June 7, 1955 C. DE GANAHL ET AL 2,710,136

AXIAL FLOW COMPRESSOR

Filed Dec. 28, 1948 3 Sheets-Sheet 1

Fig. 1,

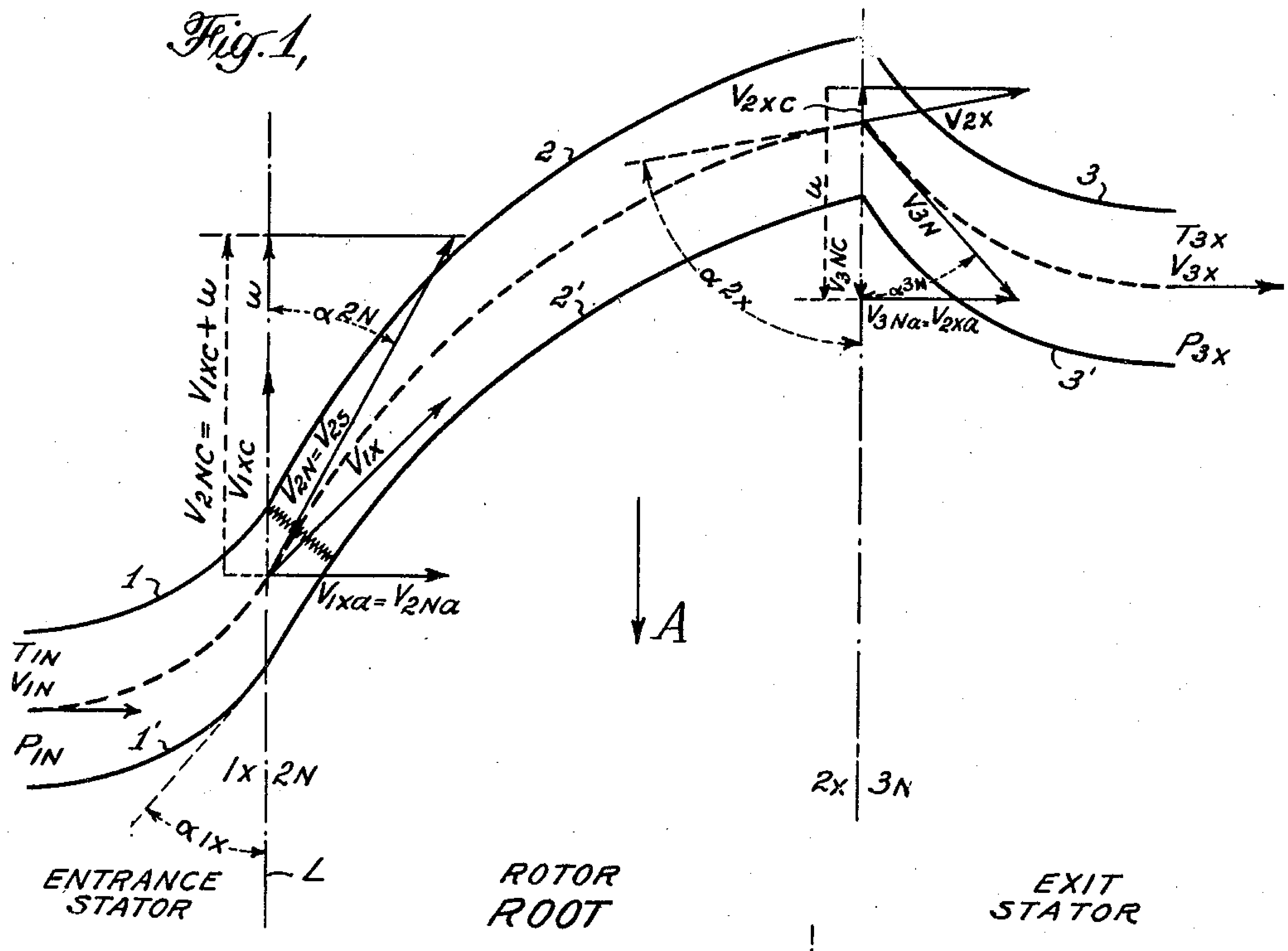
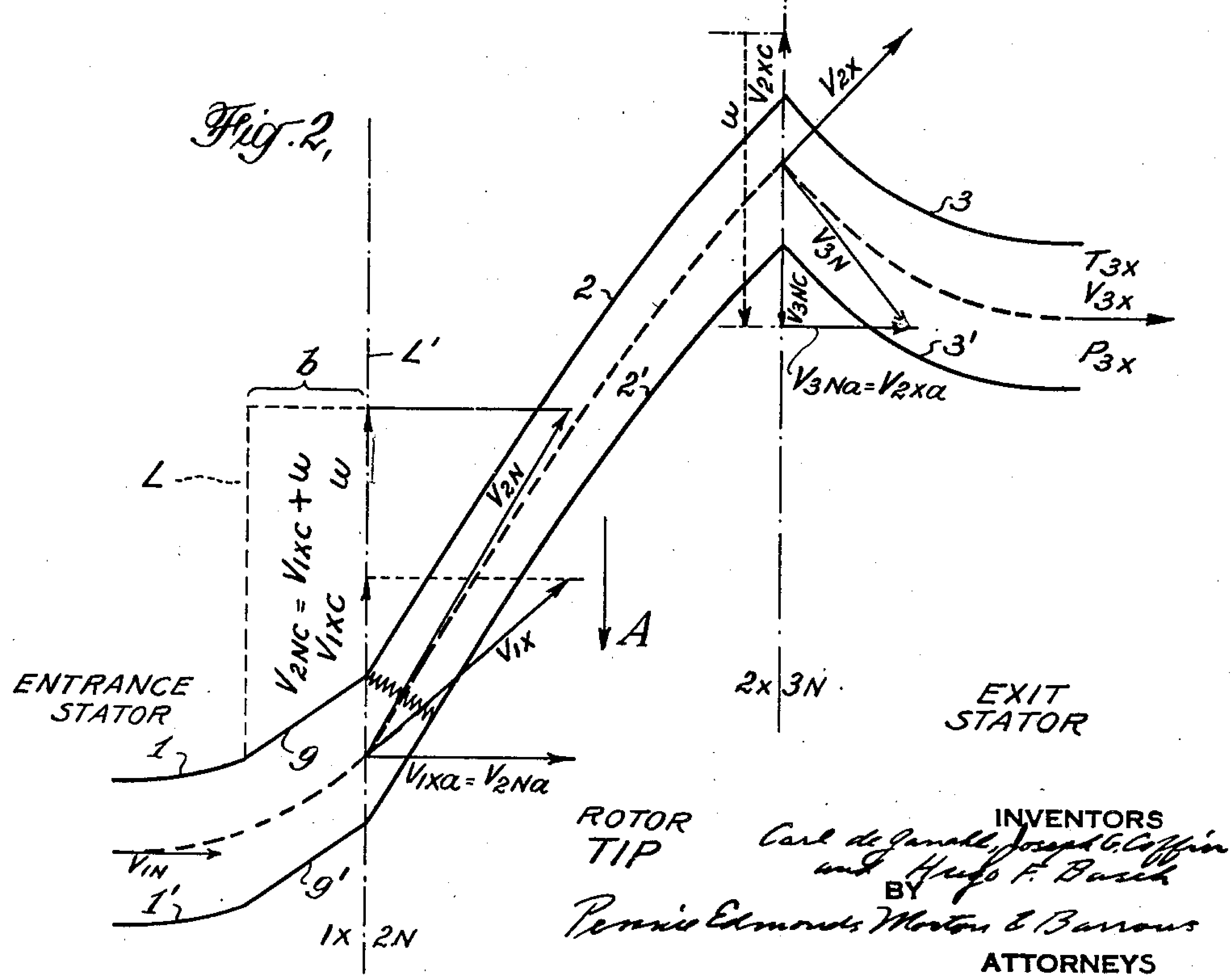


Fig. 2,



INVENTORS

Carl de Ganahl, Joseph G. Coffin and Hugo F. Barich

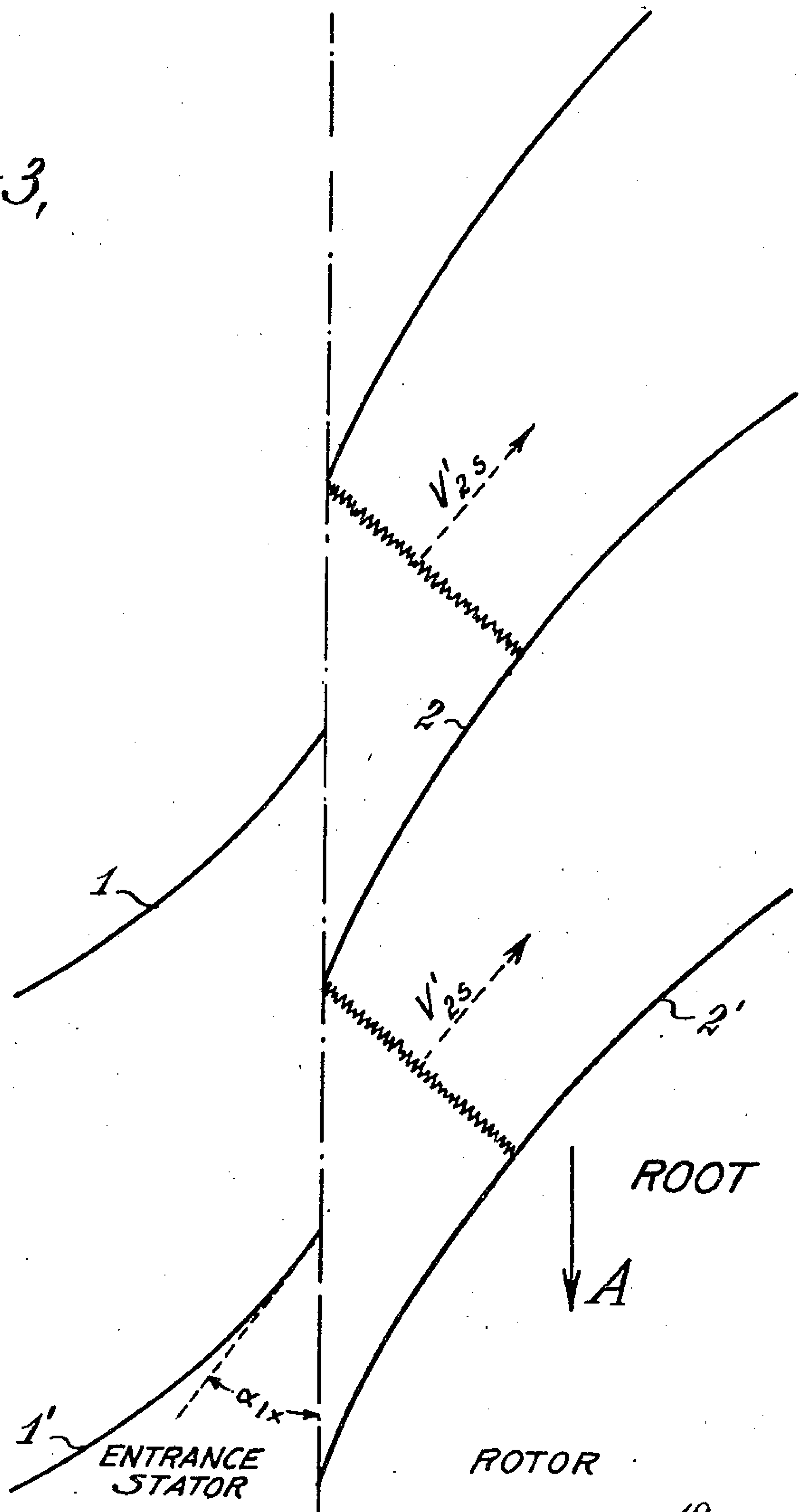
BY

Pennie Edmonds Morton & Barrows

ATTORNEYS

AXIAL FLOW COMPRESSOR

Fig. 3,



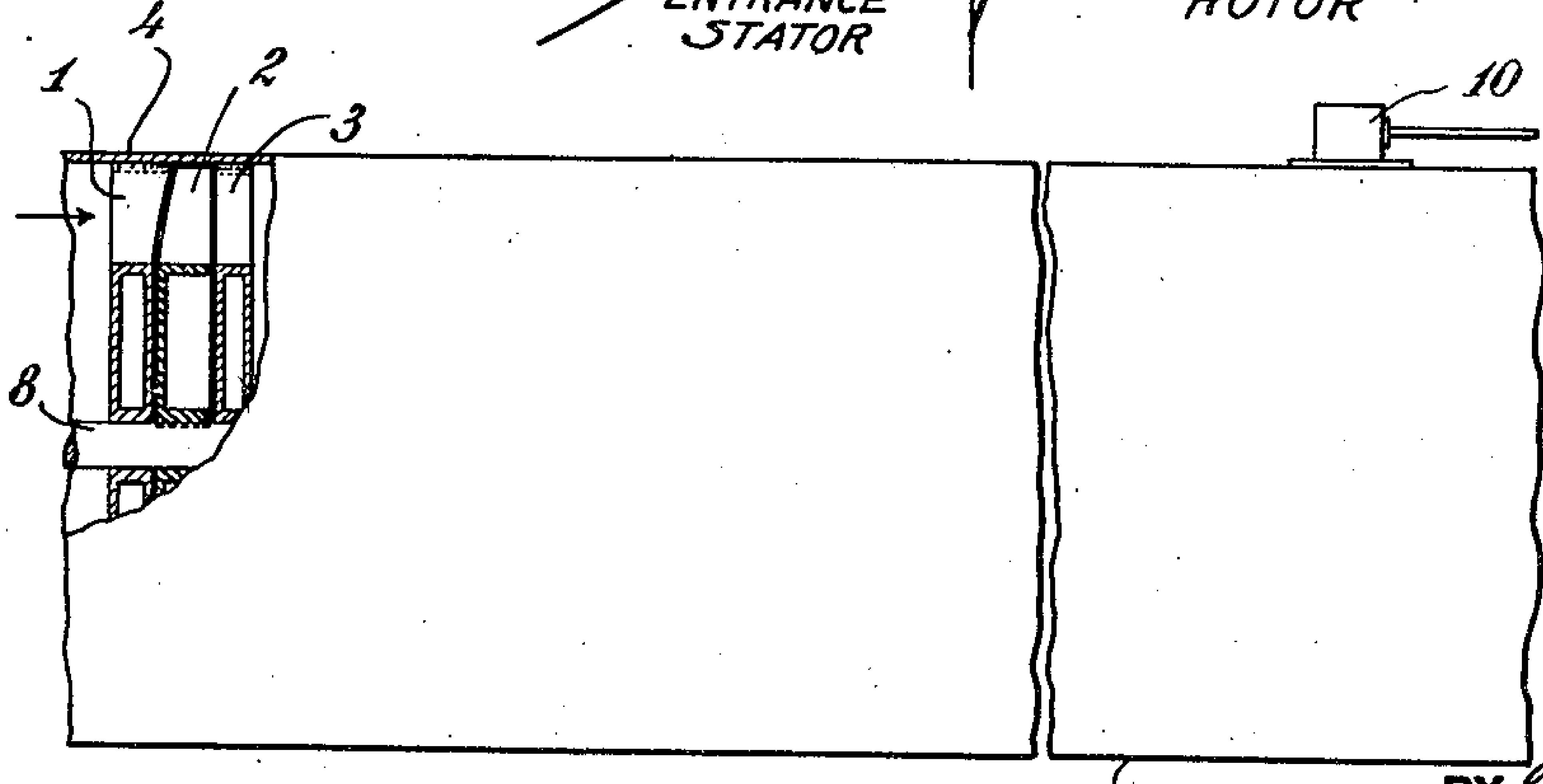


Fig. 8.

INVENTORS
Carl de Ganahl
Joseph E. Coffin
BY Hugo F. Bach

Pennie Edmonds Morton & Barrows
ATTORNEYS

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AXIAL FLOW COMPRESSOR

Filed Dec. 28, 1948 3 Sheets-Sheet 3

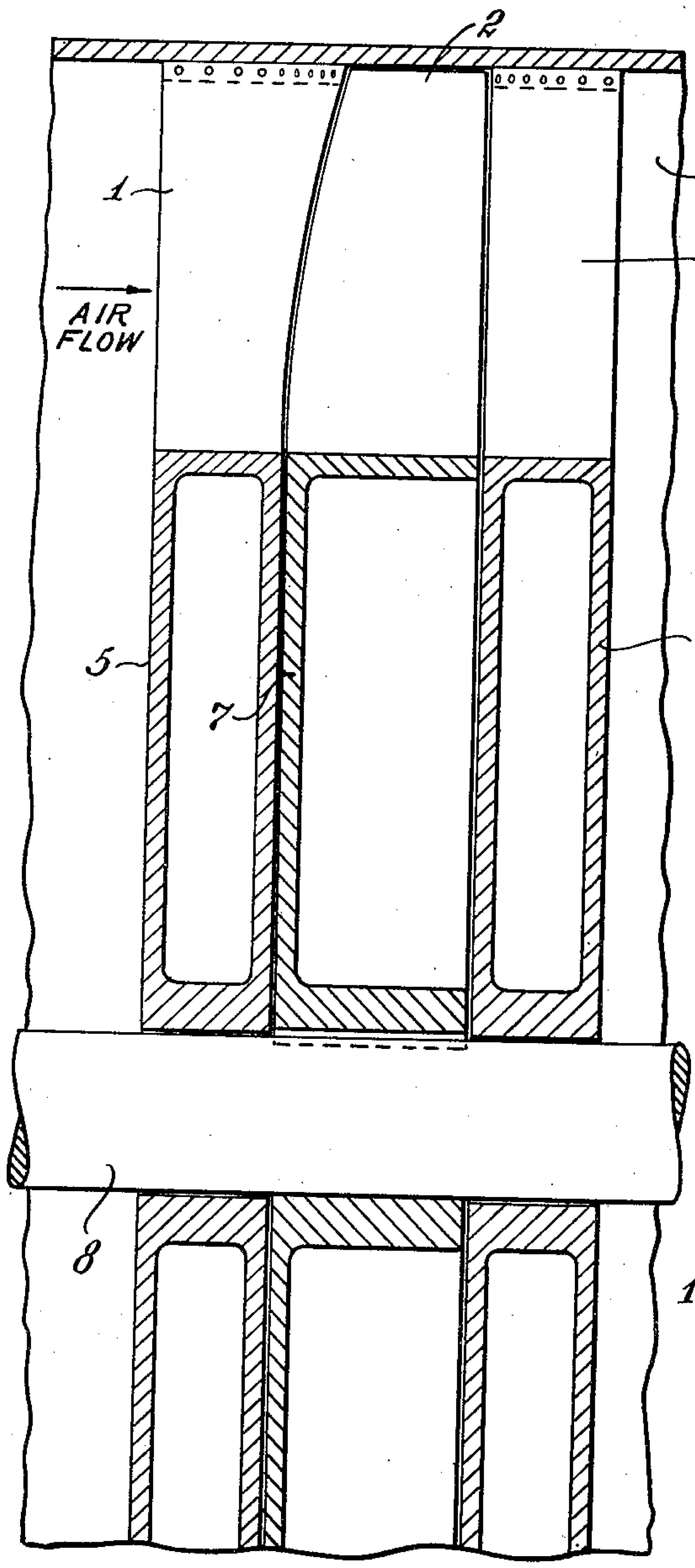


Fig. 4,

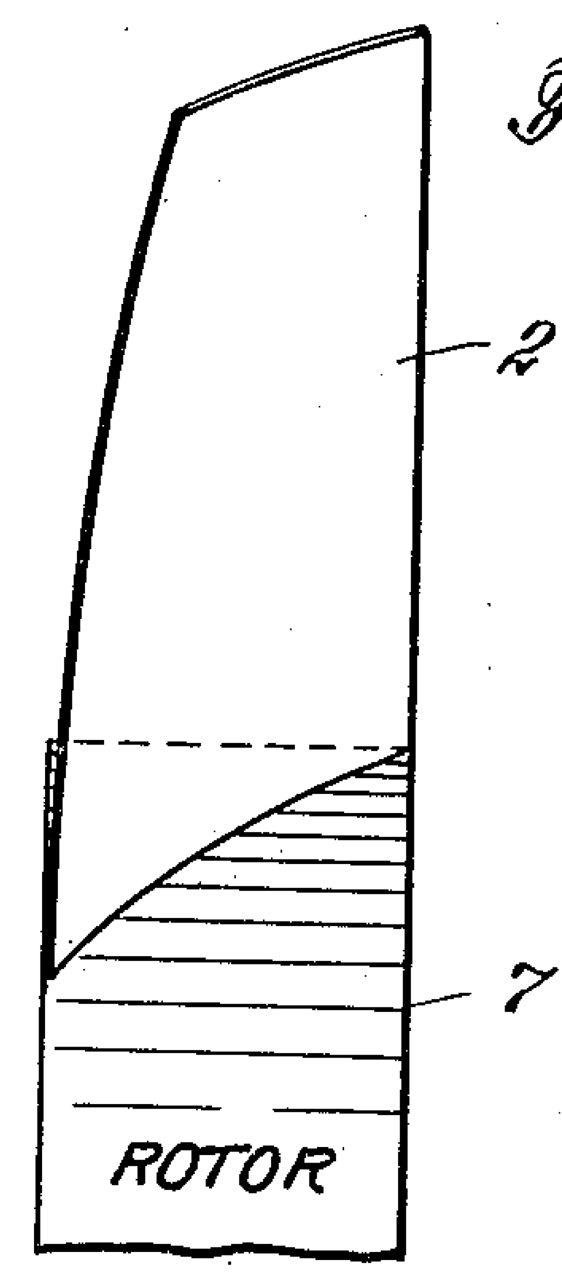


Fig. 5,

Fig. 6,

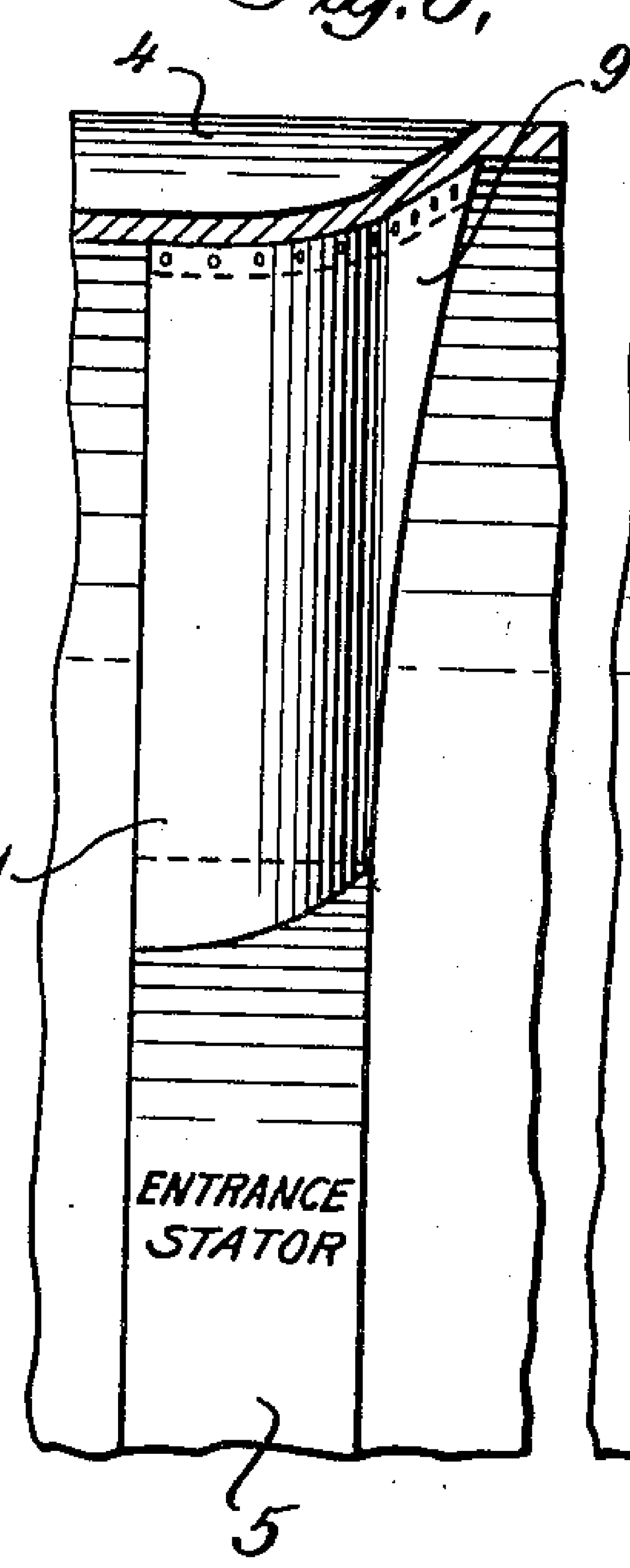
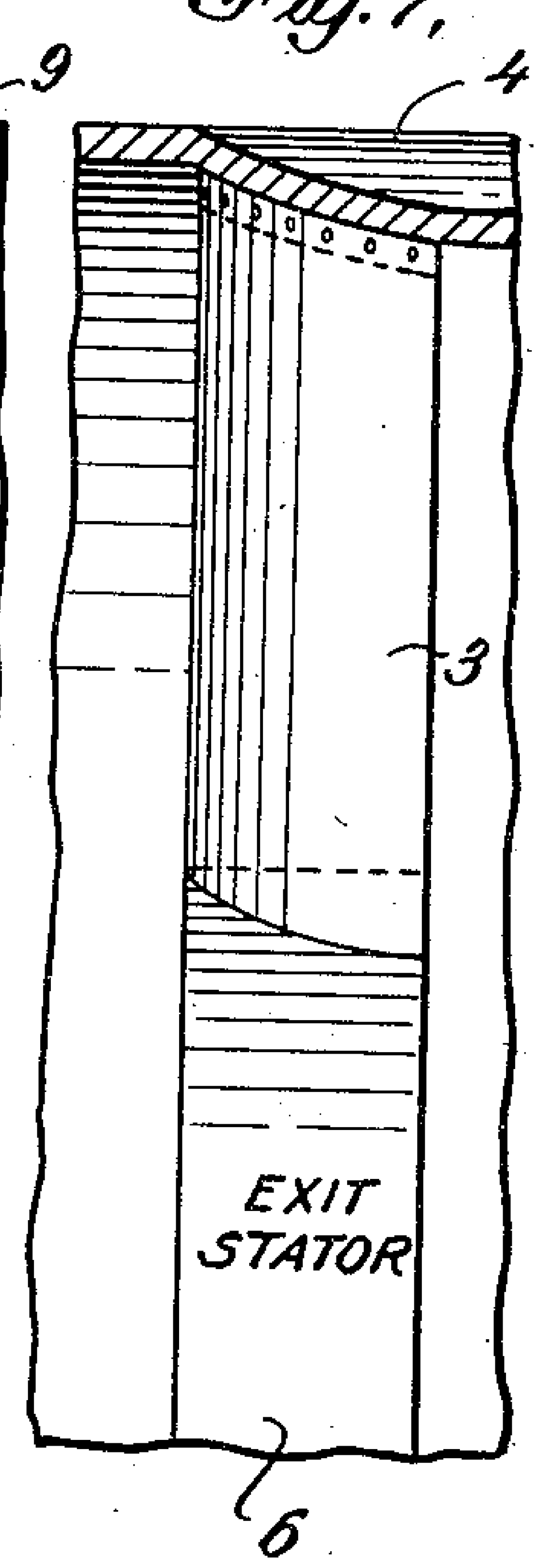


Fig. 7,



INVENTORS

Carl de Ganahl, Joseph G. Coffin and Hugo F. Baeck

BY

Pennie Edmonds Morton & Barrows

ATTORNEYS

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AXIAL FLOW COMPRESSOR

Carl De Ganahl, Trenton, Joseph G. Coffin, Vincentown, and Hugo F. Basch, Moorestown, N. J., assignors to Kaiser Metal Products, Inc., a corporation of California

Application December 28, 1948, Serial No. 67,732

6 Claims. (Cl. 230—122)

This invention relates to axial flow compressors for compressing air or other compressible fluids.

A single stage axial flow compressor, as distinguished from a centrifugal compressor, consists essentially of a compressor casing and a rotor comprising a central shaft or hub carrying a set of vanes or blades, the casing carrying a set of stationary blades constituting entrance stator blades and an additional set of blades which may be characterized as exit stator blades, the air flow being in a generally axial direction. A single stage compressor includes only one set of rotor blades associated with a set of entrance stator blades and with a set of exit stator blades. In a multi-stage compressor there are several sets of rotor blades and corresponding sets of stator blades.

This invention relates primarily to improvements in axial flow supersonic compressors, the compressor being either a single stage or a multistage compressor. In a subsonic compressor the air flow through the compressor is such that the Mach number of the flow is everywhere less than 1. In a supersonic compressor embodying the present invention, the air flow through the compressor exceeds a flow Mach number 1 within the rotor at its entrance, and a shock wave is produced which is utilized to produce compression of the compressible fluid supplementing compression produced in other parts of the compressor where the flow is or has become subsonic.

The principal objects of this invention include the provision of rotor and stator blades forming fluid ducts for conducting the fluid in a generally axial direction through the compressor, and the provision of blades of such character and so formed or shaped that the compressed fluid is directed axially in the exit stator with a fluid pressure substantially constant from the root to the tip portions of the blades, the configuration of the compressor blades also being such that the fluid velocity does not have any radial component at any point.

Other objects of this invention as embodied in a supersonic compressor comprise the production of a stable normal shock wave in the rotor, in other words, a normal shock wave, the location of which in the rotor ducts remains substantially constant during normal operation of the compressor. We prefer to employ means for maintaining the shock wave in a stable position at or near the rotor entrance, and to provide rotor blades of such a character that as the fluid advances from the shock wave it is further compressed to provide a rotor exit pressure substantially greater than the pressure obtaining at the location of the shock wave. Thus in a preferred embodiment of our supersonic compressor as much as 30% of the pressure increase through the rotor represents the pressure increase produced in the rotor ducts following the shock wave.

According to one embodiment of our invention tht rotor blades may be of such configuration and so disposed with respect to each other that they define rotor ducts gradually diverging from rotor inlet to rotor outlet and having a length several times the minimum width of the duct between adjacent blades. The rotor ducts constitute diffusers gradually diverging from rotor inlet toward the rotor outlet whereby the location of a shock wave in the rotor ducts may be stabilized by controlling the back pressure. In our improved supersonic compressor the entrance stator blades, the rotor blades and the rotor speed are such that the fluid enters the rotor with a velocity with respect to the rotor that is supersonic. A normal shock wave is formed in the rotor ducts because the fluid, flowing at a velocity that is supersonic with respect to the rotor ducts, in progressing into or through the rotor diffuser passages, reaches an area of increased cross section where the controlled back pressure is adequate, consistent with the Mach number of the supersonic fluid velocity, to prevent further advance of the fluid at supersonic velocity. In passing across the shock wave the fluid velocity is abruptly reduced from its supersonic value to a subsonic value and its pressure is substantially increased.

A further object of our invention is to provide an axial flow compressor which will operate as a subsonic compressor at rotor speeds below the normal operating speed, and as a supersonic compressor when the rotor operates at its normal speed, the transition being gradual and not requiring at any time the utilization of power in excess of that required for normal operation of the compressor. In general this object is attained by employing rotor blodes forming diffuser passages within the rotor, and blade entrance and exit angles such that streamlined flow of the fluid is provided into the rotor and into the exit stator.

A further object of our invention is to provide a rotor structure of such a character that vortex-free flow of the fluid through the rotor is accommodated to the maximum extent.

Another object of this invention is the provision of a supersonic axial flow compressor in which the length of the rotor blades does not exceed a critical value, thereby insuring maximum efficiency and maximum stability of the shock wave produced at or near the entrance of the rotor.

Our invention makes it possible to provide an axial flow compressor having numerous advantages, including the following:

a. The compressible fluid, as it flows axially in the exit stator ducts, has substantially the same pressure at all radially spaced points between root and tip portions of the blades forming the exit stator ducts and accordingly the tendency to cross current flow from a relatively high pressure region to a relatively low pressure region can be avoided.

b. Only a few stages of compression need be employed to produce the required increase in pressure; pressure ratios per stage may be in the order of 1.7 to 2.0 with high efficiencies approaching 90% or better.

c. The rotational speeds of the rotor can be kept low enough to reduce the centrifugal tension forces in the material to practical values.

d. The weight and cost of the compressor can be reduced to a minimum because of simplification of manufacture and the fact that the over-all dimensions may be reduced to a minimum for a given capacity.

e. The supersonic compressor will act as a subsonic compressor on starting, thus eliminating the necessity for high power starter devices.

f. The back pressure is controlled so as to locate the shock wave at or near the entrance of the rotor and the rotor blades are so shaped and arranged that a substantial increase in pressure occurs in the rotor between the shock wave and the rotor exit.

According to our invention, the rotor and stator blades are constructed or formed in the manner hereinafter described to provide a supersonic velocity of the air at the rotor entrance inside the rotor duct and to produce a standing shock wave of constant strength from root to tip of the rotor blade at or near the rotor entrance, whereby a part of the total pressure rise is obtained by shock at high efficiency and the remainder of the total pressure rise by diffuser action in the rotor and exit stator ducts. Substantially constant pressure may be obtained at all radially spaced points between the root and tip portions of the blades where the fluid flows axially, the blades also being preferably formed so that the fluid velocity does not have any radial component at any point.

In general, the rotor and stator blades may be formed so that the same amount of energy per unit of compressible fluid is transmitted by the rotor to the fluid at the root portion of the rotor blades as at the tip portion of the rotor blades, with a standing shock wave of constant strength at or near the rotor entrance inside the rotor, even though the circumferential speed of the rotor blade tips is substantially greater than the circumferential speed of the root portions of the rotor blades.

The required configuration of the rotor and stator blades to accomplish this purpose may be determined with the highest degree of precision by first selecting appropriate operating conditions for the entrance to the compressor, and then ascertaining by thermodynamic equations for shock wave behavior, the proper angular values of the entering and exiting angles of the inlet stator blades, the rotor blades and the exit stator blades at all concentric zones or regions between the root and tip portions of the blades.

The amount of energy imparted to the fluid by the rotor is a function of the circumferential speed of the rotor and of the velocity of the fluid entering the rotor, and according to this invention the entrance stator blades and the rotor blades are shaped so as to provide progressively decreasing rotor entrance velocities from root to tip to to compensate as completely as may be desired for the increase in circumferential rotor speed between the root and tip of the rotor blades, whereby the root portions of the rotor blades, travelling at relatively low circumferential speed, may impart to the fluid, entering the rotor with its higher circumferential velocities, the same amount of energy as the tip portions of the rotor blades, the exit portions of the rotor blades being correspondingly shaped at the rotor exit, taking into consideration the standing shock wave near the rotor entrance inside the rotor duct, so that as the compressed fluid flows axially in the exit stator, it may have the same pressure at all radially spaced points in the exit stator.

In a compressor embodying this invention, there is a pressure stage or a multiplicity of pressure stages in which each stage is a combination of an entrance stator, a rotor and exit stator. In succeeding stages, the entrance stator to the next stage becomes a part of or is contiguous with the exit stator of the preceding stage. The flow is axial between stages, i. e., the flow is purely axial as the fluid enters the entrance stator of one stage from the exit stator of the preceding stage.

The rotor blades and stator blades are preferably made of thin sheet metal. The spaces between the blades form channels for the flow of the fluid which are either converging to form nozzles or diverging to form diffusers as hereinafter explained.

The cross-sectional area of the channel or duct is proportional to the sine of the angle the blade makes with a plane normal to the rotor axis. The solidity, i. e., the circumferential spacing of the blades, determines the axial space rate change of angle between the required calculated entrance angle and exit angle of a duct, whether a nozzle or converging duct or a diffuser or expanding duct. Consecutive blades are preferably equidistant from each other at all points as measured in a circumferential direction.

In the axial flow supersonic compressor of our invention, the entrance stator directs the fluid against the direction of rotation of the rotor rather than with this direction

of rotation, thereby increasing the relative velocity between the rotor blades and the entering air thus greatly reducing the required speed of the rotor.

In this supersonic compressor of our invention the rotor blade speed is such that the velocity of the air as it enters the rotor, relative to the rotor, is above the local velocity of sound at that point. This rotor speed may be adjusted so as to obtain a Mach number of flow of any desired amount greater than 1. Also, the back pressure is controlled so as to insure the production of a standing shock wave of the desired constant strength just at the rotor entrance at the root and tip and very near to the entrance of the rotor duct at intermediate points along the rotor blade.

The angle of the blades at the rotor entrance is such as to provide streamlined flow of the fluid from the entrance stator into the rotor and the angle of the blades at the exit stator entrance is such as to provide streamlined flow into the exit stator.

According to one embodiment of our invention, the circumferential velocity at the exit of the entrance stator and the circumferential velocity at the entrance of the exit stator can be kept equal. However, it is usually preferred to let the ratio of these two circumferential velocities be of such a value as to satisfy other desired requirements of greater import, as will be explained.

The angle between the airflow leaving the entrance stator and the plane normal to the rotor axis is greatest at the tip and decreases toward the root. This progressive change in angle tends to increase the relative velocity between the rotor and the entering airflow from tip to root. The entrance stator and the rotor are so formed that the rotor puts into the air the same amount of energy at the tip as it does at the root so that as the fluid is directed axially in the exit stator, there will be the same pressure and velocity from tip to root. To accomplish this end, the direction of the air flow is bent through a lesser angle of arc at the tip of the entrance stator than at the root, the exact amount of this bending being dependent upon the combined effect of the thermodynamic conditions of the flow and the strength of the standing shock wave.

This increases the ratio between exit and entrance area at each radius of the rotor towards the root, again properly compensating for the decreased rotor velocity at the root. It also increases the amount of whirl velocity delivered to the air flow at the root, further compensating for the decreased rotor velocity at the root.

The angle between the airflow entering the exit stator and a plane normal to the axis of the compressor is largest at the tip, and decreases towards the root, providing the necessary difference of ratio between exit area and entrance area at each radius to convert the different velocities into pressure. The general conditions above described can be repeated through succeeding stages.

In the accompanying drawings, we have illustrated elements of one stage of an axial flow compressor embodying our invention.

Fig. 1 represents a diagrammatic development of a cylindrical section taken at the root portion of the stator and rotor blades;

Fig. 2 represents a similar diagrammatic development of a cylindrical section taken at the tip of the stator and rotor blades;

Fig. 3 represents an enlarged diagrammatic development of a cylindrical section taken at the root portion of the stator and rotor blades at the rotor entrance;

Fig. 4 is a longitudinal section of a portion of a single stage compressor embodying our invention;

Fig. 5 is an elevation of a rotor blade;

Figs. 6 and 7 are fragmentary elevations of stator blades of a compressor embodying our invention, and

Fig. 8 is a fragmentary elevation, partly in section, of compressor apparatus embodying our invention.

In Figs. 1, 2 and 3 of the accompanying drawings, a pair of entrance stator blades are shown at **1** and **1'**, a pair of rotor blades at **2** and **2'** and a pair of exit stator blades at **3** and **3'**. These Figs. 1, 2 and 3 thus illustrate in diagrammatic form a single stage of an axial flow supersonic compressor and it will be understood that in a multi-stage compressor the exit stator blades are extended to form the entrance stator for the next succeeding set of rotor blades.

Figs. 1 and 2 show respectively, the root and tip circumferential sections of the same pairs of stator and rotor blades and accordingly the same reference characters for the blades are used in both these figures.

The stator and rotor blades are preferably made of thin metal stampings (the sheet metal being of uniform thickness throughout). All blades in any entrance stator are identical in shape and contour. This is also true for any rotor set or any exit stator set.

Figs. 5 to 8, inclusive, show an entrance stator blade **1**, a rotor blade **2** and an exit stator blade **3**. The stator blades **1** and **3** have their tip portions secured to an outer casing **4** and their root portions secured to stationary members **5** and **6**. The rotor blades **2**, **2'**, etc., are secured to a hub **7** fixed to the compressor shaft **8**.

The rotor and stator blades define ducts extending from the root to the tip and also extending in a generally axial direction through the rotor and stator blades. Each set of rotor blades and stator blades consists of blade elements bent to form ducts or channels and because of the fact that all of the blades are identical in shape and thickness each duct or channel existing between these blades whether stator or rotor blades is also identical in shape to any other duct lying between stator or rotor blades respectively.

The cross-sectional flow areas of the ducts formed by the blades are at all points proportional to the sine of the angle that the duct axis makes with a plane normal to the axis of the rotor, regarding the blades as made of thin material of constant thickness.

The entrance stator blades **1** and **1'** are shaped to form a nozzle or in other words, so that the cross-sectional area normal to the duct axis at the exit of the entrance stator is smaller than the corresponding cross-sectional area at the entrance to the entrance stator. The duct defined by each pair of rotor blades as illustrated in the accompanying drawings, is a diffuser, for the cross-sectional area normal to the duct axis at the rotor exit is larger than the cross-sectional area normal to the duct axis at the rotor entrance. Each pair of exit stator blades likewise defines a diffuser, the exit of the exit stator duct directing the fluid axially.

The thermodynamic conditions of flow between two curved blades are such that the relative fluid temperature, pressure and velocity conditions at the entrance and exit portions of the duct are determined solely by the ratio of the cross-sectional areas normal to the duct axis at the entrance and exit respectively.

The angle of divergence (or convergence) between any pair of consecutive blades made of uniform thin material is a function of the constant circumferential distance between the blades (or solidity), the angle at any axial location between two consecutive blades of a duct either stator or rotor being greater when the circumferential distance is large than when this distance is small. The angle of divergence approaches zero as the two blades are brought nearer together.

The cross-sectional area of the duct defined by any pair of blades is a function of the shape of the blades whereas the angle of divergence (or convergence) is a function of the circumferential distance between the blades and these two factors are entirely independent of each other.

In Figs. 1 and 2, the air or other compressible fluid is shown entering the entrance stator duct in a direction substantially parallel to the axis of the rotor, the air at this point having a local temperature T_{1n}, a velocity V_{1n} and a pressure P_{1n}. The air leaves the entrance stator nozzle with a velocity V_{1x} which may be considered to have an axial component V_{1xa} and a circumferential component V_{1xc}.

It will be understood that the rotor section or element illustrated in Figs. 1, 2 and 3 is moving in the direction indicated by the arrow A, with respect to the stationary entrance stator and exit stator, and in Fig. 1 the arrow u represents the reverse of the circumferential velocity or linear speed of the root of the rotor blade.

The velocity of the air entering the rotor V_{2n} is the vectorial sum of the velocity V_{1x} and the reverse of the rotor speed u. If the rotor speed at the root is sufficiently great this velocity V_{2n} relative to the rotor is supersonic in value.

It will be shown that this supersonic velocity will produce a shock wave somewhere in the rotor diffuser duct, its position being controlled by the exit pressure in the exit of the rotor duct which, in turn, is controlled by the pressure at the exit of the exit stator. This pressure is adjusted to locate the shock right at the rotor duct entrance at the root.

The greater V_{1x} and u are, the greater will be the Mach number of flow at the rotor duct entrance and consequently the greater the strength of the shock located there.

The ratios of the velocities, temperatures, pressures, and total pressures fore and aft of a shock wave are functions of the shock strength alone, the total temperature fore and aft remains the same. By means of the knowledge of these ratios, the velocity, temperature, pressure, and total pressure aft of the shock wave may be determined when the velocity, temperature, pressure and total pressure are known in front of the shock. Conditions just in front of a shock wave are designated with a subscript s and conditions just aft the shock by a subscript s with the quantity "primed." Thus V_{2s} denotes the velocity just in front of a shock and V'_{2s} the velocity just after the shock.

The supersonic velocity just in front of the shock suddenly diminishes to a value which is subsonic just aft of the shock. The local temperature just in front of the shock suddenly increases just aft of the shock.

These changes do not take place at 100% efficiency; in other words, the transition is not isentropic although it is adiabatic. However, the efficiency is high, higher than obtaining the same pressure rise through diffuser action where the diffuser duct efficiency factor is taken into account. For example, through a shock of strength $M_{2s}=1.4$ the static pressure ratio increase is 2.12, or a pressure of 1 atm. in front of the shock becomes a pressure of 2.12 atm. just after the shock, this rise being obtained at a pressure recovery efficiency of 93.5%.

From just aft of the shock at the entrance to the rotor the flow is subsonic. The subsonic flow equations determine the air conditions at the rotor exit, for a given area ratio between the rotor duct exit area and the rotor duct entrance area.

The air leaves the entrance stator nozzle at an angle with respect to a plane normal to the rotor axis, this angle being designated α_{1x} in Fig. 1, and it will be noted that the direction of the air entering the rotor duct at the rotor entrance, as indicated by the arrow V_{2n} is parallel to the rotor blades at this point, thus providing streamlined flow into the rotor. In other words, as the air enters the rotor duct which is travelling at high speed in the direction indicated by the arrow A in Fig. 1, the air is not immediately subjected to any impact by the rotor blades. The velocity of the air entering the rotor, with respect to the rotor, i. e., V_{2n} is supersonic. The rotor duct at this point is a diffuser, or gradually enlarging passage, and, if the back pressure is adequate, a shock wave is formed at or near the rotor entrance. The velocity of the air is now diminished from just in front of the shock (M_{2s}) where its velocity is V_{2n} or V_{2s} to a new value V_{2s}' which is in the same direction as V_{2s} and of a magnitude determined by the shock strength. Similarly, the quantities T_{2s}', P_{2s}', P_{T2s}' are determined from the shock wave formulas.

As the air now progresses in the rotor duct, however, its direction of flow is progressively changed by the curved rotor blades, the air being discharged from the rotor duct in a direction indicated by the arrow V_{2x} (relative to the rotor). The air discharged from the rotating rotor ducts enters the stationary ducts formed by the exit stator blades and the direction of flow into the exit stator is represented by the arrow V_{3n} which represents the vector sum of the vector velocity V_{2x} and the rotor circumferential velocity u.

The air entering the exit stator in the direction indicated by the arrow V_{3n} flows parallel to the exit stator blades at this point, thus providing streamlined flow into the exit stator. The air entering the exit stator may be said to have a circumferential velocity component V_{3nc} and an axial component V_{3na}.

Five independent conditions determine the construction of the supersonic compressor, as follows:

1. The flight Mach number of the compressor or M_0. This number indicates the ratio of the velocity of the compressor relative to the air it is compressing, to the local velocity of sound in the still air. It thus is dependent upon the flight speed of the compressor V_0 and the temperature of the still air. The pressure of the still air may be assumed to be 1 atmosphere in all cases, because if the compressor increases this pressure to any other value, then this initial pressure, differing from 1, will be increased in the same proportion by the compressor.

If the compressor is at rest in still air, the value of M_0 is zero. If M_0 is zero, then T_0 is T_{T1} and P_0 is P_{T1}. If, actually the compressor has a relative velocity with respect to the air, then M_0 is not zero, and

$$T_{T1}=T_0+\frac{\overline{V_0}^2}{K}$$

and

$$P_{T1}=P_0\left(\frac{T_{T1}}{T_0}\right)^{3.5}$$

2. The strength of the constant standing shock M_{2s} wave at or near the entrance to the rotor duct. M_{2s} is preferably between 1.2 and 1.5.

3. The angle α_{2n} (which is α_{2s} at the root). α_{2n} is preferably between 10° and 30° at the root.

4. The ratio of the area of the exit of the rotor duct at the root to the area of the entrance to the rotor duct at the root; i. e.,

$$\frac{A_{2x}}{A_{2n}}$$

This value is chosen at values low enough to give good subsonic diffuser efficiencies. For example, values between 1.5 and 2.5 are appropriate.

Since the areas are proportional to the sines of the angles the axis of the duct makes with a plane normal to the axis of the compressor, this condition amounts to

$$\frac{A_{2x}}{A_{2n}}=\frac{\sin\alpha_{2x}}{\sin\alpha_{2n}}$$

and since α_{2n} is given then α_{2x} may be determined.

5. The ratio of the area of the exit of the exit stator at the root to the area of the entrance to the exit stator at the root, i. e.

$$\frac{A_{3x}}{A_{3n}}$$

$$\frac{A_{3x}}{A_{3n}}=\frac{\sin\alpha_{3x}}{\sin\alpha_{3n}}=\frac{\sin 90°}{\sin\alpha_{3n}}$$

and this determines α_{3n}. The ratio

$$\frac{A_{3x}}{A_{3n}}$$

is preferably between 1.5 and 2.5.

The above five conditions thus determine the following: M_0, M_{2s}, α_{2n}, α_{3x} and α_{3n}.

As hereinafter explained, the root rotor velocity u_r which will satisfy these data is given by the equation

$$\overline{u_r}^2=\frac{KT_{T1}}{\frac{\sin^2\alpha_{3n}}{\sin^2(\alpha_{2x}+\alpha_{3n})}\cdot\frac{1+\beta_{2x}}{\beta_{2x}}-\frac{2\sin\alpha_{3n}\cos\alpha_{2n}}{\sin(\alpha_{2x}+\alpha_{3n})}\cdot A+1}$$

where

$$A^2=\frac{1+\beta_{2x}}{\beta_{2x}}\cdot\frac{\beta_{2n}}{1+\beta_{2n}}$$

and

$$\beta=\frac{\Delta T}{T}=\frac{\frac{\overline{V}^2}{K}}{T}$$

as hereinafter explained.

All velocities, temperatures, pressures and flow Mach numbers are determinate as soon as u_r is found.

The total temperature T_{T1} at the entrance to the entrance stator is

$$T_{T1}=T_0+\frac{\overline{V_0}^2}{K}$$

$$\text{If } V_0=0 \quad T_{T1}=T_0 \text{ (say } 520°R)$$

The above equation is derived from the conservation of energy equation, sometimes called Bernoullis' equation, where the constant

$$K=2J_gC_p=12000$$
$$=5k_gR$$

J being equal to 778, the mechanical equivalent of heat in foot pounds per B. t. u.; g equals 32.2 the acceleration of gravity in feet per second per second, and C_p equals .24 which is the specific heat of air at constant pressure in B. t. u.'s per ° F. per lb. of air. k is the ratio of the specific heats of air, viz.

$$\frac{C_p}{C_v}$$

the specific heat at constant pressure divided by the specific heat at constant volume. R is the gas constant for 1 lb. of air and is equal to 53.3, as used in the gas equation

$$Pv=RT$$

where v is the specific volume of the air in cubic feet per pound and P is the pressure in pounds per square foot. The total pressure P_{T1} is found from the adiabatic relation between pressure and temperature

$$P_{T1}=P_0\left(\frac{T_{T1}}{T_0}\right)^{3.5}$$

where

$$3.5=\frac{k}{k-1}\text{ when }k=1.400$$

The circumferential velocity component of the air leaving the entrance stator is

$$V_{1xc}=\frac{(T_{T2}-T_{T1})K}{2u}-\frac{u}{2} \qquad \text{(A)}$$

This can be proved by combining the general energy equations

$$T_{T2}=T_{1x}+\frac{\overline{V^2}_n}{K} \qquad (1)$$

and

$$T_{T1}=T_{1x}+\frac{\overline{V_{1x}}^2}{K} \qquad (2)$$

By subtraction

$$(T_{T2}-T_{T1})K=\overline{V_{2n}}^2-\overline{V_{1x}}^2 \qquad (3)$$

With the geometry in Fig. 1

$$\overline{V_{2n}}^2=\overline{V_{1x}}^2+\overline{u}^2+2uV_{1x}\cos\alpha_{1x} \qquad (4)$$

and

$$V_{1x}\cos\alpha_{1x}=V_{1xc} \qquad (5)$$

so that

$$\overline{V_{2n}}^2=\overline{V_{1x}}^2+\overline{u}^2+2uV_{1xc} \qquad (6)$$

Substituting in (3)

$$(T_{T2}-T_{T1})K=\overline{u}^2+2uV_{1xc}$$

Solving for V_{1xc} gives Equation A above.

Likewise, the circumferential velocity component of the air entering the exit stator is

$$V_{3nc}=\frac{(T_{T3}-T_{T2})K}{2u}+\frac{u}{2} \qquad (B)$$

Adding the two Equations A and B we find

$$uV_{1xc}+uV_{3nc}=\frac{(T_{T3}-T_{T1})K}{2} \qquad (C)$$

or

$$V_{1xc}\left(1+\frac{V_{3nc}}{V_{1xc}}\right)=\frac{(T_{T3}-T_{T1})K}{2u} \qquad (C')$$

It follows from Equation C that according to our invention, the sum

$$uV_{1xc}+uV_{3nc}$$

is a constant for any predetermined values of T_{T1} and T_{T3}, this sum being exactly commensurate with the difference between the total temperature of the outgoing air T_{T3} and the total temperature of the entering air T_{T1}. Thus to obtain any given compression as indirectly measured by the value of T_{T3} there are a multiplicity of values of V_{1xc} and V_{3nc} which will satisfy the requirements. If V_{1xc} is small V_{3nc} is large and vice-versa.

The above conditions determine the ratio

$$\frac{V_{1xc}}{V_{3nc}}$$

In a supersonic compressor the value of V_{1xc} is greater than V_{3nc}, and the ratio

$$\frac{V_{1xc}}{V_{3nc}}$$

has a high value. Since

$$\frac{(T_{T3}-T_{T1})K}{2}$$

is a measure of the energy imparted to the air, it follows that the terms of its equal as in Equation C are measures of the energy imparted to the air also. The term uV_{1xc} is the energy relation at the rotor entrance and uV_{3nc} that at the rotor exit.

To have the same energy imparted to the fluid entering the rotor from root to tip, we must have

$$uV_{1xc}\Big|_r^t=C_1 \text{ (A constant)}$$

and

$$uV_{3nc}\Big|_r^t=C_2 \quad \text{constant)}$$

Thus

$$\frac{uV_{1xc}}{uV_{3nc}}\Bigg|^t=\frac{V_{1xc}}{V_{3nc}}\Bigg|^t=C_3 \text{ (A constant)}$$

The air entering the compressor is vortex-free. If the flow of air is vortex-free, i. e., irrotational (assuming an ideal compressible fluid) it remains vortex-free throughout its motion. This fact leads to the conditions that

$$uV_{1xc}$$

must be constant from root to tip and likewise

$$uV_{3nc}$$

must be constant from root to tip for ideal adiabatic flow.

The weight flow of air as it enters the entrance to the entrance stator or pounds of air per square foot per second is constant from root to tip. Since the air flow across the compressor is vortex-free there will be no cross flow, otherwise this condition would be violated, and the weight flow computed at any station of any duct must show the same value of weight flow as at the entrance to the compressor.

A compressor designed to ensure this condition will compress the fluid with greater efficiency than any other design. This is true of the subsonic compressor as well as the supersonic compressor.

A standing shock wave of given strength may be stably held at any desired section of a diffuser such as the rotor ducts of our design. Its position is controlled by the pressure P_{3x}, the discharge pressure of the compressed air. On the other hand, a standing shock wave of given strength cannot be held stably in a nozzle by the control of the exit pressure. It suddenly pops out the larger end of the duct. In a compressor this means that if a portion or all of each rotor passage forms a nozzle in which a shock wave is produced, its location would not be stable and it would pop out and be dissipated.

The calculations for flow through nozzles and diffusers, whether subsonic or supersonic, are based on the following considerations:

Bernoullis equation for adiabatic flow between two stream tube stations 1 and 2 may be expressed as

$$T_1+\frac{\overline{V_1}^2}{5kgR}=T_2+\frac{\overline{V_2}^2}{5kgR}$$

where

$$\frac{V^2}{5kgR}$$

is the impact pressure rise of a compressible fluid when its velocity V is arrested.

The above equation shows that the total temperature

$$T_{T2}=T_2+\frac{\overline{V_2}^2}{K}$$

is the same as the total temperature

$$T_{T1}=T_1+\frac{\overline{V_1}^2}{K} \qquad (14)$$

We may write the equation further as

$$T_1+\Delta T_1=T_2+\Delta T_2 \qquad (15)$$

where

$$\Delta T=\frac{\overline{V}^2}{K} \qquad (16)$$

or

$$T_1\left(1+\frac{\Delta T_1}{T_1}\right)=T_2\left(1+\frac{\Delta T_2}{T_2}\right) \qquad (17)$$

or

$$T_{T1}=T_{T2} \qquad (18)$$

We define a quantity β as

$$\beta=\frac{\Delta T}{T}=\frac{\frac{\overline{V}^2}{K}}{T} \qquad (19)$$

so that (17) becomes

$$T_1(1+\beta_1)=T_2(1+\beta_2)=T_T \qquad (20)$$

Since the Mach number of flow M is defined by

$$M^2=\frac{V^2}{a^2} \qquad (21)$$

where a is the local velocity of sound, and

$$a^2=kgRT=\frac{KT}{5} \qquad (22)$$

it follows that

$$M^2=\frac{V^2}{kgRT}=5\frac{\frac{\overline{V}^2}{K}}{T}=5\beta \qquad (23)$$

The adiabatic relation between temperatures and pressures is

$$\frac{P_1}{P_2}=\left(\frac{T_1}{T_2}\right)^{3.5} \qquad (24)$$

giving

$$P_1(1+\beta_1)^{3.5}=P_2(1+\beta_2)^{3.5} \qquad (25)$$

The air velocity V_1 is related to T_T and T_1 by Bernoullis equation

$$V_1=\sqrt{K(T_T-T_1)} \qquad (26)$$

as seen from (14).

The continuity equation for equal weight flow through the two sections 1 and 2 is

$$w=A_1V_1\rho_1=A_2V_2\rho_2 \qquad (27)$$

where A_1 is the entrance area, A_2 the exit area and ρ_1 and ρ_2 the weight densities at the entrance and exit respectively. The general gas equation gives

$$\rho_1=\frac{P_1}{RT_1} \text{ and } \rho_2=\frac{P_2}{RT_2}$$

which substituted in (27) gives

$$w=\frac{A_1V_1P_1}{RT_1}=\frac{A_2V_2P_2}{RT_2}=\frac{A_3V_3P_3}{RT_3} \text{ etc.} \qquad (28)$$

While it is always true that the total temperature does not change throughout the flow, it is not true that the total pressure is consant throughout the flow. For ideal flow

$$P_{T2}=P_{T1}$$

but for non-ideal flow, i. e., flow with friction and/or with a shock wave existing in the tube

$$P_{T2}<P_{T1}$$

In any case, if the tube efficiency is known and has a value e then

$$P_{T2}=eP_{T1}+(1-e)P_1 \qquad (29)$$

If there is a shock of known strength, then

$$\frac{P_{T2s}}{P^1_{T2s}}$$

is purely a function of the shock strength alone.

If Equations 20, 25, and 26 are substituted in 28 and obvious simplifications made, there results, in general

$$A_1P_{T1}\frac{\beta_1^{1/2}}{(1+\beta_1)^3}=A_2P_{T2}\frac{\beta_2^{1/2}}{(1+\beta_2)^3} \qquad (30)$$

We now define ϕ, a function of β alone, as

$$\phi=\frac{\beta^{1/2}}{(1+\beta)^3}$$

then (30) may be written

$$A_1P_{T1}\phi_1=A_2P_{T2}\phi_2 \qquad (31)$$

As mentioned above, if the flow is ideal and without shock (31) becomes

$$A_1\phi_1=A_2\phi_2 \qquad (32)$$

It is convenient to employ a table of the function ϕ versus β for finding β_2 where A_1, P_{T1}, β_1 and A_2 are known. For example, in such a table, if

β=.09050 then ϕ=.23198

β=.14970 then ϕ=.25460

β=.10943 then ϕ=.24225

As soon as β_2 is determined, then

$$T_2=\frac{T_{T2}}{(1+\beta_2)} \qquad (33)$$

$$P_2=\frac{P_{T2}}{(1+\beta_2)^{3.5}} \qquad (34)$$

$$V_2=\sqrt{K(T_{T2}-T_2)}=\sqrt{KT_{T2}\frac{\beta_2}{1+\beta_2}} \qquad (35)$$

The values thus obtained give the same w at entry and exit, viz.,

$$w\ (\text{entry})=\frac{A_1V_1P_1}{RT_1}=\frac{A_2V_2P_2}{RT_2} \qquad (36)$$

The equation for u_r is derived for the root duct when the five conditions mentioned above are assigned. These conditions are $M_0=0$, M_{2s}, α_{2n}, α_{2x}, and α_{3n}. ($\alpha_{1n}=90°$ and $\alpha_{3x}=90°$.)

Since

$$\overline{M}^2_{2s}=5\beta_{2s}, \beta_{2s}$$

is known, and ϕ_{2s} is known.

According to shock wave theory, the β'_{2s} just after shock is related to the β_{2s} just before shock by the relation

$$\beta'_{2s}=\frac{1+\beta_{2s}}{35\beta_{2s}-1} \qquad (37)$$

so that β'_{2s} is known and hence ϕ'_{2s}.

Equation 32 then gives for ideal flow from aft of shock wave at rotor root entrance to rotor root exit.

$$\sin\alpha_{2s}\phi'_{2s}=\sin\alpha_{2x}\phi_{2x}$$

and

$$\phi_{2x}=\frac{\sin\alpha_{2s}}{\sin\alpha_{2x}}\phi'_{2s} \qquad (38)$$

We thus know β_{2x} (from β–ϕ table) and all air conditions at the rotor exit are found from (33) (34) and (35).

The geometry of Fig. 1 shows that

$$V_{2x}=u\,\frac{\sin\alpha_{3n}}{\sin(\alpha_{2x}+\alpha_{3n})} \qquad (39)$$

We have also, using Bernoullis theorem as in (35)

$$T_{T2}=\frac{\overline{V_{2x}}^2}{K}\,\frac{1+\beta_{2x}}{\beta_{2x}} \qquad (40)$$

and

$$T_{T2}=\frac{\overline{V_{2n}}^2}{K}\,\frac{1+\beta_{2s}}{\beta_{2s}} \qquad (41)$$

Equating (40) and (41) and replacing V_{2x} by (39) we find

$$\overline{V_{2n}}^2=\overline{u}^2\,\frac{\sin^2\alpha_{3n}}{\sin^2(\alpha_{2x}+\alpha_{3n})}\cdot\frac{1+\beta_{2x}}{\beta_{2x}}\cdot\frac{\beta_{2s}}{1+\beta_{2s}}$$

$$=\overline{u}^2\,\frac{\sin^2\alpha_{3n}}{\sin^2(\alpha_{2x}+\alpha_{3n})}\times A^2$$

where

$$A^2=\frac{1+\beta_{2x}}{\beta_{2x}}\,\frac{\beta_{2s}}{1+\beta_{2s}}$$

But the geometry also shows that, remembering that at the root $\alpha_{2n}=\alpha_{2s}$

$$V_{2nc}=V_{2n}\cos\alpha_{2n}$$

$$=u\frac{\sin\alpha_{3n}}{\sin(\alpha_{2x}+\alpha_{3n})}A\cos\alpha_{2n}$$

and

$$V_{1xc}=V_{2nc}-u$$

so that

$$V_{1xc}=u\,\frac{\sin\alpha_{3n}\cos\alpha_{2n}}{\sin(\alpha_{2x}+\alpha_{3n})}A-u \qquad (42)$$

Using Equation A as demonstrated in the preceding,

$$V_{1xc}=\frac{T_{T2}K}{2u}-\frac{T_{T1}K}{2u}-\frac{u}{2} \qquad (A)$$

Using Equations 39 and 40 in (A) it becomes

$$V_{1xc}=\bar{u}^2\,\frac{\sin^2\alpha_{3n}}{\sin^2(\alpha_{2x}+\alpha_{3n})}\,\frac{(1+\beta_{2x})}{2u\beta_{2x}}-\frac{T_{T1}K}{2u}-\frac{u}{2} \qquad (43)$$

Equating (41) and (43) and solving for $\bar{u}^2$ we have finally

$$\bar{u}^2_{root}=\frac{KT_{T1}}{\frac{\sin^2\alpha_{3n}}{\sin^2(\alpha_{3x}=\alpha_{3n})}\,\frac{1+\beta_{2x}}{\beta_{2x}}-\frac{2\sin\alpha_{3n}\cos\alpha_{2n}}{\sin(\alpha_{2x}+\alpha_{3n})}A+1} \qquad (44)$$

in which everything is known. This value of u_r is the only rotor velocity at the root which will satisfy the five conditions. For supersonic compressors $M_{2s}>1$ and β_{2s} *is* $>.2$.

A numerical example will clarify the procedure for finding all conditions in the root duct. The conditions are

$$M_0=0 \text{ and } T_{T1}=T_0=520°\text{ R.}$$

$$M_{2s}=1.400$$

$$\alpha_{2n}=25°$$

$$\frac{A_{2x}}{A_{2n}}=2$$

and

$$\frac{A_{3x}}{A_{3n}}=2,\text{ hence }\alpha_{3n}=30°$$

$$\sin\alpha_{2n}=\sin 25°=.42262$$

$$\sin\alpha_{2x}=2\sin\alpha_{2n}=.84524$$

$$\alpha_{2x}=57.698°$$

$$\cos\alpha_{2x}=.90631$$

$$\sin(\alpha_{2x}+\alpha_{3n})=\sin 87.698°=.99919$$

$$\sin^2(\alpha_{2x}+\alpha_{3n})=.99838$$

$$\sin\alpha_{3n}=\tfrac{1}{2}\sin^2\alpha_{3n}=\tfrac{1}{4}$$

$$M_{2s}=1.4$$

$$\beta_{2s}=\sqrt{\frac{M_{2s}^2}{5}}=.39200$$

$\phi_{2s}=.23213$ from β–ϕ table

$\beta'_{2s}=.10943$

from shock Equation 37.

Then $\phi'_{2s}=.24225$ from β–ϕ table. Equation 38 gives

$$\phi_{2x}=\tfrac{1}{2}\phi'_{2s}=.12113$$

$\beta_{2x}=.016152$ from β–ϕ table

$$A=\sqrt{\frac{1+\beta_{2x}}{\beta_{2x}}\cdot\frac{\beta_{2s}}{1+\beta_{2s}}}=4.2092$$

$$\frac{1+\beta_{2x}}{\beta_{2x}}=62.912$$

Substituting the values in (44)

$$\bar{u}^2_{root}=482410$$

$$u_{root}=694.56$$

From (39)

$$V_{2x}=u\,\frac{\sin\alpha_{3n}}{\sin(\alpha_{2x}+\alpha_{3n})}=347.57$$

From (40)

$$T_{T2}=\frac{\bar{V}^2_{2x}}{K}\cdot\frac{1+\beta_{2x}}{\beta_{2x}}=633.31$$

$$V_{2n}=\sqrt{KT_{T2}\frac{\beta_{2s}}{1+\beta_{2s}}}=1462.9$$

$$T_{1x}=\frac{T_{T2}}{1+\beta_{2s}}=454.96$$

As a check

$$M_{2s}=M_{2n}=\frac{1462.9}{2400\times 454.96}=1.400$$

as assumed above.

$$V_{2nc}=V_{2n}\cos\alpha_{2n}=1325.9$$

$$V_{1xc}=V_{2nc}-u_r=631.34$$

$$V_{1xa}=V_{2n}\sin\alpha_{2n}=618.25$$

$$\tan\alpha_{1x}=\frac{V_{1xa}}{V_{1xc}}=.97927$$

$$\alpha_{1x}=44.40°$$

$$V_{1x}=\frac{V_{1xa}}{\sin\alpha_{1x}}=883.63$$

$$\beta_{1x}=\frac{T_{T1}-T_{1x}}{T_{1x}}=.14296$$

$$M_{1x}=\sqrt{5\beta_{1x}}=.84543$$

$$P_{1x}=P_{T1}\left(\frac{T_{1x}}{T_{T1}}\right)^{3.5}=.62651$$

$$w_{(1x)}=\frac{V_{1xa}P_{1x}}{RT_{1x}}=33.799$$

$$P_{T2s}=P_{T1}\left(\frac{T_{T2}}{T_{T1}}\right)^{3.5}=1.9937$$

$$P'_{T2s}=.9582\times P_{T2s}=1.9104$$

This factor .9582 comes from the shock for

$$\beta_{2s}=.3920$$

The formula is

$$\frac{P'_T\text{ (after sh.)}}{P_T\text{ (before sh.)}}=\left(\frac{6}{35\beta-1}\right)^{5/2}\left(\frac{6\beta}{1+\beta}\right)^{7/2} \qquad (45)$$

a function of shock strength only.

$$P'_{2s}=\frac{P'_{T2s}}{(1+\beta'_{2s})^{3.5}}=1.3286$$

$$P_{2x}=\frac{P'_{T2s}}{(1+\beta_{2x})^{3.5}}=1.8062$$

$$T_{2x}=\frac{T_{T2}}{(1+\beta_{2x})}=623.24$$

$$V_{3n}=\frac{u_r\sin\alpha_{2x}}{\sin(\alpha_{2x}+\alpha_{3n})}=587.54$$

$$T_{T3}=T_{2x}+\frac{\bar{V}^2_{3n}}{K}=652.01$$

$$\beta_{3n}=\frac{\frac{V_{3n}^2}{K}}{T_{2x}}=.04616(T_{2x}=T_{3n})$$

$$M_{3n}=.4804$$

$$P_{T3}=P'_{T2s}\left(\frac{T_{T3}}{T_{T2}}\right)^{3.5}=2.1153$$

If the Equations 20, 25, and 26 are substituted in Equation 28

$$w=\frac{A_3V_3P_3}{RT_3}$$

$$\frac{wR\sqrt{T_{T3}}}{\sqrt{K}P_{T3}}=A_3\phi_{3x}=\sin\alpha_{3x}\phi_{3x} \quad (46)$$

$$=\sin 90\,\phi_{3x}=\phi_{3x}=.09382$$

using the values of w, T_{T3}, P_{T3}, found above. Then from the (β–ϕ) table we have

$$\beta_{3x}=.009305$$

and

$$(1+\beta_{3x})^{3.5}=1.03285$$

so that

$$P_{3x}=\frac{P_{T3}}{(1+\beta_{3x})^{3.5}}=2.048$$

$$T_{3x}=\frac{T_{T3}}{1+\beta_{3x}}=646.00$$

and

$$V_{3x}=\sqrt{K(652.01-646.00)}$$
$$=268.55$$

The air enters the entrance stator axially and leaves the exit stator axially and accordingly $\alpha_{1n}=90°$ and $\alpha_{3x}=90°$.

Thus in the foregoing example, the root angles are as follows:

$$\alpha_{1n}=90°$$
$$\alpha_{1x}=44.40°$$
$$\alpha_{2n}=25°$$
$$\alpha_{2x}=57.698°$$
$$\alpha_{3n}=30°$$
$$\alpha_{3x}=90°$$

The rotor blades may be formed so that at the root a development of the blade shape is a segment of a circle of constant radius R, and

$$R=\frac{1}{\cos\alpha_{2n}-\cos\alpha_{2x}}$$

If the axial blade width is taken as unity, values b may be taken as the axial distance from entrance, $b=0$, to $b=1$, all for the root, we can find by the above formula the angles α_2 at any value of b, as

$$\cos\alpha_2=\cos\alpha_{2n}-\frac{b}{R}$$

From the β–ϕ relation, the ϕ'_2 belonging to any b or its corresponding angle α_2 we have

$$\phi'_2=\frac{\phi'_{2n}\sin\alpha_{2n}}{\sin\alpha_2}$$

The primes denote values downstream of the shock which is placed at the root duct entrance. The strength of this shock M_{2n} is given and hence β_{2n}.

From β_{2n} we find β'_{2n} as

$$\beta'_{2n}=\frac{1+\beta_{2n}}{35\beta_{2n}-1}$$

from shock wave formula.

From β'_{2n} we find ϕ'_{2n} from the β–ϕ table.

The relative velocity V'_2 is given by

$$V'_{2\,(rel)}=\sqrt{KT_{T2}\frac{\beta'_2}{1+\beta'_2}}$$

Then

$$V'_{2c(rel)}=V'_{2(rel)}\cos\alpha_2$$

and

$$V'_{2c(abs)}=V'_{2c(rel)}-u_r$$

A plot of these absolute whirl velocities at the root for any particular case may be made. For example, if

$$u_r=623.88 \text{ and } w=20.897$$

then at the entrance there is a positive value to the whirl velocity, and this value decreasing to zero at $b=.17710$, and becomes negative for larger values of b.

If we now consider a duct at a u larger than u_r, for example at a u corresponding with a radius larger than that of the root, the β–ϕ relation gives

$$\sin\alpha_{2s}=\frac{wR\sqrt{T_{T2(u)}}}{\sqrt{K}P_{T2(u)}\phi_{2s}}$$

Since

$$P_{T2\,(u)}=P_{T1}\left(\frac{T_{T2(u)}}{T_{T1}}\right)^{3.5}$$

and substituting this value in the above, we find

$$\sin\alpha_{2s}=\frac{wR\sqrt{T_{T1}}}{\sqrt{K}P_{T1}\phi_{2s}}\left(\frac{T_{T1}}{T_{T2u}}\right)^3$$

Since

$$T_{T2(u)}=T_{T2\,(root)}-\frac{\bar{u}_r^2}{K}+\frac{\bar{u}^2}{K}$$

we find $\sin\alpha_{2s}$ and hence α_{2s} and $\cos\alpha_{2s}$.

We then have

$$V_{2s(u)\,(rel)}=\sqrt{K\,T_{T2(u)}\frac{\beta_{2s}}{1+\beta_{2s}}}$$

$$V_{2sc(u)\,(rel)}=V_{2s(u)\,(rel)}\cos\alpha_{2s}$$

This is the value in front of the shock in the u-duct. The value behind the shock is found from the shock wave formula for velocities behind (V′) when the velocity in front (V) is given. For a normal shock this applies also to all components of the velocity.

$$\frac{V'}{V}=\frac{1+\beta}{6\beta}$$

giving

$$V'_{2sc(u)\,(rel)}=\frac{1+\beta_{2s}}{6\beta_{2s}}V_{2sc(u)\,(rel)}$$

Then

$$V'_{2sc(u)\,(abs)}=V'_{2sc(u)\,(rel)}-u$$

The vortex-free condition requires that this absolute whirl velocity in the u-duct be situated directly over an absolute whirl velocity in the root duct of amount

$$V'_{2c\,(r)\,(abs)}=\frac{V'_{2sc(u)\,(abs)}\,xu}{u_r}$$

By referring to the plot of absolute whirl velocities we find for what value of b this occurs.

For all other points on the rotor blade downstream from this Mach line location as found above for any u, the situation is the same as for the subsonic compressor, except that P'_{T2} behind the shock must be used instead of P_{T2} in front. This contingency does not occur in subsonic flow.

The circumferential velocity component of the air leaving the entrance stator is

$$V_{1xc(u)}=\frac{(T_{T2(u)}-T_{T1})K}{2u}-\frac{u}{2}$$

and knowing $T_{T2(u)}$ we can find $V_{1xc(u)}$.

The value of α_{1x} for any radial point having a rotor speed u is

$$\alpha_{1x}=\tan^{-1}\left\{\frac{KT_{T1}}{\bar{V}_{1xc}^2}\left(1-\frac{T_{1x}}{T_{T1}}\right)-1\right\}^{1/2}$$

where

$$\frac{T_{1x}}{T_{T1}}$$

is the larger positive root of

$$\left(\frac{T_{1x}}{T_{T1}}\right)^6-\left(\frac{T_{1x}}{T_{T1}}\right)^5\left(1-\frac{\overline{V_{1xc(u)}}^2}{K\,T_{T1}}\right)+\left(\frac{wR\sqrt{T_{T1}}}{\sqrt{K}\,P_{T1}}\right)^2=0 \qquad (G)$$

and solving for

$$\frac{T_{1x}}{T_{T1}}$$

we find $T_{1x(u)}=T_{2n(u)}$, then

$$\beta_{2n(u)}=\frac{T_{T2(u)}-T_{1x(u)}}{T_{1x(u)}}=\frac{\Delta T_{(u)}}{T_{1x(u)}}$$

and

$$M_{2n(u)}=\sqrt{5\beta_{2n(u)}}$$

If $M_{2n(u)}$ is slightly less than M_{2s} it shows that a small amount of supersonic acceleration must take place between entrance to rotor duct at u and the position of the standing shock wave in the same duct.

It is found that as higher u's are used, the values of M_{2n} decrease from M_{2s} and then increase again.

When the $M_{2n}=M_{2s}$ at a point B, we have the maximum usable blade length for vortex free flow. To find the u belonging to B, and the position of B along the axial width of the blade at the root we proceed as follows:

If $M_{2n}=M_{2s}$, then $\beta_{2n}=\beta_{2s}$, and T_{1x} for this point is given by

$$T_{1x}=\frac{T_{T2}}{1+\beta_{2s}}$$

Inserting this value of T_{1x} in the (G) formula we have

$$\left(\frac{1}{1+\beta_{2s}}\,\frac{T_{T2}}{T_{T1}}\right)^6-\left(\frac{1}{1+\beta_{2s}}\,\frac{T_{T2}}{T_{T1}}\right)^5\left(1-\frac{\overline{V_{1xc}}^2}{KT_{T1}}\right)+\left(\frac{wR\sqrt{T_{T1}}}{\sqrt{K}\,P_{T1}}\right)^2=0$$

We are using this equation to determine a u_{max} for which $M_{2n}=M_{2s}$ which occurs both at the root (A) and at the point (B). Let

$$Z=T_{T2\,(root)}-\frac{\overline{u_r}^2}{K}$$

then

$$T_{T2}=Z+\frac{\overline{u}^2}{K}$$

Let

$$\frac{V_{1xc}}{V_{3nc}}=S\Big|_{root}^{tip}$$

This value of S must be constant from root to tip and

$$V_{1xc}=\frac{S}{1+S}\,\frac{(T_{T3}-T_{T1})K}{2u}$$

If these quantities T_{T2}, V_{1xc}, are substituted in the above formula we shall find it to become

$$(a+x)^6-(a+x)^5\left(1-\frac{d}{x}\right)+c=0$$

where

$$\frac{1}{(1+\beta_{2s})}\,\frac{Z}{T_{T1}}=a$$

$$\frac{\overline{u}^2}{K\,T_{T1}}\,\frac{1}{(1+\beta_{2s})}=x$$

$$\left(\frac{wR\sqrt{T_{T1}}}{\sqrt{K}\,P_{T1}}\right)^2=c$$

$$\frac{\left[\frac{S}{2(1+S)}\left(\frac{T_{T3}}{T_{T1}}-1\right)\right]^2}{1+\beta_{2s}}=d$$

For example, let

$$T_{T1}=520$$

$$T_{T3}=628.48$$

Z, as found in root duct caluclation, and constant from root to tip,

$$=T_{T2(r)}-\frac{\overline{U_r}^2}{K}=597.90$$

$$S=\frac{V_{1xc}}{V_{3nc}}=\frac{749.22}{274.95}=2.7250$$

$$c=.012007$$

$$a=.84266$$

$$d=.0041113$$

The equation is then

$$(.84266+x)^6-(.84266+x)^5 1-\frac{.0041113}{x}+.01201=0$$

where

$$x=\frac{\overline{u}^2}{12000\times 520-1.3645}=\frac{\overline{u}^2}{8514500}$$

A few trials for u around 925 will give $u_t=923.6$ for

$$x=.100186$$

Then

$$T_{T2(tip)}=597.90+\frac{\overline{923.6}^2}{K}=668.99$$

$$V_{2sc\,(rel)\,tip}=\frac{(T_{T2(tip)}-T_{T1})K}{2u_{tip}}+=\frac{U_{tip}}{2}$$

$$=\frac{148.99\times 6000}{923.6}+461.8$$

$$=1429.69$$

$$V'_{2sc\,(rel)(tip)}=1429.69=\frac{1.3645}{2.1870}=892.00$$

$$V'_{2sc\,(abs)\,(tip)}=892.00-923.6-31.60$$

$$V'_{2c\,(abs)(root)}=\frac{-31.60\times 923.6}{623.88}=046.781$$

This value of $V'_{2c(abs)(root)}$ occurs at $b=.247$.

The Mach line then starts at $b=0$ at the root and ends at $b=.247$ at the maximum $u_{tip}=923.6$.

Fig. 2 of the drawings shows how the entrance to the rotor tip is displaced downstream with respect to the entrance to the root portion of the rotor. In Figs. 1 and 2 the line L indicates the point at which the fluid leaves the entrance stator and enters the rotor at the root and the line L′ indicates the corresponding point at the tip and in Fig. 2 it will be noted that these lines L and L′ are separated or displaced by an amount b. Under normal operating conditions the shock line, indicated by a wavy line normal to the rotor duct pasage in Figs. 1, 2 and 3, substantially intersects the edge of the rotor entrance both at the root and at the tip, and at intermediate points it is slightly displaced in the downstream direction from the rotor entrance by an amount sufficient to produce area ratio equal to

$$\frac{\sin\alpha_{2n}}{\sin\alpha_{2s}}$$

at the rate of convergence of the duct at the shock toward the rotor entrance. The preferred shape of the rotor blades, with their entrance tip portions displaced downstream with respect to their entrance root portions, is diagrammatically illustrated in Figs. 4 and 5, and Fig. 6 best illustrates in diagrammatic form the preferred configuration or shape of the entrance stator blades, showing how the tip portions thereof are displaced downstream with respect to the root portions thereof to accommodate the corresponding configuration of the rotor blades. Portions of the entrance stator blades designated by the reference characters 9 and 9' in Figs. 2 and 6 constitute straight duct sections having parallel walls defined by the entrance stator blades, these straight passages serving to interconnect those portions of the rotor ducts displaced radially outward from the root portion of the rotor with the corresponding converging or nozzle portions of the entrance stator ducts, these nozzle sections of the entrance stator ducts extending from the entrance to the entrance stator to a plane normal to the axis of rotation and coinciding with the line L indicating the point of separation between the root portion of the entrance stator exit and the root portion of the rotor entrance.

As soon as u_t has been determined the calculations for α_{1x}, α_{2n}, α_{2x}, α_{3n} may be made for any desired u equal to or less than u_t.

For any such u, the V_{1xc} for that value is given by the vortex-free condition

$$V_{1xc_{(u)}}=\frac{V_{1xc_r}\cdot u_r}{u}$$

Similarly, the

$$V_{3nc_{(u)}}=\frac{V_{3nc}r\cdot u_r}{h}$$

Again

$$T_{T2_u}=T_{T2_r}-\frac{\bar{u}_r^2}{K}+\frac{\bar{u}^2}{K}$$

We shall now derive equations defining the proper angles for all desired u's, or at any radius along the blades up to the radius corresponding to u_t.

We have (for air)

$$P_{1x}=P_{T1}\left(\frac{T_{1x}}{T_{T1}}\right)^{3.5}$$

the well known adiabatic relation between pressure and temperature.

$$\overline{V}_{1x}^2=K(T_{T1}-T_{1x})$$

$$=KT_{T1}\left(1-\frac{T_{1x}}{T_{T1}}\right)$$

By the geometry of Figs. 1 and 2

$$V_{1xa}=\sqrt{\overline{V}_{1x}^2-\overline{V}_{1xc}^2}$$

and we have

$$V_{1xa}\sqrt{KT_{T1}\left(1-\frac{T_{1x}}{T_{T1}}\right)-\overline{V}_{1xc}^2}$$

Using the continuity equation

$$w=\frac{\sin\alpha_{1x}V_{1x}P_{1x}}{RT_{1x}}=\frac{V_{1xa}P_{1x}}{RT_{1x}}$$

because

$$V_{1xa}=V_{1x}\sin\alpha_{1x}$$

Substituting the above values of P_{1x} and V_{1xa} in the above equation for w, squaring and rearranging, we have

$$\left(\frac{T_{1x}}{T_{T1}}\right)^6-\left(\frac{T_{1x}}{T_{T1}}\right)^5\left(1-\frac{\overline{V}_{1xc}^2}{KT_{T1}}\right)+\left(\frac{wR\sqrt{T_{T1}}}{\sqrt{K}P_{T1}}\right)^2=0 \qquad \text{(G)}$$

Also, since

$$\tan\alpha_{1x}=\frac{V_{1xa}}{V_{1xc}}$$

and dividing the above equation for V_{1xa} by V_{1xc} we have

$$\tan\alpha_{1x}=\left\{\frac{KT_{T1}}{\overline{V}_{1xc}^2}\left(1-\frac{T_{1x}}{T_{T1}}\right)-1\right\}^{1/2} \qquad \text{(F)}$$

where

$$\left(\frac{T_{1x}}{T_{T1}}\right)$$

is the larger positive real root of G, and hence α_{1x} is determined.

Similarly, we find

$$\tan\alpha_{2x}=\left\{\frac{KT_{T2}}{(u-V_{3nc})^2}\left(1-\frac{T_{2x}}{T_{T2}}\right)-1\right\}^{1/2} \qquad \text{(H)}$$

where

$$\left(\frac{T_{2x}}{T_{T2}}\right)$$

is the larger positive real root of

$$\left(\frac{T_{2x}}{T_{T2}}\right)^6-\left(\frac{T_{2x}}{T_{T2}}\right)^5\left(1-\frac{(U-V_{3nc})^2}{KT_{T2}}\right)+\left(\frac{wR\sqrt{T_{t1}}}{\sqrt{K}EP_{T1}}\right)^2\left(\frac{T_{T1}}{T_{T2}}\right)^6=0 \qquad \text{(J)}$$

by use of

$$PT_2=T_{T1}\left(\frac{T_{T2}}{T_{T1}}\right)^{3.5}$$

where

$$E=\frac{P'_{T2s}}{P_{T2s}}=\left(\frac{6}{35\beta_{2s}-1}\right)^{5/2}\left(\frac{6\beta_{2s}}{1+\beta_{2s}}\right)^{7/2}$$

and α_{2x} is thus determined.

Again, in a similar manner, we find

$$\tan\alpha_{2n}=\left\{\frac{KT_{T2}}{(u+V_{1xc})^2}\left(1-\frac{T_{1x}}{T_{T2}}\right)-1\right\}^{1/2} \qquad \text{(K)}$$

where

$$\left(\frac{T_{1x}}{T_{T2}}\right)$$

is the smaller positive real root of

$$\left(\frac{T_{1x}}{T_{T2}}\right)^6-\left(\frac{T_{1x}}{T_{T2}}\right)^5\left(1-\frac{(u+V_{1xc})^2}{KT_{T2}}\right)+\left(\frac{wR\sqrt{T_{T1}}}{\sqrt{K}P_{T1}}\right)^2\left(\frac{T_{T1}}{T_{T2}}\right)^6=0 \qquad \text{(L)}$$

and α_{2n} is determined.

We have also

$$\tan\alpha_{3n}=\left\{\frac{KT_{T3}}{\overline{V}_{3nc}^2}\left(1-\frac{T_{2x}}{T_{T3}}\right)-1\right\}^{1/2} \qquad \text{(M)}$$

where

$$\frac{T_{2x}}{T_{T3}}$$

is the larger positive real root of

$$\left(\frac{T_{2x}}{T_{T2}}\right)^6-\left(\frac{T_{2x}}{T_{T3}}\right)^5\left(1-\frac{\overline{V}_{3nc}^2}{KT_{T3}}\right)+\left(\frac{wR\sqrt{T_{T1}}}{\sqrt{K}P_{T1}E}\right)^2\left(\frac{T_{T1}}{T_{T3}}\right)^6=0 \qquad \text{(N)}$$

and α_{3n} is determined.

The angle α_{1x} and the corresponding rotor angle α_{2n} producing streamlined flow into the rotor, are seen to be related by the geometrical equation (see Figs. 1 and 2):

$$\tan\alpha_{2n}=\frac{V_{1xc}}{u+V_{1xc}}\tan\alpha_{1s} \qquad (65)$$

$$=\frac{V_{2nc}-u}{V_{2nc}}\tan\alpha_{1x}$$

and also

$$\tan\alpha_{2x}=\frac{V_{3nc}}{u-V_{3nc}}\tan\alpha_{3n} \qquad (66)$$

$$=\frac{u-V_{2xc}}{V_{2xc}}\tan\alpha_{3n}$$

In the equations G, J, L, and N, the exponents are simple whole numbers (6, and 5) because of the value used for $k=1.4$. However, if k is not equal to 1.4 there will be a similar equation with exponents

$$\frac{k+1}{k-1} \qquad (67)$$

instead of 6 and

$$\frac{2}{k-1} \qquad (68)$$

instead of 5

All of these equations G, J, L, and N are of the form

$$x^6-x^5b+c=0$$

b is a quantity slightly less than 1 and x is a quantity less than 1.

There are four consecutive missing terms in the equation and accordingly there are two pairs of conjugate imaginary roots, and there are two positive roots; one larger than

$$\frac{5}{6}$$

for equation G, and one smaller than

$$\frac{5}{6}$$

A close approximation to the root of any of these equations is found from

$$x\ (\text{approx.})=\frac{b-6c}{1-5c}$$

A trial or two will give an exact solution. For example,

$$x^6-x^5(.98165)+.01000=0$$

has an approximate root

$$x\ (\text{approx.})=\frac{.98165-.06}{1-.05}=.97017$$

The exact root found by a trial is

$$x(\text{exact})=.97001$$

The exact value of the exponents does not affect this approximation to any great extent.

It will be noted that two of these equations G and N, are simpler to use than the ones containing T_{T2}, and are sufficient to determine all angles using Equations 65 and 66. The angles determined by these equations; viz., $\alpha_{1n}=90°$, α_{1x}, α_{2n}, α_{2_x}, α_{3n}, and $\alpha_{3x}=90°$ give the structure required and will insure a constant weight flow across the compressor from root to tip of the blade without cross flow and produce a pressure at the exit of the exit stator which is constant from root to tip.

All centrifugal forces and pressures of the air are in perfect balance as the total temperatures and total pressures satisfy everywhere the vortex-free conditions.

As explained above, the location of the shock wave in the rotor ducts is controlled by what may be called the back pressure on the shock wave or in other words the pressure obtaining in the rotor duct downstream of the shock wave. This back pressure can be varied by controlling or regulating the pressure downstream of the rotor, for example at the exit of the exit stator or in some portion of the compressor apparatus communicating with the exit to the exit stator. As an illustrative example we have shown in Fig. 8 a compressor apparatus or installation embodying our invention wherein a pressure control valve **10** of any well known type may be adjusted as desired to control the compressor back pressure, it being understood that the details of the valve construction constitute no part of this invention. In general, any adjustable valve mechanism or means for controlling the value of the pressure P_{3_x} either directly, or indirectly as by controlling the pressure downstream from the exit portion of the compressor exit stator, may be used for controlling or regulating the back pressure whereby the shock wave is located in the desired position near or at the entrance to the compressor rotor.

It is to be understood that our invention is not limited to the specific embodiments thereof described above in detail as illustrative examples of the invention but includes such modifications thereof as fall within the scope of the appended claims. In the appended claims the various angles specified, such as the entrance angle to the rotor α_{2n}, the angle in the rotor duct α_{2s}, the exit angle of the rotor blade α_{2_x}, the exit angle of the entrance stator α_{1x}, the entrance angle of the exit stator α_{3n} and the intermediate angles α_1, α_2 and α_3, are all angles with respect to a plane normal to the axis of the rotor.

We claim:

1. An axial flow fluid compressor comprising entrance stator blades, rotor blades and exit stator blades constituting stator and rotor units in closely related tandem relation, the rotor and stator blades being spaced apart and radially extending and forming fluid ducts extending from the root to the tip portions of the blades and extending in a generally axial direction through the stator and rotor blades, the blades of the entrance stator being shaped to allow axial flow of the fluid into the entrance stator, the exit stator blades being shaped to direct exit of the fluid axially, the rotor blades forming ducts diverging in the downstream direction and the entrance edge of the rotor blades being progressively displaced downstream from root to tip, wherein the entrance angle to the rotor α_{2n} is given by

$$\alpha_{2n}=\tan^{-1}\left\{\frac{KT_{T2}}{\overline{V}_{2nc}^{2}}\left(1-\frac{T_{1x}}{T_{T2}}\right)-1\right\}^{1/2}$$

where

$$\left(\frac{T_{1x}}{T_{T2}}\right)$$

is the smaller real positive root of

$$\left(\frac{T_{1x}}{T_{T2}}\right)^6-\left(\frac{T_{1x}}{T_{T2}}\right)^5\left(1-\frac{\overline{V}_{2nc}^{2}}{KT_{T2}}\right)+\left(\frac{wR\sqrt{T_{T1}}}{\sqrt{K}P_{T1}}\right)^2\left(\frac{T_{T1}}{T_{T2}}\right)^6=0$$

and α_{2s}, the angle in the rotor duct where a shock occurs, is given by

$$\alpha_{2s}=\sin^{-1}\left(\frac{wR\sqrt{T_{T1}}}{\sqrt{K}P_{T1}\phi_{2s}}\right)\left(\frac{T_{T1}}{T_{T2}}\right)^3$$

in which

$$\phi_{2s}=\frac{\beta_{2s}^{1/2}}{(1+\beta_{2s})^3}$$

and

$$\beta_{2s}=\frac{M_{2s}^2}{5}$$

M_{2s} being the strength of the shock, wherein α_{2s} equals α_{2n} at the root and at the tip of the rotor blade and α_{2s} is slightly greater than α_{2n} at all points between root and tip; wherein the exit angle of the rotor blade α_{2_x} is greater than α_{2n} and is given by

$$\alpha_{2x}=\tan^{-1}\left\{\frac{KT_{T2}}{\overline{V}_{2xc}^{2}}\left(1-\frac{T_{2x}}{T_{T2}}\right)-1\right\}^{1/2}$$

where

$$\left(\frac{T_{2x}}{T_{T2}}\right)$$

is the larger real positive root of

$$\left(\frac{T_{2x}}{T_{T2}}\right)^6-\left(\frac{T_{2x}}{T_{T2}}\right)^5\left(1-\frac{\overline{V}_{2xc}^{2}}{KT_{T2}}\right)+\left(\frac{wR\sqrt{T_{T1}}}{\sqrt{K}P_{T1}E}\right)^2\left(\frac{T_{T1}}{T_{T2}}\right)^6=0$$

in which E is the ratio of the value of the total pressure after the shock of strength M_{2s} to the value of the total pressure before the shock as found from the expression

$$E=\frac{P'_{T2s}}{P_{T2s}}=\left(\frac{6}{35\beta_{2s}-1}\right)^{5/2}\left(\frac{6\beta_{2s}}{1+\beta_{2s}}\right)^{7/2}$$

wherein the exit angle of the entrance stator α_{1x} is given by the slipless flow condition

$$\alpha_{1x}=\tan^{-1}\left(\frac{V_{2nc}}{V_{2nc}-u}\tan\alpha_{2n}\right)$$

and the entrance angle to the exit stator α_{3n} is given by the slipless flow condition

$$\alpha_{3n}=\tan^{-1}\left(\frac{V_{2xc}}{u-V_{2xc}}\tan\alpha_{2x}\right)$$

wherein

$$\frac{V_{1xc}}{V_{3nc}}=\frac{V_{2nc}-u}{u-V_{2xc}}=S$$

and

$$T_{T2}-\frac{\bar{u}^2}{K}=Z$$

S and Z being constant from root to top; wherein u_r and u_t are determined by the two real positive roots x_r and x_t of

$$(a+x)^6-(a+x)^5\left(1-\frac{d}{x}\right)+c=0$$

wherein

$$a=\frac{1}{1+\beta_{2s}}\,\frac{Z}{T_{T1}}$$

$$x=\frac{\bar{u}^2}{KT_{T1}}\cdot\frac{1}{1+\beta_{2s}}$$

$$c=\left(\frac{wR\sqrt{T_{T1}}}{\sqrt{K}P_{T1}}\right)^2$$

$$d=\frac{\left[\frac{S}{2(S+1)}\left(\frac{T_{T3}}{T_{T1}}-1\right)\right]^2}{1+\beta_{2s}}$$

thus giving

$$u_t=\sqrt{x_tKT_{T1}(1+\beta_{2s})}$$

and

$$u_r=\sqrt{x_rKT_{T1}(1+\beta_{2s})}$$

wherein the absolute subsonic whirl velocity $V'_{2sc(abs)}$ just after the shock at points along the shock line matches the absolute whirl velocity in the root duct $V'_{2cr(abs)}$, satisfying the relation

$$uV'_{2sc(abs)}=u_rV'_{2cr(abs)}$$

wherein the distance of edge of the rotor blade upstream from the shock line, is zero at the root and at the tip, and at intermediate points its amount is sufficient to produce an area ratio equal to

$$\frac{\sin\alpha_{2n}}{\sin\alpha_{2s}}$$

at the rate of convergence of the duct at the shock toward the rotor entrance; wherein the intermediate angles α_2 in the rotor from shock line to exit are determined by

$$\alpha_2=\tan^{-1}\left\{\frac{KT_{T2}}{\overline{V'}_{2c}^{2}}\left(1-\frac{T_2}{T_{T2}}\right)-1\right\}^{1/2}$$

where

$$\frac{T_2}{T_{T2}}$$

is the larger real positive root of

$$\left(\frac{T_2}{T_{T2}}\right)^6-\left(\frac{T_2}{T_{T2}}\right)^5\left(1-\frac{\overline{V'}_{1c}^{2}}{KT_{T2}}\right)+\left(\frac{wR\sqrt{T_{T1}}}{\sqrt{K}P_{T1}E}\right)^2\left(\frac{T_{T1}}{T_{T2}}\right)^6=0$$

and in which E is the

$$\frac{P'_{T2s}}{P_{T2s}}$$

for a shock of strength M_{2s} and

$$T_{T2}=T_{T2r}-\frac{\bar{u}_r^2}{K}+\frac{u^2}{K}=Z+\frac{\bar{u}^2}{K}$$

wherein the intermediate angles α_1 in the entrance stator from the entrance to points in a plane normal to the axis of rotation, at the exit of the entrance stator at the root, are given by

$$\alpha_1=\tan^{-1}\left\{\frac{KT_{T1}}{\bar{V}_{1c}^{2}}\left(1-\frac{T_1}{T_{T1}}\right)-1\right\}^{1/2}$$

where

$$\frac{T_1}{T_{T1}}$$

is the larger real positive root of

$$\left(\frac{T_1}{T_{T1}}\right)^6-\left(\frac{T_1}{T_{T1}}\right)^5\left(1-\frac{\bar{V}_{1c}^{2}}{KT_{T1}}\right)+\left(\frac{wR\sqrt{T_{T1}}}{\sqrt{K}P_{T1}}\right)^2=0$$

wherein the entrance stator ducts extend from said points toward the rotor without change in angle; and wherein the intermediate angles α_3 of the exit stator are given by

$$\alpha_3=\tan^{-1}\left\{\frac{KT_{T3}}{\bar{V}_{3c}^{2}}\left(1-\frac{T_3}{T_{T3}}\right)-1\right\}^{1/2}$$

where

$$\frac{T_3}{T_{T3}}$$

is the larger real positive root of

$$\left(\frac{T_3}{T_{T3}}\right)^6-\left(\frac{T_3}{T_{T3}}\right)^5\left(1-\frac{\bar{V}_{3c}^{2}}{KT_{T3}}\right)+\left(\frac{wR\sqrt{T_{T1}}}{\sqrt{K}P_{T1}E}\right)^2\left(\frac{T_{T1}}{T_{T3}}\right)^6=0$$

whereby the weight flow of air per unit cross-sectional area in any plane normal to the axis of rotation from the entrance to the exit of the compressor is constant and the flow is directed against the rotation of the rotor by the entrance stator in a vortex-free whirl and the speed of the rotor is such as to create a supersonic velocity of the entering air to the rotor relative to the rotor, and a normal shock wave of constant strength occurs near the entrance to the rotor, the shock wave being substantially at the entrance at the tip and root of the rotor and slightly downstream of the entrance to the rotor at intermediate points, and vortex-free flow occurs immediately following the shock in all planes normal to the axis of rotation from the entrance to the rotor to its exit, the flow being supersonic on entering the rotor and dropping to subsonic immediately after the shock line with a sudden increase in pressure and a continued increase in pressure by subsonic diffusion to the exit of the rotor, and the fluid enters the exit stator by slipless flow in a vortex-free whirl and continues to increase in pressure by subsonic diffusion until it leaves the exit stator axially at substantially constant pressure from root to tip.

2. An axial flow fluid compressor according to claim 1, in which the rotor entrance angle α_{2n} is between 10° and 30°, with the ratio

$$\frac{\sin\alpha_{2x}}{\sin\alpha_{2n}}$$

between 1.5 and 2.5 and the ratio

$$\frac{\sin\alpha_{3x}}{\sin\alpha_{3n}}$$

between 1.5 and 2.5 where α_{3x} is the exit angle of the exit stator, the compressor having during normal operation a stable normal shock wave in the rotor with a shock strength M_{2s} between 1.2 and 1.5.

3. An axial flow supersonic air compressor for opera-

tion at a speed such that the relative velocity of the air entering the rotor is supersonic, the compressor comprising stator blades, rotor blades, and exit stator blades constituting stator and rotor units in closely related tandem relation, the rotor and stator blades being spaced apart and radially extending and forming fluid ducts extending from the root to the tip portions of the blades and extending in a generally axial direction through the stator and rotor blades, the blades of the entrance stator being shaped to allow axial flow of the air into the entrance stator, the exit stator blades being shaped to direct exit of the air axially, the rotor blades forming ducts diverging in the downstream direction to provide a stable normal shock wave of constant strength from root to tip of the rotor blade substantially at the diverging duct entrances, with vortex-free flow through the compressor from root to tip in all planes normal to the axis of rotation from compressor entrance to exit, whereby a constant weight flow of air per unit area is obtained as it issues axially from the exit stator at a constant exit pressure from root to tip which locates the shock wave substantially at the rotor entrance, the rotor speed at the root of the rotor, u_r, being given by

$$\bar{u}_r^2=\frac{KT_{T1}}{\frac{\sin^2\alpha_{3n}}{\sin^2(\alpha_{2x}+\alpha_{3n})}\frac{1+\beta_{2x}}{\beta_{2x}}-\frac{2\sin\alpha_{3n}\cos\alpha_{2n}}{\sin(\alpha_{2x}+\alpha_{3n})}A+1}$$

where

$$A=\left(\frac{1+\beta_{2x}}{\beta_{2x}}\cdot\frac{\beta_{2s}}{1+\beta_{2s}}\right)^{1/2}$$

with the shock strength

$$\beta_{2s}=\frac{M_{2s}^2}{5}$$

and the angles α_{2x} and α_{3n} having values such that with an assumed value of α_{2n}, the area ratios at the root are

$$\frac{\text{rotor exit area}}{\text{rotor entrance area}}=\frac{\sin\alpha_{2x}}{\sin\alpha_{2n}}$$

and

$$\frac{\text{exit stator exit area}}{\text{exit stator entrance area}}=\frac{\sin\alpha_{3x}}{\sin\alpha_{3n}}$$

4. An axial flow fluid compressor comprising entrance stator blades, rotor blades, and exist stator blades constituting stator and rotor units in closely related tandem relation, the rotor and stator blades being spaced apart and radially extending and forming fluid ducts extending from the root to the tip portions of the blades and extending in a generally axial direction through the stator and rotor blades, the blades of the entrance stator being shaped to allow axial flow of the fluid into the entrance stator, the rotor blades being shaped to constitute a diverging duct to provide a stable normal shock wave from root to tip, the exit stator blades being shaped to direct exit of the fluid axially, the rotor having an entrance angle α_{2n} having the value of

$$\alpha_{2n}=\tan^{-1}\left\{\frac{KT_{T2}}{V_{2nc}^2}\left(1-\frac{T_{1x}}{T_{T2}}\right)-1\right\}^{1/2}$$

where

$$\frac{T_{1x}}{T_{T2}}$$

is the smaller real positive root of

$$\left(\frac{T_{1x}}{T_{T2}}\right)^6-\left(\frac{T_{1x}}{T_{T2}}\right)^5\left(1-\frac{V_{2nc}^2}{KT_{T2}}\right)+\left(\frac{wR\sqrt{T_{T1}}}{\sqrt{K}P_{T1}}\right)^2\left(\frac{T_{T1}}{T_{T2}}\right)^6=0$$

and

$$\alpha_{2s}=\sin^{-1}\left(\frac{wR\sqrt{T_{T1}}}{\sqrt{K}P_{T1}\phi_{2s}}\right)\left(\frac{T_{T1}}{T_{T2}}\right)^3$$

where $\alpha_{2s}=\alpha_{2n}$ at the root and the tip of the rotor blade and $\alpha_{2s}>\alpha_{2n}$ at the other portions of the rotor blade, the rotor having an exit angle α_{2x} having the value

$$\alpha_{2x}=\tan^{-1}\left\{\frac{KT_{T2}}{V_{2xc}^2}\left(1-\frac{T_{2x}}{T_{T2}}\right)-1\right\}^{1/2}$$

with $\alpha_{2x}>\alpha_{2s}>\alpha_{2n}$

where

$$\frac{T_{2x}}{T_{T2}}$$

is the larger real positive root of

$$\left(\frac{T_{2x}}{T_{T2}}\right)^6-\left(\frac{T_{2x}}{T_{T2}}\right)^5\left(\frac{1-V_{2xc}^2}{KT_{T2}}\right)+\left(\frac{wR\sqrt{T_{T1}}}{\sqrt{K}P_{T1}E}\right)^2\left(\frac{T_{T1}}{T_{T2}}\right)^6=0$$

with

$$\frac{V_{2nc}-u}{u-V_{2xc}}=S$$

constant from root to tip and

$$V_{2nc}-u=V_{1xc}=\frac{S}{1+S}\,\frac{(T_{T3}-T_{T1})K}{2u}$$

and

$$V_{2nc}=\frac{(T_{T2}-T_{T1})K}{2u}+\frac{u}{2}$$

and E being the ratio of total pressure after the shock to total pressure in front of the shock

$$E=\frac{P'_{T2s}}{P_{T2s}}$$

and the entrance stator having an exit angle α_{1x} varying progressively from root to tip and having the value

$$\alpha_{1x}=\tan^{-1}\left(\frac{V_{2nc}}{V_{2nc}-u}\tan\alpha_{2n}\right)$$

and the exit stator having an entrance angle α_{3n} varying progressively from root to tip and having the value

$$\alpha_{3n}=\tan^{-1}\left(\frac{V_{2xc}}{u-V_{2xc}}\tan\alpha_{2x}\right)$$

whereby streamlined flow of the fluid is provided as the fluid enters the rotor and as it enters the exit stator, the relative velocity of the fluid to the rotor being in excess of the speed of sound as the fluid enters the rotor, whereby the pressure in the rotor after the shock wave may be so controlled that the shock occurs substantially at the entrance to the rotor at the root and tip of the rotor blades and the strength of the shock is constant from root to tip, whereby vortex-free flow is maintained in all planes normal to the axis of the compressor, and whereby the velocity of the fluid as it leaves the rotor varies progressively from root to tip and consists of circumferential and axial components each varying progressively from root to tip, and the pressure of the fluid as it is directed axially by the exit stator is constant from root to tip.

5. An axial flow fluid compressor comprising entrance stator blades, rotor blades, and exit stator blades constituting stator and rotor units in closely related tandem relation, the rotor and stator blades being spaced apart and radially extending and forming fluid ducts extending from the root to the tip portions of the blades and extending in a generally axial direction through the stator and rotor blades, the blades of the entrance stator being shaped to allow axial flow of the fluid into the entrance stator, the rotor blades being shaped to form ducts diverging in the downstream direction to provide a shock wave of constant strength from root to tip with its extremities at the root and tip portions of the rotor entrance, with the intermediate portion of the shock wave downstream near the entrance of the rotor, the exit stator blades being shaped to direct exit of the fluid axially, the ratio of blade length to the tip radius of the rotor being equal to the ratio of the tip speed minus the root speed, to the tip speed, the tip speed and root speed

being derived from the two real positive roots x_r and x_t of the equation

$$(x+a)^6-(x+a)^5\left(1-\frac{d}{x}\right)+c=0$$

6. An axial flow supersonic air compressor comprising stator blades, rotor blades, and exit stator blades constituting stator and rotor units in closely related tandem relation, the rotor and stator blades being spaced apart and radially extending and forming fluid ducts extending from the root to the tip portions of the blades and extending in a generally axial direction through the stator and rotor blades, the blades of the entrance stator being shaped to allow axial flow of the air into the entrance stator, the exit stator blades being shaped to direct exit of the air axially, wherein the entrance edge of each rotor blade is gradually displaced downstream from root to tip, the absolute whirl velocities at all points just behind a shock line matching those in the rotor duct at the corresponding root points in a vortex-free manner such that at all values of u, from root to tip

$$(uV'_{2sc})_u=(uV'_{2c})_r$$

thus determining the position of the shock line from root to tip, with the distance between the shock line and the rotor entrance determined by the value of the supersonic Mach number of flow having at the rotor entrance the same value $M_{2n}=M_{2s}$ at the root and tip and smaller values at intermediate points, with the value of u at any point downstream, from the shock line to the exit edge of the rotor duct, determined by

$$(J)\left(\frac{T_2}{T_{T2}}\right)^6-\left(\frac{T_2}{T_{T2}}\right)^5\left(1-\frac{\overline{V_{2c}}^2}{KT_{T2}}\right)+\left(\frac{wR\sqrt{T_{T2}}}{\sqrt{K}\,P'_{T2}}\right)^2=0$$

and

$$\tan\alpha_2=\left\{\frac{KT_{T2}}{\overline{V_{2c}}^2}\left(1-\frac{T_2}{T_{T2}}\right)-1\right\}^{1/2}$$

where

$$\frac{T_2}{T_{T2}}$$

is the larger real positive root of (J) and P'_{T2} is the reduced value of P_{T2} in front of the shock as found from the shock wave equation

$$\frac{P'_{T2}}{P_{T2}}=\left(\frac{6}{35\beta_{2s}-1}\right)^{5/2}\left(\frac{6\beta_{2s}}{1+\beta_{2s}}\right)^{7/2}$$

and β_{2s} is

$$\frac{M_{2s}^2}{5}$$

and where

$$(T_{T2})_u=(T_{T2})_r-\frac{\overline{u_2}^2}{K}+\frac{u^2}{K}$$

whereby vortex-free flow resulting in a constant weight flow of air per unit area and a constant exit pressure P_{3x} are maintained from root to tip.

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