulders of the cyloid is just such as to offset the charge concentrations on the shoulders of the semiellipse so that, with the cycloidal contour, the charge density on the surface of the conductor, and hence the field intensity outside of it, are uniform over the entire contour.

It is plain that the concentrations of charge on the shoulders of the ellipse can be reduced, and brought to equality with the charge concentration at its midpoint, by a small reduction in its flatness; i.e., by conforming the contour of the conductor to a semiellipse of which the ratio of major axis to minor axis is greater than unity but less than $\pi/2$. This however, would leave three regions of equal charge concentrations, and hence three regions of equal field intensities, at the midpoint of the contour and at its two shoulders, with regions of less field intensity between. Hence the field intensity would not be uniform over the entire contour. Moreover, such a contour involves the removal of a greater mass of the material of the conductor than does the cycloidal contour.

As is well known, the cycloid curve is the plane locus of a point which is fixed on the circumference of a circle, as the circle rolls along a straight base line as a track; i.e., the path described by a point on the rim of a wheel. The track, the path, and the manner of its generation, are shown in FIG. 7. The entire curve is thus periodic, with one wavelength or arch for each full revolution of the generating circle. Taking the track as extending along the (horizontal) X-axis from an origin at which the point fixed to the circle of radius a touches the base line, and designating the angle through which the circle has rolled as θ, measured from the downward-pointing radius at the origin, the equations of the cycloid are

$$X=a(\theta-\sin\theta)$$
$$Y=a(1+\cos\theta) \quad (1)$$

These equations are in the form most commonly reproduced in texts and tabulations; e.g., "Mathematics Dictionary," by Glenn James and Robert C. James (Van Nostrand, 1959). When the origin is shifted to a point of the base line midway between two cusps of the curve, and measuring θ from the upward-pointing radius, the equations become

$$X=a(\theta+\sin\theta)$$
$$Y=a(1+\cos\theta) \quad (2)$$

Other forms are readily obtainable by other shifts of the origin, and still others by choosing a nonhorizontal base line; i.e., a vertical base line or a base line at 45 degrees to the horizontal.

To fit the cycloid curve of Equations 1 or 2 to the narrow side of the strip conductor, of thickness D, it is only necessary so to choose the radius a of the rolling circle that the length of the base line between adjacent cusps is equal to the thickness of the strip; i.e.,

$$D=2\pi a \quad (3)$$

It can readily be verified by differentiation that the slope of the cycloid curve is infinite at each of its cusps. Hence the contour not only coincides with the wider sides of the strip conductor but is tangent to them.

At points of the cycloid slightly displaced from its cusps, inwardly toward the midpoint of the base line, the cycloid has equal and opposite finite slopes. This observation leads naturally to the inquiry whether that part of the cycloid arch that is included between these points is the optimal contour for rounding the edge of a conductor whose sides are not parallel; e.g., a conductor whose cross section, before rounding, is triangular. Such a conductor is shown in FIG. 9. Pursuit of this inquiry leads to the further observation that the cycloid is a special case of the prolate trochoid, in which the generating point is fixed to the end of an extension of the radius of the rolling circle, and for which the portions having zero altitude do not coincide with the portions having infinite slope. It turns out that, while such contours are favorable for the purpose, they are not optimal, and that the optimal contour, in such case, is that of the hypocycloid: the plane locus of a point fixed on a circle which rolls on the inside of another, larger, circle. As commonly presented in texts and tabulations such as the Mathematics Dictionary referred to above, and with an interchange of the parameters a and b as given in that text, the equations of the hypocycloid, stated with respect to an origin at the center of the fixed circle, are

$$X=(b-a)\cos\theta+a\cos\left[\frac{b-a}{a}\theta\right]$$

$$Y=(b-a)\sin\theta-a\sin\left[\frac{b-a}{a}\theta\right] \quad (4)$$

where, as before, a is the radius of the rolling circle and b is the radius of the (larger) fixed circle. A hypocycloid of four cusps, and hence of four arches, is shown on page 195 of the Dictionary, and is here reproduced, for ready reference, as FIG. 8. Any one of its four arches is suitable to serve as a rounding contour to interconnect two plane sides of a conductor that, before rounding, intersect at an angle of ninety degrees. For other angles, an arch of a hypocycloid of more, or fewer, cusps should be chosen. If the ratio b/a is not an integer, the number of cusps is no longer defined, but a single arch of the curve is still defined by Formulas 4. In any case, the center of the fixed circle is to be taken at the point at which the straight sides of the cross section of the conductor would meet, if extended. Such a circle is perpendicular to each of the straight sides of the cross section at the points at which it intersects them. Hence the arch of the hypocycloid, being perpendicular to the fixed circle at these points, is coincident with and parallel to the straight cross section sides at these points.

The hypocycloidal contour curve is to be selected in the following fashion. First, the angle to be rounded is measured. Thus, in FIG. 10, the angles are

$$\alpha_1=90 \text{ degrees}$$
$$\alpha_2=36 \text{ degrees}$$
$$\alpha_3=54 \text{ degrees}$$

These angles determine the number of arches of the complete curve, in each case; i.e.,

$$n_1=\frac{360}{\alpha_1}=4$$

$$n_2=\frac{360}{\alpha_2}=10$$

$$n_3=\frac{360}{\alpha_3}=6.67$$

Next, a suitable length is chosen for the radius b of the fixed circle, as a compromise between the advantage of generous rounding of the edge and the disadvantage of removing an unnecessary amount of the material of the conductor. Suppose that this compromise leads to the radii b_1, b_2, b_3, respectively. Now a hypocycloid has two arches when $b=2a$, three arches when $b=3a$ and, in general, n arches when $b=na$. Hence, in the cases of the three angles of the illustrative figure, the radii of the rolling circles should be

$$a_1=\frac{b_1}{4}$$

$$a_2=\frac{b_2}{10}$$

$$a_3=\frac{b_3}{6.67}$$

It remains merely to roll each of the (smaller) generating circles along the arcuate track constituted of the associated larger circle, allowing a point fixed to the smaller circle to trace a curve, and the curve thus traced is the required single arch of the desired hypocycloid.

The cycloid may be regarded as a special case of the hypocycloid in which the radius b of the fixed circle increases without limit so that an arc of this circle becomes, in the limit, the straight base line of the cycloid. Hence the processes of generating the cycloid and the hypocycloid are, in the end, the same: having chosen an appropriate track, straight or arcuate, that intersects the two sides of the cross section to be rounded at ninety degrees, choose a generating circle of radius such that a point fixed to it touches the track at one of these two end points and, after the circle has rolled the full length of the track, touches it again, and for the first time, at the other end.

Once the edges of an extended conductor strip have been rounded in the fashion described above its ends, unless terminated in an already rounded object, should be domed. This is because, for a given potential to which the conductor is raised, the electric field intensity at a corner at which three edges meet is even higher than that at any of the edges. Optimal rounding at such corners, however, is not critical, since the material requiring removal is negligible.

The principles of the invention, described in connection with an elongated strip conductor apply, as well, to an object having the shape of a figure of revolution. A porcelain insulator-support for a high voltage conductor is an example. Especially when wet, such an insulator can carry enough surface current to make for serious power losses through corona discharge from its sharp edges. Thus the term "conductor," as employed in this specification, is employed in the electrostatic sense: it denotes any object that carries electric charges, and along the surface of which such charges can migrate under the influence of their mutual repulsions. The term has no connotation of high volume conductivity or of any particular material of which the conductor may be fabricated. Accordingly, each sharp edge of an insulator support may advantageously be replaced by a cycloidal or a hypocycloidal contour.

The mathematical aspects of electrostatic potential theory, from which stems the optimal character of the contours of the invention, are similar to those of the theory of velocity potential which describes the conditions which arise when a solid and a fluid are in relative movement. Accordingly, it is advantageous to replace a sharp angle in a canal, or the edge in which the nose cone of a projectile meets the projectile body, by a rounded contour of cycloidal or hypocycloidal configuration.

What is claimed is:

1. An extended conductor, for carrying electric power at high voltage, having a cross section composed of two plane parallel longer sides and two similar outwardly convex curved shorter sides, each of said shorter sides conforming to a single arch of a cycloid curve between adjacent cusps, said curve being tangent at said cusps to said plane longer sides, respectively.

2. A conductor of electric charges bounded by surfaces incuding two nonparallel plane surfaces and an outwardly convex rounded contour interconnecting said plane surfaces, said surfaces defining an angle of α degrees between them, the cross section of said contour conforming to a single arch of a hypocycloid curve between adjacent cusps, said hypocycloid being defined by the radius a of a generating circle that rolls on the inside of a circular track of radius b and of which the center lies at the point at which said plane surfaces would intersect, if extended, wherein a, α, and b are in the relation

$$\frac{a}{b}=\frac{\alpha}{360^\circ}$$

said curve being tangent at said cusps to said plane surfaces, respectively.

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