ACOUSTIC DEEP WELL PUMP WITH FREE COMPRESSION COLUMN

Filed Jan. 13, 1958 2 Sheets-Sheet 1

FIG. 5

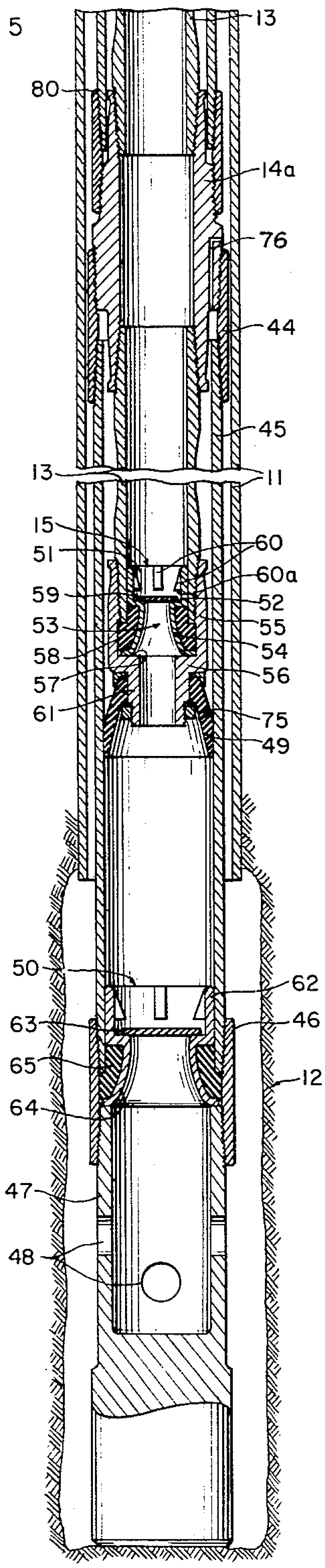


FIG. 1

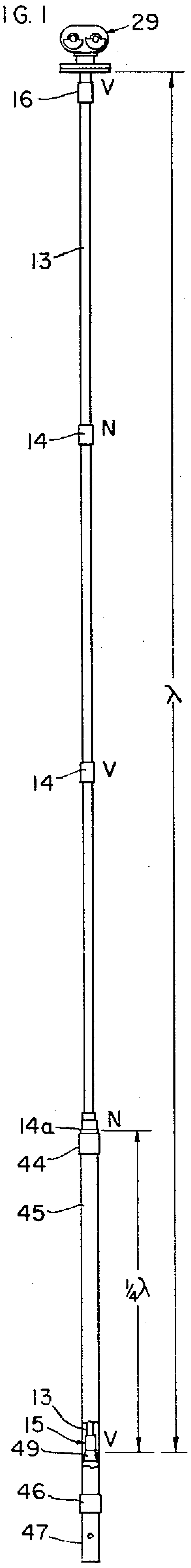


FIG. 3a

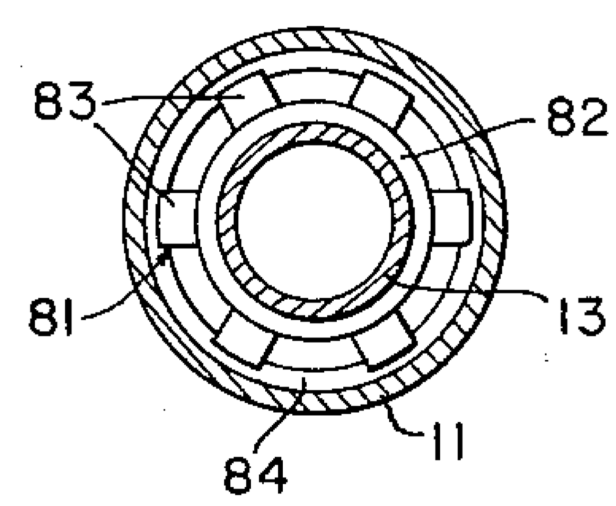
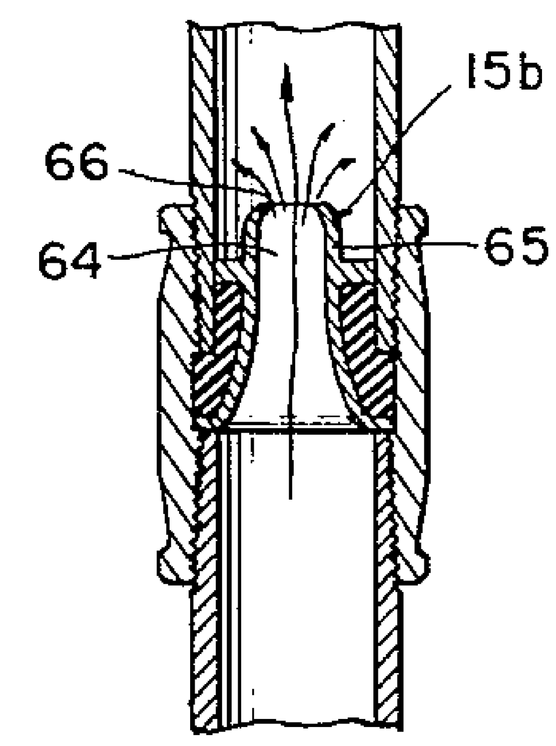


FIG. 6



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ACOUSTIC DEEP WELL PUMP WITH FREE COMPRESSION COLUMN

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FIG. 2

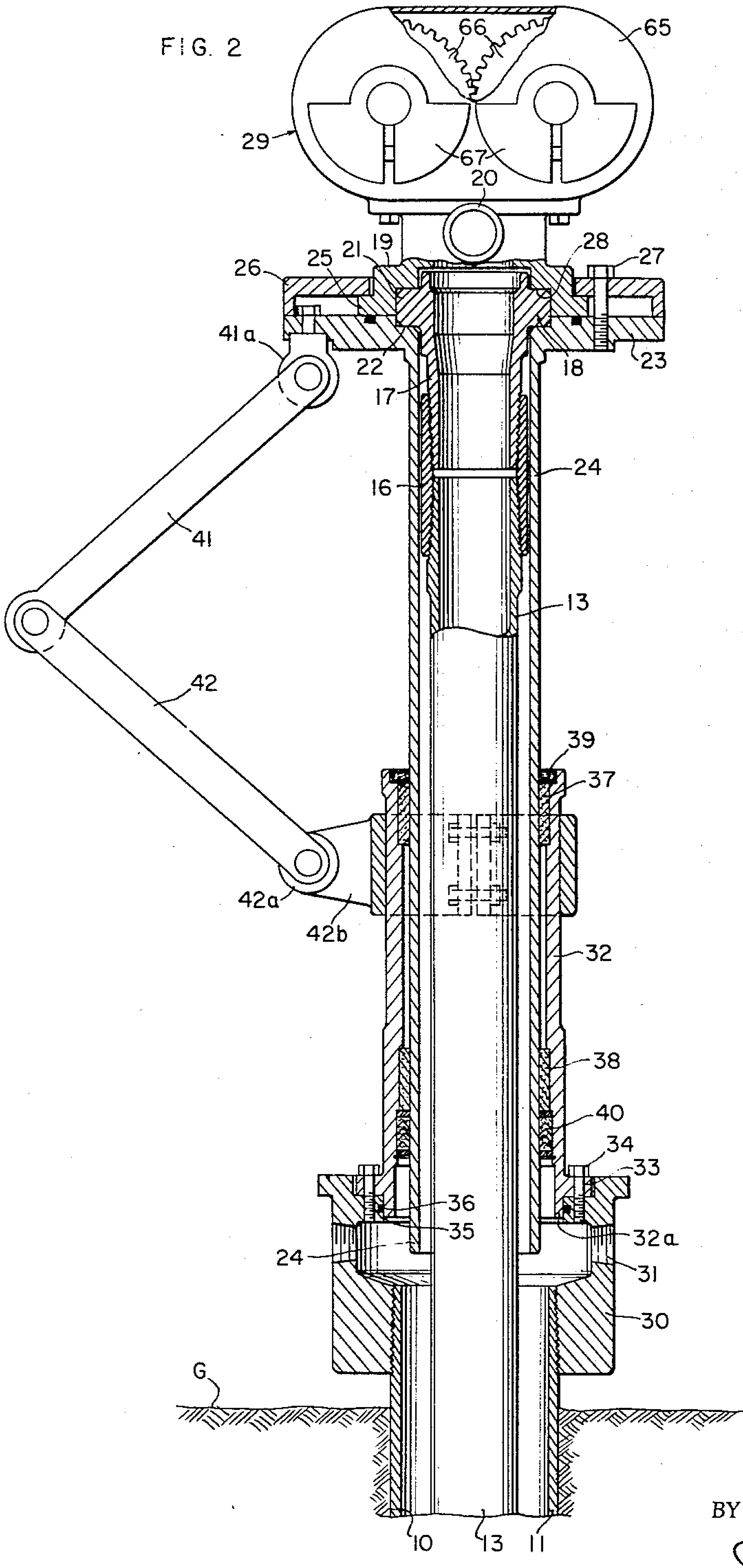


FIG. 3

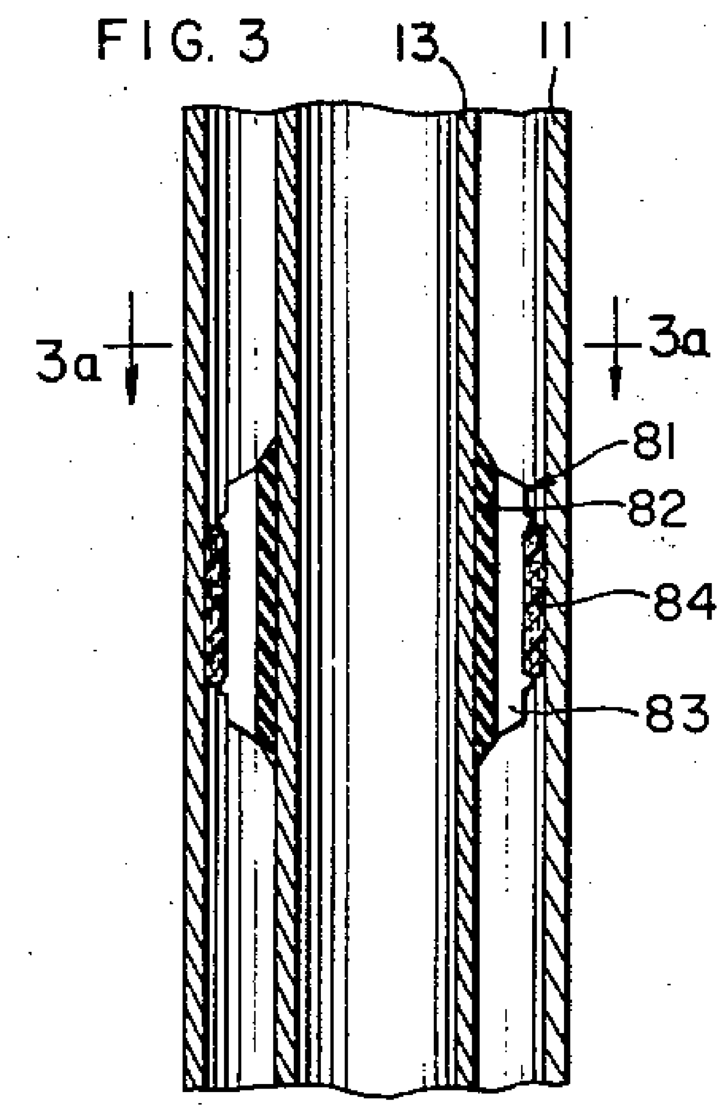
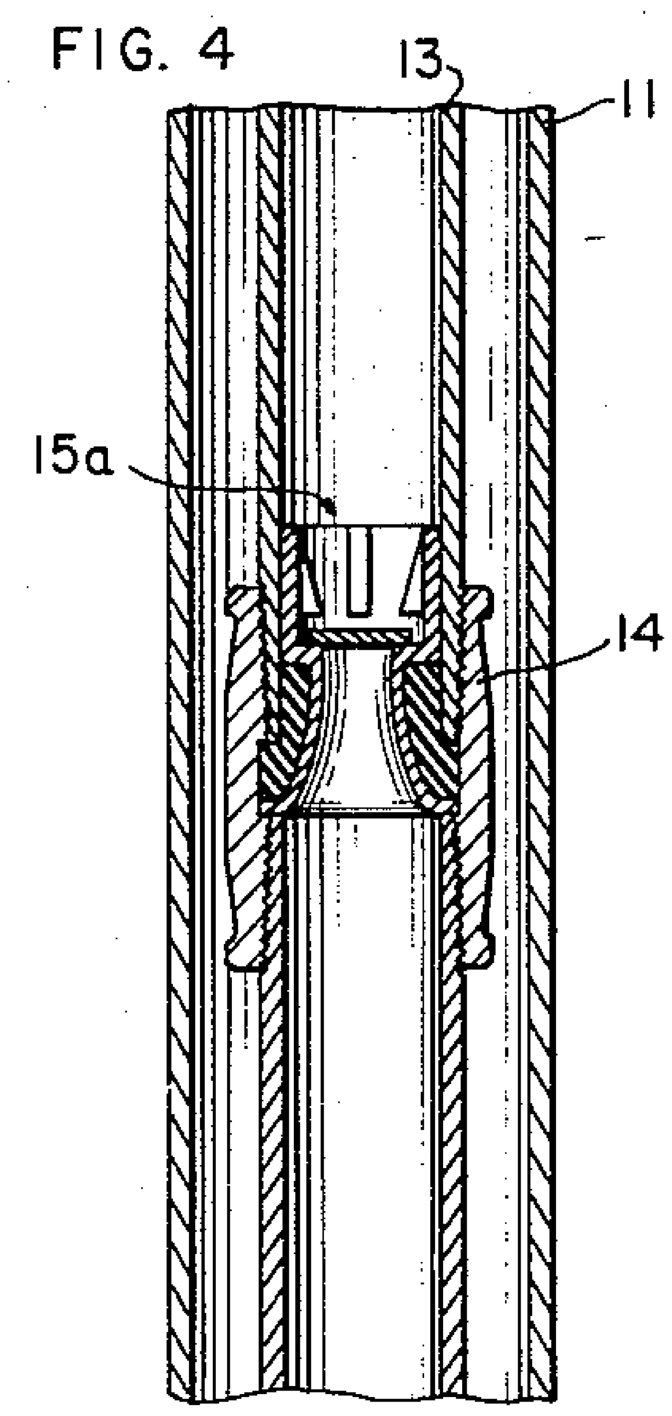


FIG. 4



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ACOUSTIC DEEP WELL PUMP WITH FREE COMPRESSION COLUMN

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5 Claims. (Cl. 103—1)

This invention relates generally to sonic deep well pumps, of the general class disclosed in my prior Patent No. 2,444,912, and deals especially with improvements in such pumps with particular reference to attaiment of greatly enhanced ability to withstand elastic fatigue.

The basic principle of the sonic type well pump is the generation and transmission of longidutinal elastic waves, i.e., powerful sound waves, in a solid elastic column (pump tubing, rod, etc.), with the elastic vibrations of such column actuating a fluid impelling means in the pump tubing. In many forms of the pump, this fluid impelling means includes or is associated with a check valve. In the simplest form, as heretofore known, the sonic pump consists of an elastic column in the form of an elastic (e.g. steel) string of pump tubing, one or more fluid impelling means in the tubing, a wave generator mounted atop the tubing, whereby to deliver alternating force pulses to the tubing and thereby send waves of elastic deformation down the same, and a support means suspending the pump tubing from its upper end.

A characteristic of all prior sonic pumps of the class described is that the elastic deformation waves are transmitted through an elastic column which is suspended primarily in tension from the upper portion of the well. The elastic waves traveling along the column accordingly involve periodic increases and decreases in a tension which is in the column as a consequence of its weight and its upper end support point. That is to say, there exists a tension bias in the column, zero at the lower end, and increasing to a large value at the top end where the column is supported. The wave action in the column therefore consists of periodic increases and decreases in this biasing tension. The column is thus always in tension, and experiences periodic increases and decreases in tension. Such an elastic column, with its couplings, hung in tension, and carrying periodic waves consisting of increases and decreases in tension, is subject to the threat of early fatigue and fatigue failure when very high sonic power is utilized. Careful attention to selection of materials and to the development of designs which avoid stress concentrations are required under such conditions in order to attain reasonable life prior to ultimate fatigue failure.

The primary object of the present invention is to provide a sonic pump of the class mentioned, but which avoids proneness to fatigue failure, and which therefore permits less attention to selection of materials, as well as to avoidance of stress concentrations.

My present invention embraces several concepts, the

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first and foremost of which is to stand the elastic column in compression, with a support point within the lower end region thereof. Accordingly, for the principal part of the length of the column, the elastic waves therein consist of alternating increases and decreases in compression, rather than tension. Since, as is known, periodic fluctuations in compression do not lead to fatigue failure of elastic materials, that problem is eliminated for the extent of the column that is in compression. The elastic waves transmitted down the column will, in general, create periodic tensions in the upper portion of the column, where the compression is sufficiently low that it can be exceeded on the tension half-cycles of the wave. Also, any small length of the column below the lower end support point necessarily hangs in tension. For these limited extends of the column which still experience a tension cycle, carefully selected materials and careful design against stress concentrations are still requisite to a degree. But a very large fraction of the column has been removed from the possibility of fatigue failure, with accompanying possibility of great savings in initial cost, together with greatly reduced threat of fatigue failure, resulting shut down, and replacement cost.

As disclosed in my aforementioned prior patent it is possible, by adjustment of the frequency of the elastic wave transmitted down the column, to create a standing wave along the column, with quarter-wave-spaced velocity nodes and antinodes. Assuming that such an elastic column, with a longitudinal elastic standing wave active therein, were free in space, its upper and lower ends would be antinodal regions, and one, or a number, of nodes would be found along the column. A further concept of the invention is to provide a support point for such a column at or close to the lowermost node of the standing wave. This lowermost node is located one-quarter of a wavelength up from the bottom end of the column, assuming the bottom end of the column to be unrestrained. With this type of lower end support, leakage of the driving wave energy into the surrounding formation is minimized. The column then resonates with negligible energy loss from the attachment point. On the other hand, in some cases it is desirable to radiate a pre-selected proportion of the wave energy into the formation, as for the purpose of augmenting flow of well fluids from the formation to the pump. In such case, the attachment or support point may be located at a selected distance from the lowermost node. It may be located at a node above the lowermost node, e.g., at some node above where the nodal definition is not so pronounced as it is at the lowermost node. Or it may simply be located at a selected point in the gradual transition region between a node and an antinode. Where greater wave energy extraction from the column is desired, the attachment point is moved toward an antinode; where less energy extraction is desired, the attachment point is moved toward a node, especially the lowermost node. Thus, precise selection of a desired energy extraction rate is feasible.

The invention will be better understood from the following detailed description of a present illustrative embodiment thereof, references for this purpose being had to accompanying drawings, in which:

Figure 1 is a view of an illustrative embodiment of a pumping system in accordance with the invention, the length of the installation being, of course, materially shortened for convenience of illustration;

Figure 2 is a longitudinal sectional view of the upper end portion of the pumping system of Figure 1;

Figure 3 is a longitudinal sectional view of a portion of the tubing lower down, showing a tubing guide and centering device;

Figure 3*a* is a transverse section taken on line 3*a*—3*a* of Figure 3;

Figure 4 is a longitudinal sectional view of a portion of the tubing, showing a fluid impelling and check valve unit;

Figure 5 is a longitudinal sectional view of the lower portion of the pumping system, a portion being broken away; and

Figure 6 is a longitudinal sectional view of a modified fluid impelling unit.

In the drawings, numeral **10** designates generally a well bore, lined by casing **11**, which extends down to the region of the oil bearing formation **12**.

Annularly spaced inside casing **11** is an elastic column, in this case a pump string tubing **13**, composed typically of 30-foot lengths or stands of steel or steel alloy tubing, joined by suitable couplings **14**. A good grade of steel alloy is generally desirable, though, for reasons mentioned in the foregoing, the main length of the tubing is not subject to elastic fatigue, and these regions of the tubing string may be fabricated to less stringent specifications, as will be discussed more fully hereinafter.

The lower end of tubing string **13** carries a fluid impelling assembly **15**, and there may be additional such assemblies at selected points in the tubing above. It is often found desirable to use one such assembly at each tubing coupling.

The upper end of tubing string **13**, above ground surface "G," is joined by coupling **16** to the tubular part **17** of a flanged discharge fitting **18**, which discharges production fluid upwardly into the interior of an adapter **19** fitted with discharge pipe **20**. The flange **21** of fitting **18** overhangs and engages an annular shoulder **22** of a mounting plate **23** on the upper end of a guide tubing **24** that extends down for a distance around tubing **13**. The adapter **19** rests on the upper face of plate **23**, outside shoulder **22**, and is formed with a peripheral flange **25** which is engaged and held down by means of a clamp ring **26** fastened to plate **23** as by screws **27**. The flange **21** of fitting **18** will be seen to fit snugly inside the lower portion of the adapter **25**, and the adapter has an internal downwardly-facing seat **28** which engages the upper side of flange **21**, all as clearly shown in the drawings.

Mounted on the top of adapter **19** is a vibration or elastic wave generator unit generally designated by numeral **29**, and which will be more fully described hereinafter. A casing head **30** is mounted on the upper end of casing **11**, and has a hollow interior leading to ports **31**, to which are coupled gas outlet pipes, not shown. A tubular bearing housing **32** for guide tube **24** has near its lower extremity an external ring flange **33** secured to casing head **30** as by screws **34**, and the housing portion **32*a*** below flange **33** is received inside a casing head opening **35** and sealed therein as by ring seal **36**.

Bearing housing **32** carries near its upper and lower ends suitable bearing bushings **37** and **38**, respectively, adapted for sliding support of guide tube **24**, and suitable seals are used above and below these bearings, as indicated at **39** and **40**, respectively.

It will be seen that the bearing housing **32** is supported from casing head **30**, and in turn from the casing **11** which is firmly supported by the walls of the well bore, but that the tubing string **13**, guide tube **24**, mounting plate **23**, and the adapter **19** and vibration generator **29** have no means of vertical support above the ground surface, and on the contrary, are free for vertical sliding movement within the bearings **37** and **38** carried by bearing housing **32**. The primary support place for these members is, as preliminarily described, in the lower end portion of the well, so that a large length of the tubing **13**, i.e., from the support point to the top end, will stand in compression, as an elastic compression column.

Preferably, to prevent rotation of the tubing string **13**, vibration generator **29**, etc., relative to the casing, a pair of pivotally connected links **41** and **42** are pivotally connected at **41*a*** and **42*a***, respectively, to the mounting plate **23**, and to a bracket **42*b*** on the bearing housing **32**. These links permit vertical vibratory movement of the generator, tubing string **13** and guide tube **24**, but hold said members against rotation.

A typical and preferred lower end support means for the tubing will next be described. At a selected point of the tubing string, in the lower end region thereof, in certain cases at a one-quarter wavelength distance from the bottom end of the tubing string, a special coupling **14*a*** is used, provided with external threads engaged by a second coupling **44**, into which is threaded the upper end of a support casing **45** which extends downwardly inside well casing **11** and outside tubing string **13**, as shown. This support casing **45** extends downwardly beyond the lower end of tubing string **13**, and to its lower end is coupled, as by coupling **46**, a foot member **47**, which is tubular and hollow down to and below fluid intake ports **48** formed therein, and which is solid at the bottom and rests firmly on the lower end of the well hole. The tubing string **13** is thus supported at coupling **14*a***, and stands in compression thereabove, while hanging in tension therebelow. Casing **45** stands in compression on the bottom of the well hole.

The lower end of tubing string **13**, below lowermost check valve **15**, carries, in the present illustrative embodiment, a rubber casing swab **49** which slidably engages the interior surface of casing **45**.

While not an essential in a sonic type pump, I have here shown the use of a foot check valve assembly **50** in the casing **45**, above fluid intake ports **48**.

The aforementioned vibration generator **29** may be of the same type as shown and described in my aforementioned Patent No. 2,444,912, and is hence shown in a somewhat diagrammatic fashion herein. Housing **65** contains oppositely rotating meshing spur gears **66** having shafts carrying eccentric weights **67** which balance out horizontal vibrations but which coact to produce a substantial alternating or oscillatory force in a vertical direction. Any suitable driving means, not shown, may be provided for the generator. For example, one of the gear shafts may carry a universal joint driven by a drive shaft from a suitable prime mover, which may be an internal combustion engine or an electric motor.

The above-mentioned fluid impelling assembly **15** comprises a cup **51** snugly received in the lower end portion of tubing string **13**, and having in the lower portion thereof an annular seat **52** around an intake passage **53** formed in a convergent intake member **54**. A tubular fitting **55** screwed on to the lower extremity of tubing string **13** is reduced at **56** to afford an upwardly facing annular shoulder **57**, which engages the lower end of member **54**, and a soft rubber sealing ring **58** around member **54** is compressed tightly by the member **54** and the lower extremity of tubing string **13** to effect a suitable seal when the fitting **55** is screwed tightly onto tubing string **13**. In the present illustrative embodiment, a check valve element is employed, in the form of a check valve disc **59** seating on the afore-mentioned annular seat **52**, and in the operation of the pump, is limitedly movable between said seat and an interrupted upper seat **60*a*** at the lower ends of ribs **60** formed inside cup **51**. The disc **59** is of smaller diameter than the seat **52**, so that when lifted above the seat, a fluid passageway is formed from intake passage **53** around the periphery of the disc to the space between the ribs **60**, and thence upwardly into the tubing string.

Fitting 55 on the lower end of tubing string 13 has a reduced tubular downwardly extending portion 61, to which the hub portion of the aforementioned swab 50 is firmly attached.

As mentioned earlier, a fluid impelling assembly, similar to 15, is preferably used at the various tubing joints above, and these may be the same in general construction and operation as the described assembly 15. Such an assembly is shown at 15a in Figure 4, incorporated in a tubing coupling 14.

The check valve assembly 50 at the lower end of casing 45 may also be of similar construction. Thus it may comprise a cup member 62 seated in the lower portion of casing 45, and designed to afford upper and lower seats for a vertically movable check valve disc 63. The cup 62 has a convergent inlet portion 64, and a rubber sealing ring 65, which in this instance is compressed between the lower end of casing 45 and the upper end of member 47, when the latter is screwed into coupling sleeve 46.

Operation will first be described with the assumption that foot valve 50 is omitted. When the vibration generator 29 is driven, a vertically alternating or oscillating force is applied to the upper end of the elastic tubing string 13 through adapter 19, mounting plate 23 and fitting 18. It will be recalled that the tubing string 13, supported near its bottom, stands in compression from its lower end region point of support 14a to its top end, and additional compression is imposed by reason of carrying the vibration generator 29. The tubing therefore has a compression bias, which is of course greatest at the point of support, and least at the top end. Below the point of support 14a the tubing string is suspended in moderate tension. The alternating force applied to the upper end of the elastic tubing 13 by the generator 29 causes alternating sinusoidal pulses of compression and rarefaction in the tubing relative to its described compressive bias. These pulses cause alternating elastic deformation waves, of compression and rarefaction, to be launched down the tubing. The waves so started down the tubing travel to the lower end thereof, and are reflected from the lower end to travel back up the tubing. Reaching the upper end, they are reflected back down, and so on. Assuming the case of a tubing length which is approximately equal to a whole number of half-wave lengths, at the generated wave frequency, the reflected and forwardly traveling waves interfere with and reinforce one another to produce a resonant standing wave in the tubing. For one typical example, the pump tubing may have a length equal to a full wavelength λ for the generated wave frequency. A full wavelength standing wave can then be established (Fig. 1) with velocity antinodes V at the upper and lower ends of the tubing and at the midpoint, and with a velocity node N located a quarter-wavelength distance from each end. A support means for the tubing may then be used at the lower node, as described above, permitting a strong standing wave in the tubing. Other points of support permit standing wave action, modified in accordance with the particular support point chosen, and the wave frequency, as will be understood by those skilled in the acoustics art.

The several fluid impelling assemblies 15 and 15a in the pump tubing are caused to vibrate vertically as a result of the wave action in the tubing; and in cases of standing wave operation, those located at or near velocity antinodes (which is the preferred location) will of course vibrate with greater amplitude than those in the nodal regions.

The mechanism by which the fluid impelling assemblies 15 and 15a elevate well fluids through the pump tubing as a result of the vibratory wave action has been fully set forth in my aforementioned Patent No. 2,444,912, and need not here be repeated in full detail. Briefly, on the downstroke of the vibratory motion of the tubing and

assembly 15 or 15a, the acceleration becomes greater than that of gravity, and therefore greater than that of the column of fluid in the tubing above. A suction, or void, is therefore developed in the column of liquid, above the assembly 15. Liquid below the assembly, displaced by the downward movement of the assembly, thus passes upwardly, past the check valve disc, into the void above, thus adding liquid to the internal liquid column above. On the upstroke, the check valve seats, and the column of liquid is elevated. It is not necessary that there be a definite check valve element, such as the disc 59. In Figure 6, I have shown a modified fluid impelling assembly 15b, similar to 15a, excepting that no check valve disc such as 59 is used. A continuously open nozzle passage 64 through the assembly is defined by a convergent wall 65, having nearly parallel sides near its upper end, terminating in a final contraction such as indicated at 66. Owing to the shape of the "nozzle," liquid forced upwardly therethrough on the upstroke exceeds that returning on the downstroke by a ratio of say two to one. This difference results from well-known characteristics of flow through orifices and nozzles. The pump thus operates successfully notwithstanding some back flow on the downstroke, so long as the back flow does not equal the up flow on the upstroke.

Operation may be somewhat improved by use of the swab 49 on the lower end of the pump tubing string, in combination with the foot check valve 50. On each vibratory downstroke of the pump tubing, the fluid body between the swab and the check valve 50 is placed under compression, causing the check valve disc 63 of valve 50 to seat downwardly, and so maximizing the volume of fluid forced upwardly past the check valve 15. On each upstroke of the vibratory pump tubing, a suction is developed between the swab and the valve 50, causing valve disc 63 to lift, and a quantity of well fluid to enter.

In the operation of the pump, collection of gas in the fluid space below the check valve 15 interferes with pumping. Gas in this region is accordingly bled upwardly into the space between tubing 13 and casing 45 through a port 75 in swab 49, and from the space between tubing 13 and casing 45 to the space between tubing 13 and casing 11 via a passageway 76 through coupling 14a.

As described previously, the elastic pump tubing stands in compression down to support coupling 14a, and hangs in tension therebelow. The compression "bias" is maximum at the support coupling and minimum at the top. Further, the alternating force applied to the upper end of the tubing creates traveling wave pulses of compression and rarefaction, which alternately add to and subtract from the compression bias. In the very uppermost region of the pump tubing, where the compressive bias is not large, the rarefaction pulses may exceed the compressive bias, thus placing this portion of the tubing under tension for a portion of each half cycle. A little lower down is a point at which the pulse of rarefaction just equals the compression bias, so that the tubing does not go into tension, but merely to a state of neutral stress. Below this last-mentioned point and down to the support coupling 14a, the rarefaction pulse is exceeded in magnitude by the compressive bias, so that the wave action consists entirely in alternating increases and decreases in compression. This favorable last-described condition prevails for the majority of the length of the tubing string. Throughout this length, liability to fatigue failure owing to elastic wave action in the tubing and couplings is eliminated. Accordingly, this portion of the tubing and the included couplings may be fabricated of ordinary tubing and coupling materials, and without special care to avoid such stress concentrations as might, under tension, lead to early fatigue. The very upper portion of the tubing, as well as that portion hanging in tension below the support point, are still, of course, subject to the threat of elastic fatigue failure, and should continue to be made of good steel alloys and with good fatigue resisting design. The exact location down from the upper end of the tubing at which the wave action in the tubing ceases passing through a partial tension half-cycle will of course vary in any given pump installation with the weight and force output of the wave generator.

One field installation of sonic pump in accordance with the drawings herein has a tubing length of 1200 feet, and develops a full wave length standing wave at a wave frequency of 760 cycles per minute. The nodal point support coupling for the tubing is located 300 feet up from the lower end. With deeper wells, the wave frequency will generally be adjusted to provide more than one wave length along the tubing. With the support point in such cases at a quarter-wavelength from the bottom, a substantially larger proportion of the tubing will stand in compression, and correspondingly increased benefits are realized from the practice of the invention.

As mentioned in the introductory paragraphs, in the event it should be desired to extract vibration or wave energy from the tubing, the support coupling **14***a* may be displaced from the node by a selected distance. Under such conditions, the tubing will undergo a degree of vibration at the support point, and this vibration will be transmitted into the coupling **14***a*, down the support casing **45**, and to the foot member **47**, from which this energy is radiated into the formation, as for the purpose of increasing the rate of migration of well fluids therethrough.

If it is desired to place a maximum length of the tubing string in compression, and at the same time limit the amount of sonic energy extracted from the tubing, then, I have found, it might be desirable to locate the support point close to the lower end, and use a compliant structure or material for support means **45**.

As a consequence of the pump tubing standing under considerable compression, particularly toward its lower end, the laws of column buckling must be considered. Any such buckling tendencies can be combatted by using larger diameters of tubing toward the bottom, and/or fitting the tubing closely within a guide means. For example, as suggested in Figure 5, a guide tubing **80** may be coupled onto the support coupling **14***a*, and extend upwardly a distance as necessary to guard against buckling. It may be centered in the casing in any desired fashion, not shown. Also, tubing guides such as **81** (see Figure 3) may be mounted directly on the tubing **13** at spaced points. These guides **81** may comprise a rubber sleeve **82** fitted onto the tubing and furnished with longitudinal ribs **83** which carry a fabric impregnated phenolic ring **84** adapted to engage the inside surface of the casing **11**. Such a guide does not have a tendency to wear the casing, and serves to center the tubing and to guard against buckling. The friction introduced by these guides rubbing on the casing is negligible, particularly if a small flow of output fluid is introduced at the top of the casing annulus.

The drawings and description show one present illustrative embodiment of the invention. It will be understood that various changes in design, structure and arrangement may be made without departing from the spirit and scope of the broader of the appended claims.

I claim:

1. In a deep well sonic pump, the combination of: an elastic pump tubing in the well bore, a sonic wave generator coupled to said elastic pump tubing for maintaining therein periodic longitudinal deformation waves of compression and rarefaction relative to normal mean longitudinal stress at any given point therealong, fluid impelling means in said tubing reciprocated by said waves in said tubing, and tubing support means for supporting said tubing at a support point within the lower region of said tubing but spaced substantially above the lower extremity thereof, said means including a coupling fixed to said tubing at said support point, and a leg fixed to said coupling and extending downwardly past the lower end of the tubing and including a footing at its lower end in supporting engagement with the lower end of the well bore, in such manner that the weight of said tubing is borne by said support means, and said tubing stands normally in compression above said support point, and hangs normally in tension below said support point.

2. The subject matter of claim 1, wherein said leg of said tubing support means comprises a support casing positioned within the well bore outside the tubing and coupled at its upper end to said tubing coupling.

3. In a deep well sonic pump, the combination of: an oscillating fluid impelling member adapted for placement in the lower region of the well bore, an elastic column of solid material in the well bore extending from the ground surface to said fluid impelling member and operatively coupled to the latter, a sonic wave generator coupled to said elastic column for maintaining therein periodic elastic deformation waves of compression and rarefaction relative to normal mean longitudinal stress at any given point therealong, and column support means for supporting said column at a support point within the lower region of said column but spaced substantially above the lower extremity thereof, said means including a coupling fixed to said column at said support point, and a leg fixed to said coupling and extending downwardly past the lower end of the column and including a footing at its lower end in supporting engagement with the lower end of the well bore, in such manner that the weight of said column is borne by said support means, and said column stands normally in compression above said support point, and hangs normally in tension below said support point.

4. In a deep well sonic pump, the combination of: an elastic pump tubing in the well bore, a sonic wave generator coupled to said elastic pump tubing for maintaining therein periodic longitudinal deformation waves of compression and rarefaction relative to normal mean longitudinal stress at any given point therealong, said generator being operable at a frequency to establish a standing wave in said tubing, with a nodal region in the lower portion of the tubing, above the lower end thereof, fluid impelling means in said tubing reciprocated by said waves in said tubing, and tubing support means for supporting said tubing at said nodal region, said means including a coupling fixed to said tubing at said support point, and a leg fixed to said coupling and extending downwardly past the lower end of the tubing and including a footing at its lower end in supporting engagement with the lower end of the well bore, in such manner that the weight of said tubing is borne by said support means, and said tubing stands normally in compression above said support point, and hangs normally in tension below said support point.

5. In a deep well sonic pump, the combination of: an oscillating fluid impelling member adapted for placement in the lower region of the well bore, an elastic column of solid material in the well bore extending from the ground surface to said fluid impelling member and operatively coupled to the latter, a sonic wave generator coupled to said elastic column for maintaining therein periodic elastic deformation waves of compression and rarefaction relative to normal mean longitudinal stress at any given point therealong, said generator being operable at a frequency to establish a standing wave in said column, with a nodal region in the lower portion of the column, above the lower end thereof, and column support means for supporting said column at a support point in proximity to said nodal region, said means including a coupling fixed to said column at said support point, and a leg fixed to said coupling and extending downwardly past the lower end of the column and including a footing at its lower end in supporting engagement with the lower end of the well bore, in such manner that the weight of said column is borne by said support means, and said

column stands normally in compression above said support point, and hangs normally in tension below said support point.

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