March 27, 1945. S. A. SCHELKUNOFF **2,372,228**

HIGH FREQUENCY TANKS AND RESONANT CAVITIES

Filed Dec. 9, 1939 2 Sheets-Sheet 1

FIG. 1

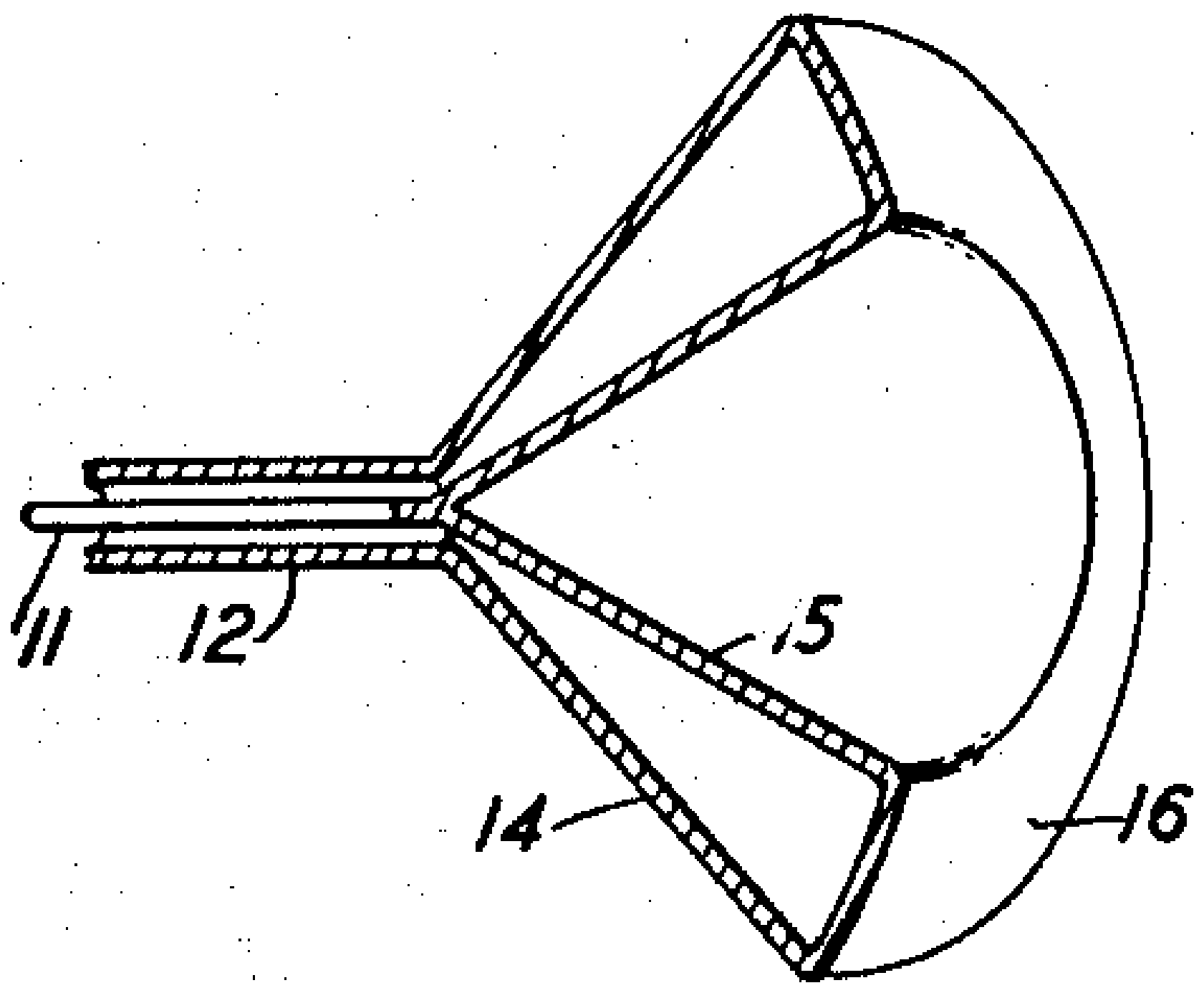


FIG. 2

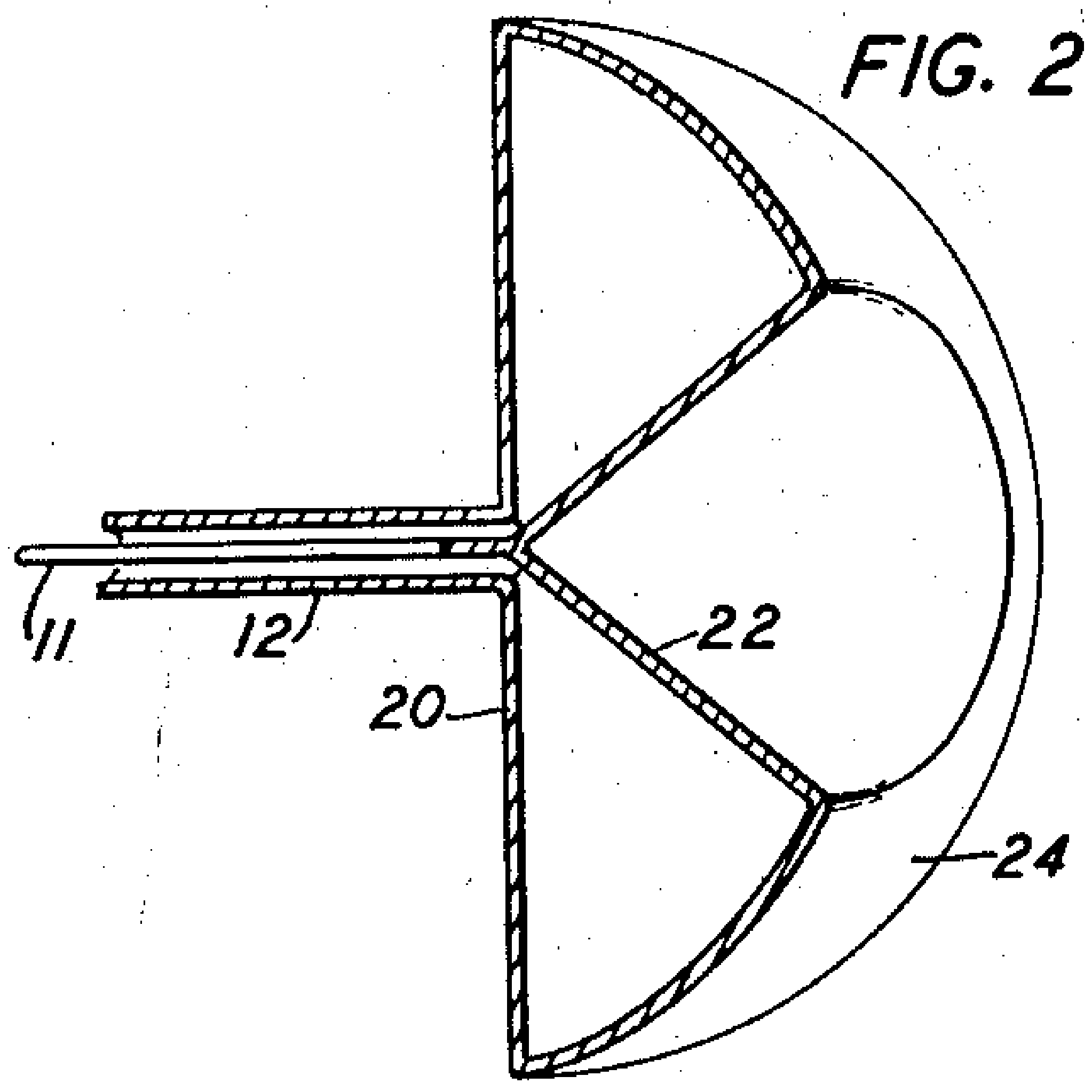


FIG. 3

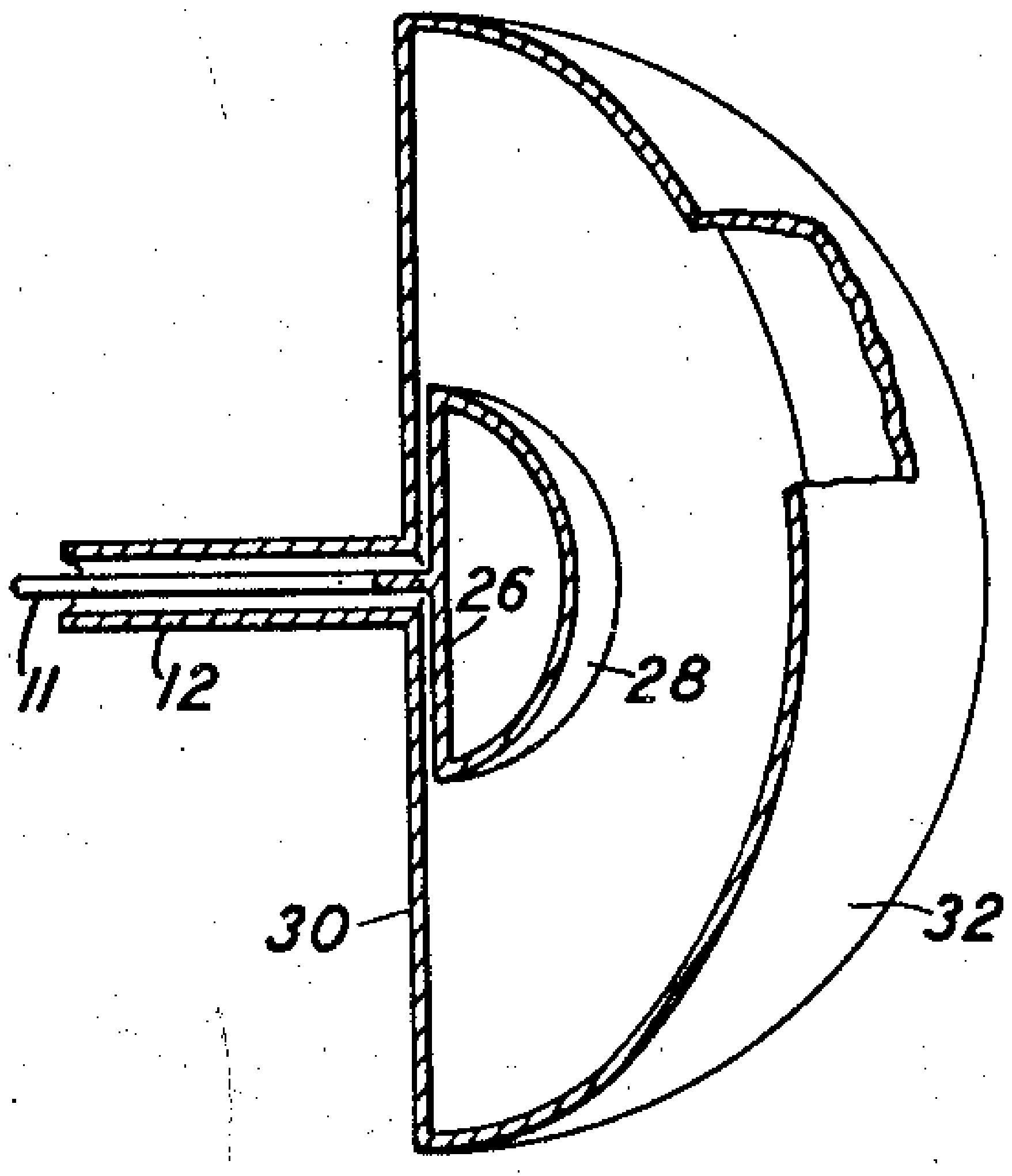


FIG. 4

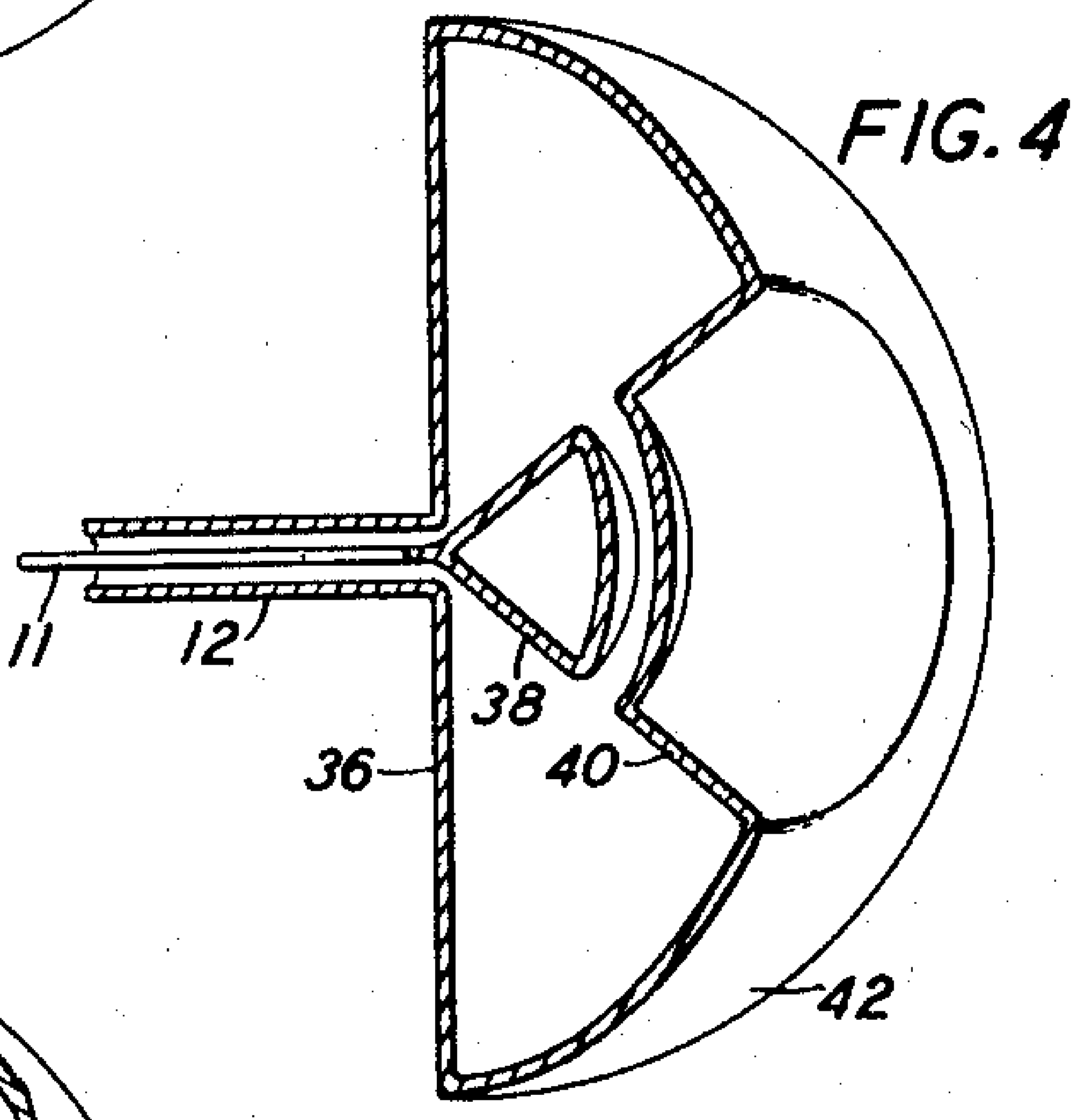


FIG. 7

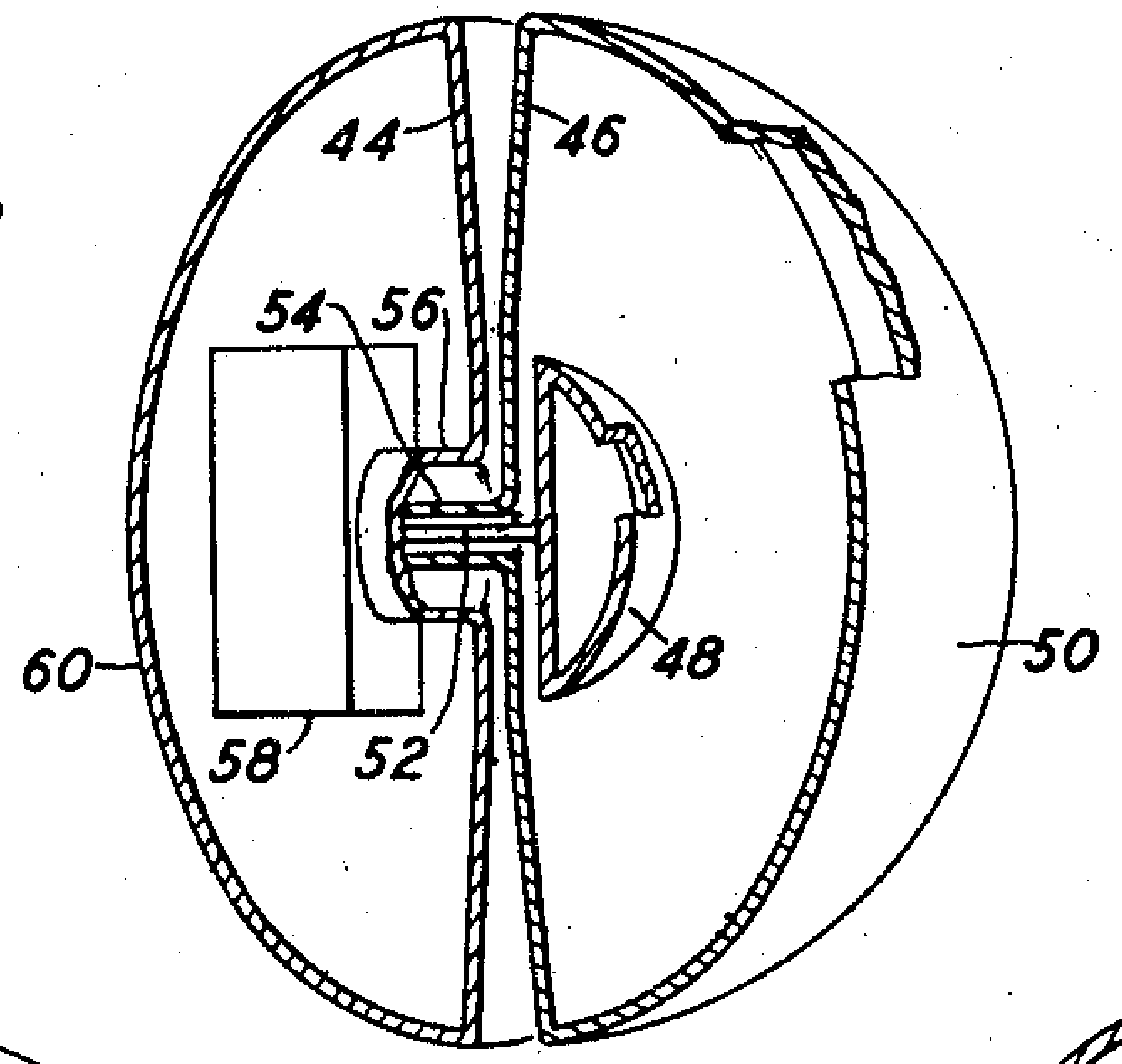


FIG. 5

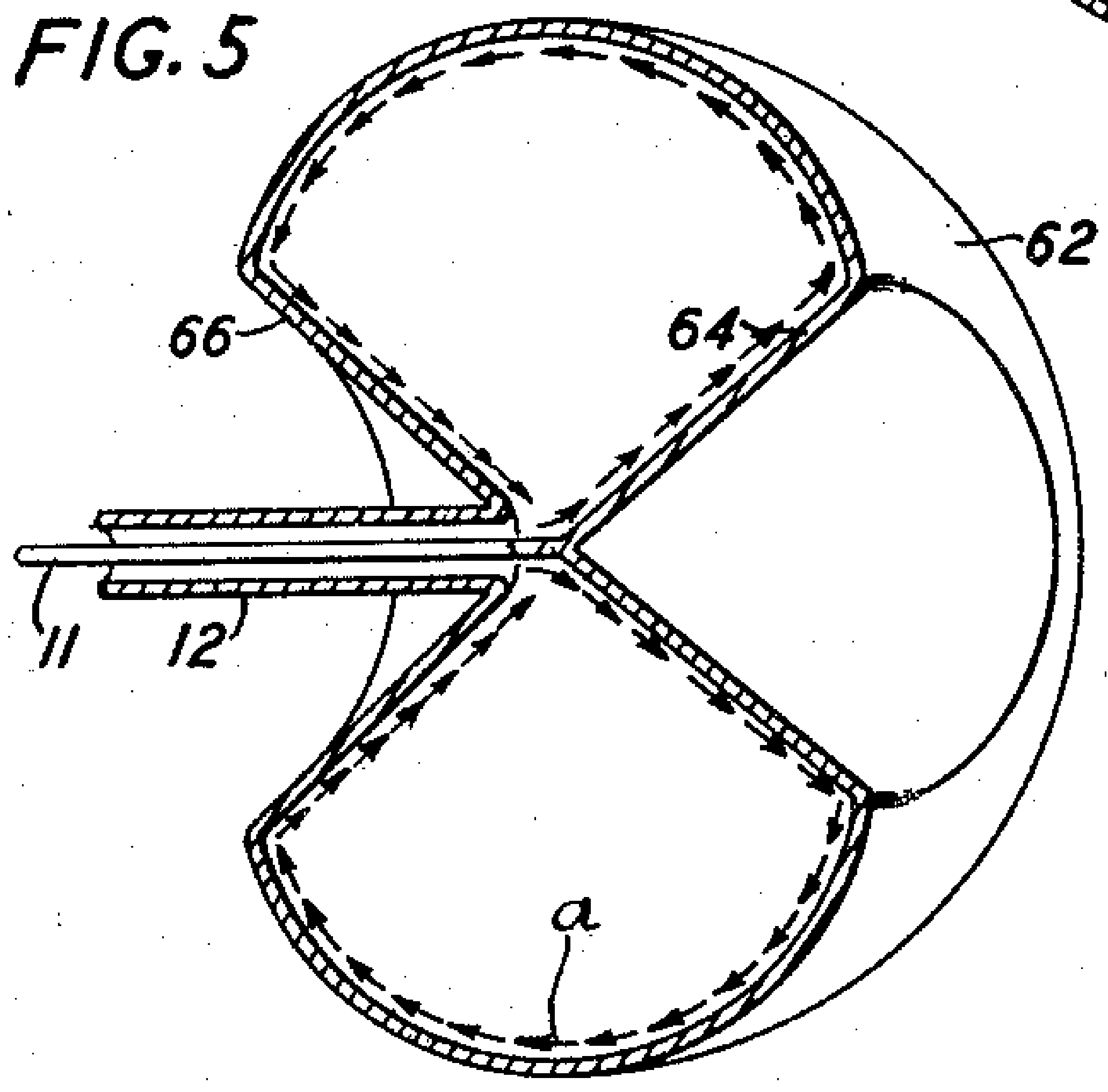
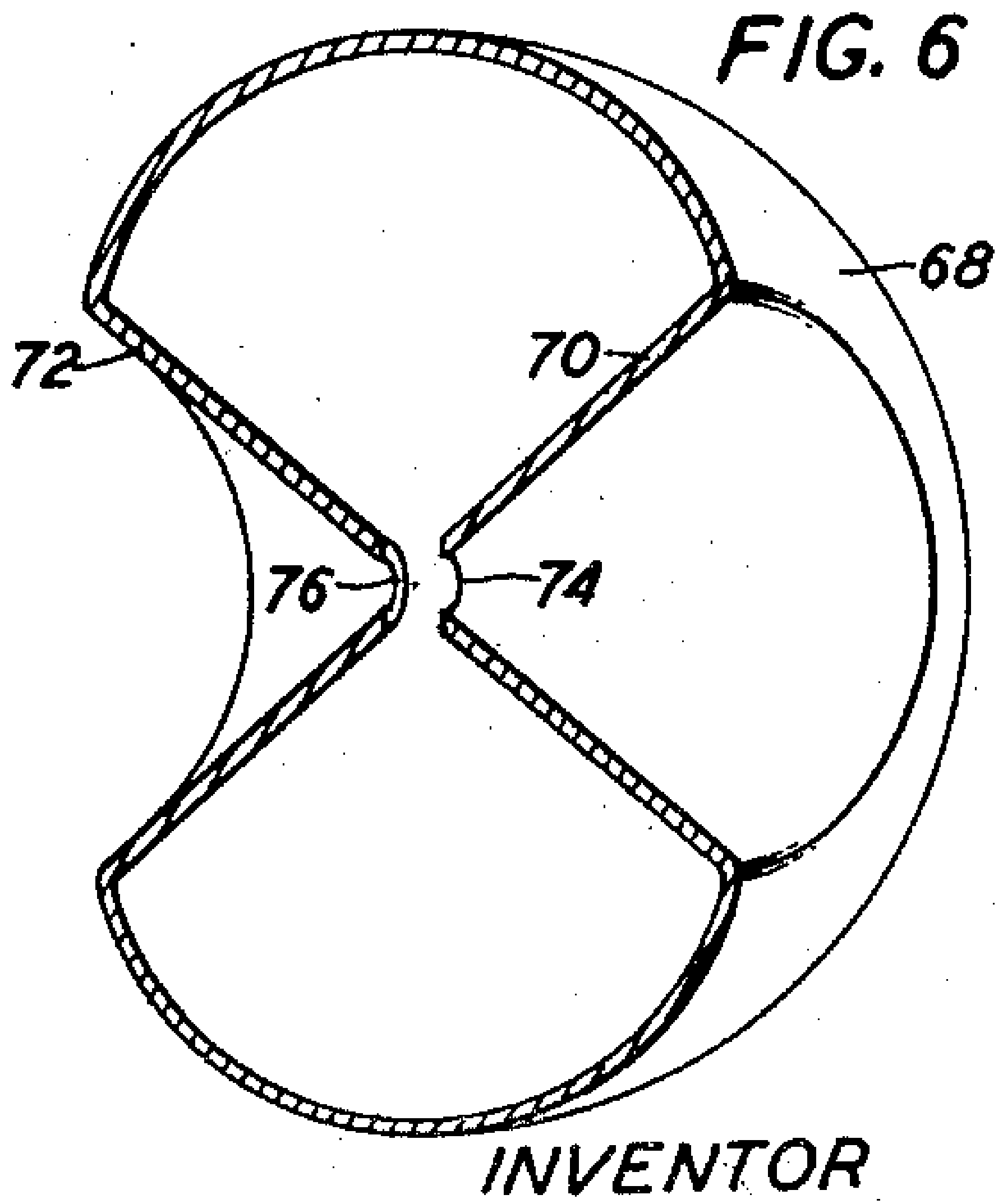


FIG. 6



INVENTOR
S.A. SCHELKUNOFF
BY H.O. Wright
ATTORNEY

March 27, 1945. S. A. SCHELKUNOFF **2,372,228**

HIGH FREQUENCY TANKS AND RESONANT CAVITIES

Filed Dec. 9, 1939 2 Sheets-Sheet 2

FIG. 8

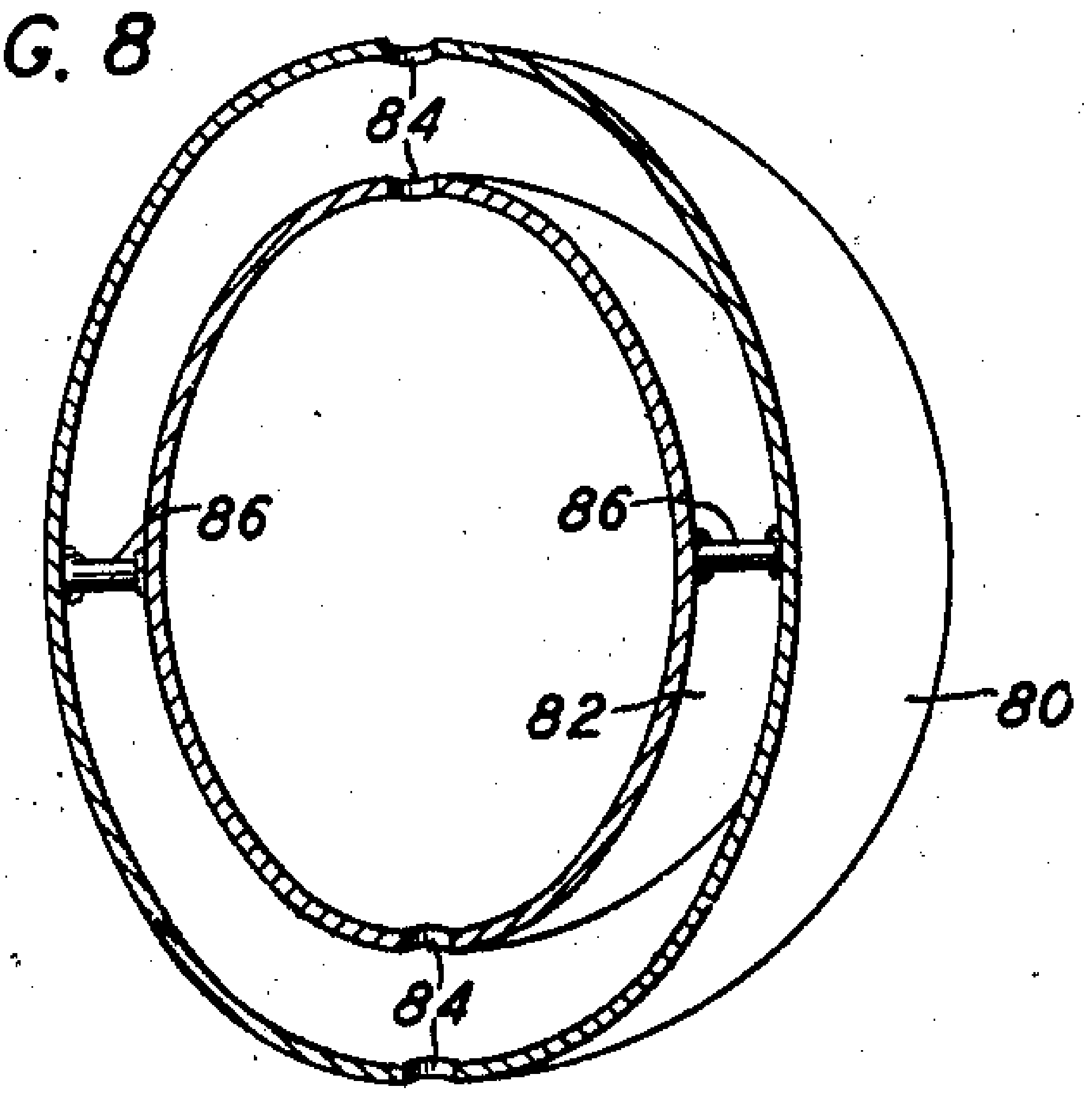


FIG. 9A

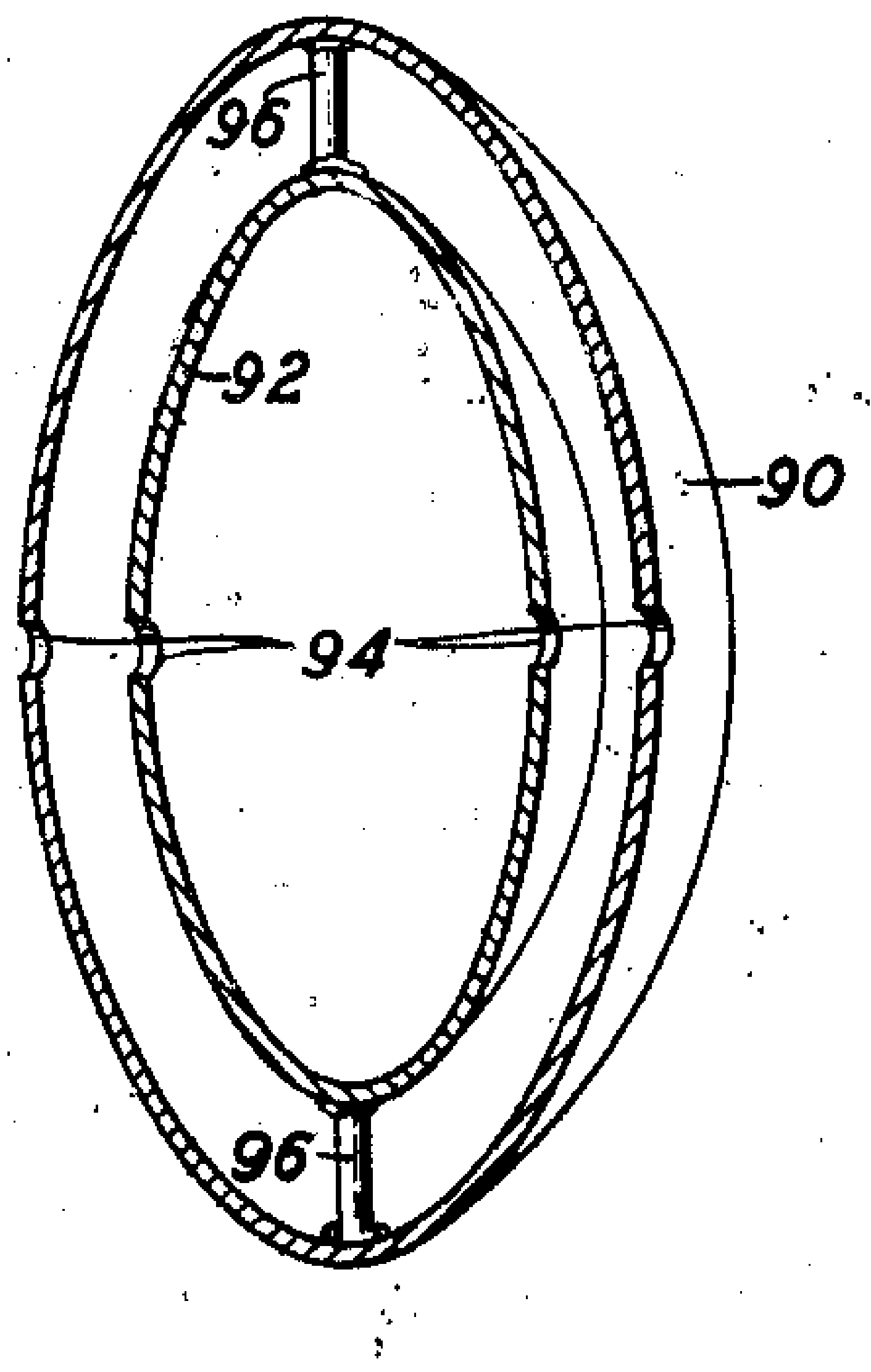
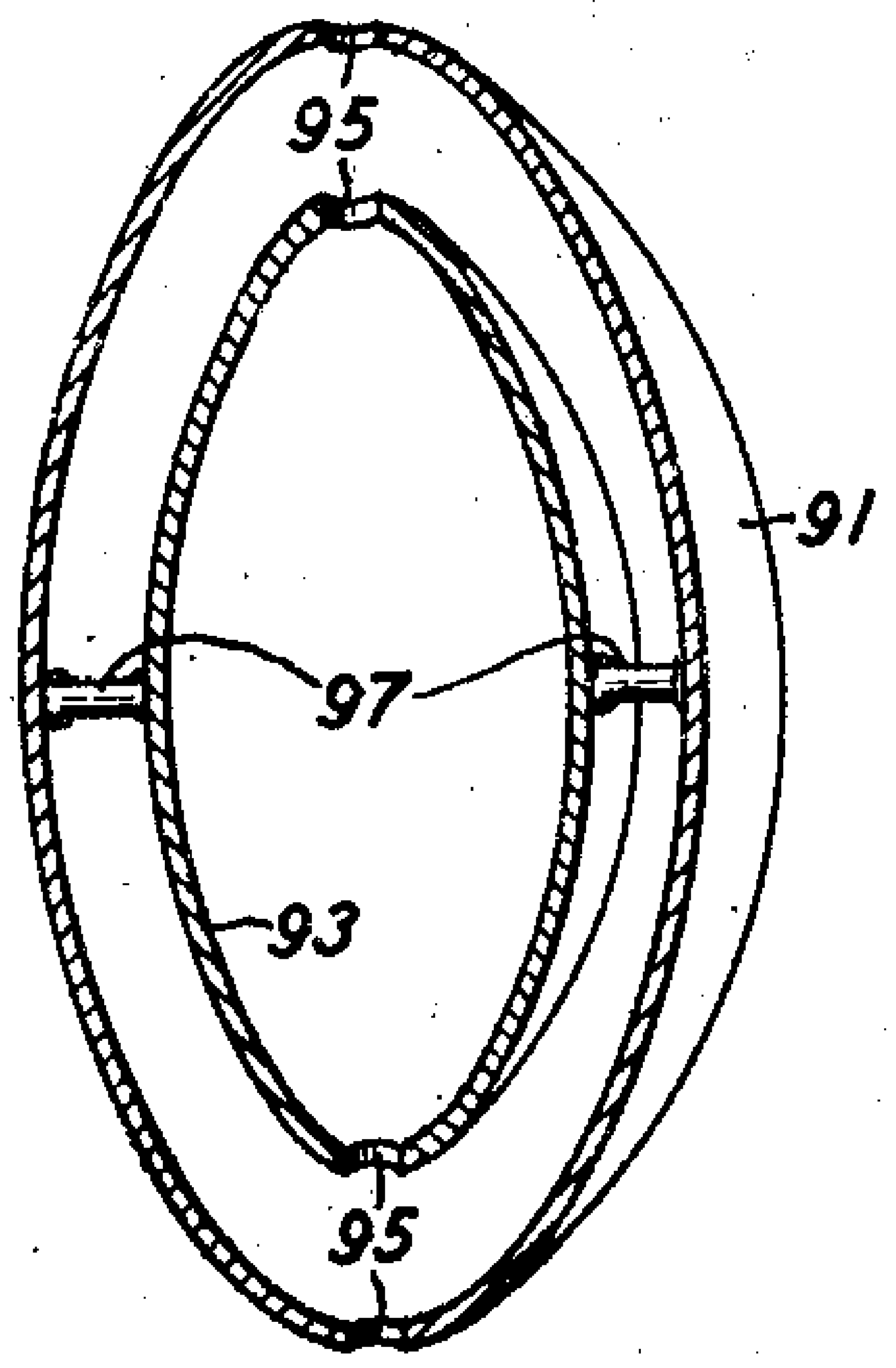


FIG. 9B



INVENTOR
S. A. SCHELKUNOFF
BY
H. O. Wright
ATTORNEY

UNITED STATES PATENT OFFICE

2,372,228

HIGH FREQUENCY TANKS AND RESONANT CAVITIES

Sergei A. Schelkunoff, New York, N. Y., assignor to Bell Telephone Laboratories, Incorporated, New York, N. Y., a corporation of New York

Application December 9, 1939, Serial No. 308,376

15 Claims. (Cl. 178—44)

This invention relates to electrically resonant tank circuits and cavities. More particularly, it relates to tank circuits and cavities bounded by conical and spherical surfaces proportioned and arranged to provide optimum electrical efficiency as resonant devices at high and ultra-high frequencies.

In my copending application Serial No. 278,032, filed June 8, 1939, which has been patented as U. S. Patent 2,235,506 March 18, 1941, the properties of conical lines are dealt with in considerable detail and certain uses of such lines in conjunction with special antennas are set forth.

In this application further uses of conical members are shown and a number of tank circuits and resonant cavities bounded by spherical segments and conic surfaces are described. Such tank circuits and cavities have greater electrical efficiencies than the tank circuits and cavities commonly employed in the prior art to precisely define the frequency response of circuits at ultra-high frequencies. This increased efficiency results principally from the fact that the structures of the invention are proportioned to provide conducting surfaces of larger area in the vicinity of points where maximum oscillating current is induced at the resonant frequencies of the structures.

The arrangements of the invention have the added advantage of lending themselves readily to interconnection with ultra-high frequency circuits of the prior art, such, for example, as the concentric line.

It is a principal object of the invention to provide electrically resonant cavities of optimum electrical efficiency.

Another object is to provide tank circuits of convenient mechanical contour and maximum electrical efficiency for stabilizing the frequency of oscillatory circuits.

Further objects will become apparent during the course of the following description and in the appended claims.

The arrangements and principles of the invention will be more readily understood in connection with the following detailed description in conjunction with the accompanying drawings in which:

Fig. 1 shows in cross-section a high frequency tank circuit comprising a pair of coaxial conical members, their bases being joined by a section of a spherical surface;

Fig. 2 shows in cross-section a high frequency tank circuit comprising a conical member enclosed within a hemispheroidal member;

Fig. 3 shows in cross-section a high frequency tank circuit comprising a hemisphere enclosed within a larger hemisphere;

Fig. 4 shows in cross-section the structure of Fig. 2 with a spherical section of the conical member of the latter figure removed intermediate the base and apex;

Fig. 5 shows in cross-section a high frequency tank circuit comprising two conical members enclosed within a spherical surface;

Fig. 6 shows in cross-section a cavity defined by a pair of conical members enclosed within a sphere;

Fig. 7 shows in cross-section a spherical doublet antenna, connected by an inverted conical line of my above-mentioned copending application, to a radio system enclosed within one doublet member, and means for stabilizing the frequency of the radio system comprising a tank circuit of the type illustrated in Fig. 3 formed within the other doublet member;

Fig. 8 shows in cross-section a pair of concentric spherical members one enclosing the other, appropriate apertures being provided in both members, the combination constituting a particularly efficient resonant cavity for systems employing the velocity modulation of electron streams; and

Figs. 9A and 9B show in cross-section pairs of concentric ovoid members arranged in a manner similar to the members of Fig. 8 and useful for similar purposes.

In more detail, in Fig. 1 conical members 14 and 15 preferably have a common apex point, a small portion of conical member 14 near its apex being removed to avoid contact with member 15 and to facilitate connection with conductor 12 as shown. Members 14 and 15 are arranged coaxially with member 14 enclosing member 15. The bases of both members 14 and 15 extend to intersect surface 16, which surface is a section of the surface of a sphere, the center of which sphere coincides with the common apex point of the members 14 and 15. The inner member 15 connects at its left end to the inner conductor 11 of concentric conductor pair 11, 12 and the outer member 14 is, as indicated above, truncated near its apex to avoid electrical contact with the apex of cone 15 and to connect to outer conductor 12 of the concentric pair. The tank circuit defined by members 14, 15 and spherical surface 16 will when energized to electrical resonance have a voltage node, that is, maximum current at its right end where the conducting surfaces are of maximum area and minimum resistance. The device of Fig. 1 will consequently be a more efficient resonant circuit than those defined by cylinders or rectangular members as proposed in the prior art.

The efficiency of the arrangement of Fig. 1 will, of course, vary with the dimensions of the component elements, particularly with those of the conical members.

The most commonly employed index of electrical efficiency for an electrical reactive device is the ratio of its reactive to its dissipative or resistive components. This index is usually designated by the capital letter Q.

For structure of the type illustrated in Fig. 1,

$$Q=\frac{30\pi^2}{\overline{R}}\;\frac{\sin\theta_1\sin\theta_2\log\left(\cot\frac{\theta_1}{2}\tan\frac{\theta_2}{2}\right)}{p(\sin\theta_1+\sin\theta_2)+\sin\theta_1\sin\theta_2\log\left(\cot\frac{\theta_1}{2}\tan\frac{\theta_2}{2}\right)}\tag{1}$$

where θ_1 is half the apex angle of member **15** of Fig. 1, θ_2 is half the apex angle of member **14**, $\overline{R}$ is the intrinsic resistance of the material of which members **14**, **15** and **16** are made and p is $\frac{1}{2}(C+\log\pi-C_i\pi)=0.824\ldots$ where C is Euler's constant $(0.577\ldots)$ and $C_i\,\pi$ is the integral cosine of π. The derivation of Equation 1 will become apparent, particularly in connection with the derivation of the closely related Equation 9 given in detail hereinunder.

By assigning a convenient value to θ_2 the value of θ_1 resulting in a maximum value for Q may be determined by conventional methods from Equation 1 above.

Considering the concentric cones **14** and **15** of Fig. 1, having a common apex point, as a conical transmission line, the inductance per unit length of such a line is given by the equation.

$$L=\frac{\mu}{2\pi}\log\left(\cot\frac{\theta_1}{2}\tan\frac{\theta_2}{2}\right)\tag{2}$$

Equation 2 follows from the theory of transverse electromagnetic waves, traveling between two coaxial cones as expounded, for example, in my paper entitled "Transmission theory of spherical waves" published in the Transactions of the American Institute of Electrical Engineers, volume 57, 1938, at page 750. Equations 37 to 43 of this paper imply the following properties: (1) The voltage between the cones, as measured along any meridian, and the conduction current in either cone vary with the distance from the apex in exactly the same manner as do the voltage and the current in a transmission line with uniformly distributed inductance and capacity, (2) the phase velocity of the waves is

$$\frac{1}{\sqrt{\mu\epsilon}}$$

and (3) the characteristic impedance is

$$\frac{1}{2\pi}\sqrt{\frac{\mu}{\epsilon}}\log\left(\cot\frac{\theta_1}{2}\tan\frac{\theta_2}{2}\right)$$

Thus, if L is the distributed inductance per unit length and C the distributed capacity,

$$\sqrt{LC}=\sqrt{\mu\epsilon}$$

and

$$\sqrt{\frac{L}{C}}=\frac{1}{2\pi}\sqrt{\frac{\mu}{\epsilon}}\log\left(\cot\frac{\theta_1}{2}\tan\frac{\theta_2}{2}\right)$$

Multiplying these last two equations together we obtain Equation 2, above. Equation 2 may, obviously, also be obtained by several other methods.

For the complete arrangement shown in Fig. 1 the input impedance at resonance is KQ where K is the reactance of the arrangement and Q is the index of electrical efficiency as above described.

If the apex angle of the outer member **14** of Fig. 1 is increased until its sides are in line and the terminating spherical surface **16** is correspondingly extended the arrangement of Fig. 2 is obtained, which is in essence a hemisphere comprising expanded conical member **20** and extended spherical surface **24**, the substantially closed hemispherical member thus formed, enclosing conical member **22**, the hemisphere being connected near the center of its base to the outer conductor **12**, a small portion of the base being omitted to form an appropriate junction, and the cone **22** being connected at its apex to inner conductor **11** of the concentric line. As the flow of current will be over the outer surfaces of inner conductor **11** and conical member **22** and the inner surfaces of the spherical surface **24** which connect the right end of member **22** to the base member **20** and thence over the inner surfaces of member **20** and conductor **12**, it is obvious that the central portion of surface **24** carries no current and may be omitted, as was the central portion of surface **16** of Fig. 1 for the same reason.

To arrive at the optimum design for an antiresonant device of the type shown in Fig. 2, the following analysis may be followed. Equation 2 above may be applied to the structure of Fig. 2 if θ_2 is made 90 degrees. Equation 2 then becomes

$$L=\frac{\mu}{2\pi}\log\cot\frac{\theta_1}{2}\tag{3}$$

The resistance per unit length at distance r from the apex point is

$$R=\frac{\overline{R}}{2\pi r\sin\theta_1}+\frac{\overline{R}}{2\pi r}\tag{4}$$

where $\overline{R}$ is the intrinsic resistance of the conducting members in ohms.

The resistance of the spherical cap member **24** joining the outer ends of the "conical" members **20** and **22** is

$$R_t=\frac{\overline{R}\log\cot\frac{\theta_1}{2}}{2\pi}\tag{5}$$

The average resistance per unit length is

$$R_{av}\int_0^{\frac{\lambda}{4}}\sin^2\beta r\,dr=\int_0^{\frac{\lambda}{4}}\frac{\overline{R}}{2\pi}\left(\frac{1}{\sin\theta_1}+1\right)\frac{\sin^2\beta r}{r}dr$$

$$\frac{\lambda}{8}R_{av}=\frac{\overline{R}}{2\pi}(1+\csc\theta_1)\int_0^{\frac{\lambda}{4}}\frac{1-\cos 2\beta r}{2r}dr\tag{6}$$

$$R_{av}=\frac{2\overline{R}(1+\csc\theta_1)}{\pi\lambda}(C+\log\pi-C_i\pi)$$

Equation 6 follows from the first of the three properties mentioned above in connection with the method described for deriving the relation expressed in Equation 2. When the tank is resonant at a certain frequency, with a current node at the apex and a current antinode at the spherical boundary, the conduction current in either cone varies as $\sin\beta r$. In view of Equation 4 the second term in Equation 6 is the total power dissipated in the conical conductors assuming the maximum effective current is one unit, i. e., one ampere. The first term is the expression for the same power on the supposition that the resistance is independent of r. In other words, the first line of Equation 6 defines the average resistance R_{av}

per unit length of the conical line, the actual resistance R, as given by (4) being a function of r.

Where

λ=wave-length,
$\beta=2\pi/\lambda$=phase constant,
r=distance from the apex of the cone,
C=Euler's constant=0.577 . . . ,
$C_i\pi$=integral cosine of π.

The average stored magnetic energy is

$$E=\frac{I^2_{\max}\lambda L}{32} \qquad (7)$$

Equation 7 follows from the well-known expression ($\frac{1}{4}LI^2$) for the average magnetic energy stored in an inductance L. In the particular case being considered, this expression is integrated between the limits O and $\lambda/4$, letting $I=I_{\max}\sin\beta r$.

The dissipated power is

$$W=\frac{I^2_{\max}\lambda R_{av}}{16}+\frac{\overline{R}}{4\pi}\left(\log\cot\frac{\theta_1}{2}\right)I^2_{\max} \qquad (8)$$

The first term in Equation 8 represents the power dissipated in the cones when the maximum amplitude is $I_{\max}$. This term could be obtained, of course, directly from the second term in Equation 6 without bringing in the average resistance R_{av}. The second term in Equation 8 represents the power dissipated in the spherical part of the tank circuit,

$$\frac{\overline{R}}{2\pi}\log\cot\frac{\theta_1}{2}\text{,}$$

being the resistance of the portion of the conducting sphere between the cone and the plane (the "cone" with θ_2=90 degrees). The latter expression follows from Equation 43 of my above-mentioned paper by letting θ_2=90 degrees, $\eta=\overline{R}(1+i)$ and taking the real part. In so doing it is assumed that the conducting spherical segment is sufficiently thick that the field at the outer surface thereof is negligibly small. Because of the well-known property commonly referred to as "skin effect," the spherical segment need not be very thick to satisfy the foregoing assumption. From Equation 6 of my above-mentioned paper it can readily be seen that η has a 45-degree phase angle for the commonly used conducting metals, assuming the dielectric constant ϵ is zero.

To define the electrical efficiency we take the commonly used expression

$$Q=\frac{2\omega E}{W}$$

(where Q is the index of electrical efficiency expressing the ratio of stored energy to dissipated energy) and substitute the above particular values for E and W.

Hence the electrical efficiency is

$$Q=\frac{2\omega E}{W}=$$

$$\frac{2\omega\lambda L}{32\left[\frac{\overline{R}}{8\pi}(1+\csc\theta_1)(C+\log\pi-C_i\pi)+\frac{\overline{R}}{4\pi}\log\cot\frac{\theta_1}{2}\right]}$$

or

$$Q=\frac{\pi^2 CL}{R\left[(1+\csc\theta_1)(C+\log\pi-C_i\pi)+2\log\cot\frac{\theta_1}{2}\right]}=$$

$$\frac{60\pi^2\log\cot\frac{\theta_1}{2}}{\overline{R}\left[(1+\csc\theta_1)(C+\log\pi-C_i\pi)+2\log\cot\frac{\theta_1}{2}\right]} \qquad (9)$$

This becomes a maximum when $\theta_1=24°\ 6'$ which means that the device of Fig. 2 has maximum efficiency when half the apex angle of the cone is equal to 24° 6'.

Applying the relation above mentioned between input impedance and Q, namely $Z=KQ$, we find that for maximum input impedance, θ_1 should be 9° 6'.

In designing structures of this type, therefore, it may be found advisable in some instances to accept a somewhat smaller Q in order to provide an appropriate input impedance.

In Fig. 3 a structure combining features of the present invention with those of the hollow tank structures disclosed in the copending application of F. B. Llewellyn, Serial No. 185,139, filed January 15, 1938, is shown. A hemisphere **28** having a base surface **26** is assembled coaxially within but not in electrical contact with a larger hemisphere **32** having a base surface **30**. Inner conductor **11** of the associated concentric line connects to surface **26** and outer conductor **12** connects to surface **30**. The structure of Fig. 3 should, as explained for the corresponding tank circuits of the above-mentioned application of F. B. Llewellyn, preferably be of such dimensions that the capacity between surfaces **26** and **30** will substantially exceed the capacity between surfaces **28** and **32**. Analogously to the hollow tank devices of the above-mentioned application of Llewellyn as compared with conventional tank circuits, structures of the type illustrated in Fig. 3 of the drawing accompanying this application will have much larger dimensions for a particular desired resonant frequency than structures of the type illustrated in Fig. 2. The former type will consequently provide frequency stabilizing resonators of convenient dimensions at frequencies which are so high that devices of the latter type become inconveniently small. Because of the configuration employed, devices of the type of Fig. 3 will have maximum conducting surface area at and adjacent to the current antinodal points for resonance as well as shorter paths for the circulating currents in the device and will therefore have even greater electrical efficiency than the hollow tank structures of the above-mentioned copending Llewellyn application.

Fig. 4 represents another method of combining features of conventional tank circuits with those of the structures of this invention.

The structure of Fig. 4 is essentially that of Fig. 2 with a narrow spherical section removed from the central cone and spherical caps provided at each side of the removed section. This has the effect of introducing a series capacity and the structure is approximately analogous to the conventional tank circuit of Fig. 3a of U. S. Reissue Patent 20,859, issued September 13, 1938, to R. K. Potter. The structure of Fig. 4 of the present application, however, by virtue of its special geometrical configuration will have a greater electrical efficiency than the approximately analogous forms of conventional tank circuits.

In Fig. 5, a further application of the principles of the invention to the design of a resonant tank circuit is shown. Fig. 5 comprises two co-axial conical members **64** and **66** extending in opposite directions from a common apex point. Member **66** is truncated near its apex so that it will not make electrical contact with member **64** at the common apex point and so that it may be conveniently joined with outer conductor **12** of the associated concentric conductor pair. The inner conductor **11** is electrically connected to the apex of member **64**. Surface **62** is that of a sphere whose center preferably coincides with the common apex point of the two cones. The circular sections of this spherical surface defined by the junctions of the bases of conical members **64** and **66** therewith respectively may be omitted since the ultra-high frequency currents introduced by concentric pair **11**, **12** will flow only over the outer surfaces of members **64** and **66** and the portion of the inner surface of spherical surface **62** which serves to make electrical connection between the bases of the conical members. The arrows *a* of Fig. 5 indicate the instantaneous current flow at a particular instant in the plane of the cross-section shown in that figure, the current being introduced on the outer surface of inner conductor **11** and flowing from the structure along the inner surface of outer conductor **12**, of the concentric pair. As a result of its more perfect symmetry the device of Fig. 5 will be even more efficient than that of Fig. 2. Where, as in Fig. 5, both conical members **64**, **66** have the same apex angle, the efficiency of the device may be expressed by the equation (analogous to Equation 1 above),

$$Q=\frac{30\pi^2}{R}\,\frac{\sin\theta_1\log\cot\frac{\theta_1}{2}}{p+\sin\theta_1\log\cot\frac{\theta_1}{2}} \qquad (10)$$

Q is maximum when $\theta_1=33°\ 30'$ and the input impedance at resonance is maximum when $\theta_1=7°\ 30'$, i. e., the apex angles of the conical members should be 67° for maximum efficiency or 15° for maximum impedance at resonance.

For a structure of the type shown in Fig. 5 but having conical members of different apex angles Equation 1, given above, is applicable and the maximum Q is determined in terms of one angle when a particular value is assigned to the other. As a general rule the maximums referred to for the structures of Figs. 1, 2 and 5 are broad, that is, they do not decrease abruptly as the optimum angle is departed from, so that if the optimum angle is approximated the efficiency or impedance, as the case may be, will be close to its maximum value.

Fig. 6 is identical with Fig. 5 except that concentric pair **11**, **12** has been omitted and the conical members **70** and **72** of Fig. 6 are both truncated near their respective apices, to provide orifices **74** and **76** normal to the common axis of the cones, the orifices being of appropriate size and spacing and the dimensions of the whole combination of Fig. 6 being chosen so that it may be employed as a resonant cavity for use in systems employing velocity modulated electron streams. Such a cavity will have very high electrical efficiency for the above-mentioned use. Equation 10 given above is applicable to structures of Fig. 6 where the cones have equal apex angles, and Equation 1 where they have different angles.

In Fig. 7 is shown the combination of a frequency stabilizer comprising members **60**, **59** closely approximating the type shown in Fig. 3, with an inverted conical line and a spherical dipole antenna of the types described in my above-mentioned copending application.

The exteriors of the substantially hemispherical members **50** and **60** serve as a doublet antenna system. The bases **44** and **46** of these members are made slightly divergent to form an inverted conical line of appropriate impedance for conveying power from the center of the system to the exterior surfaces of the doublet members **50** and **60**. The interior of member **59**, as before mentioned, serves as part of a frequency stabilizer closely approximating the type illustrated by Fig. 3. The interior of member **60** is employed to house radio apparatus **58** which may include a radio transmitter and/or receiver. A three-conductor concentric line comprising conductors **52**, **54** and **56** serves to connect apparatus **58** with the frequency stabilizer and the inverted conical line above mentioned, respectively. As is apparent from Fig. 7 the inner surface of conductor **56** and the outer surface of conductor **54** serve to make connection to the inner end of the inverted conical line formed by the adjacent surfaces of members **44** and **46** while the inner surface of conductor **54** and the outer surface of conductor **52** serve to make connection to the elements **48** and **50** of the frequency stabilizing system respectively within hemispherical member **50**. The combination has the outstanding advantages of inherently high electrical efficiency, substantially perfect shielding where coupling or radiation is not desirable, and mechanical compactness combined with the efficient utilization of several portions of the system for two or more functions.

In Fig. 8 a second form of resonant cavity having high efficiency for use in velocity modulated electron systems is shown and comprises two spherical members **80** and **82**, member **80** enclosing member **82**, each member having two orifices **84**, **90** and **86**, **88** respectively arranged along a common axis, as shown, and normal thereto. Members **86** are of insulating material and maintain member **82** in a predetermined position (usually concentric) with respect to member **80**. The system is proportioned to provide resonance at desired frequencies.

Figs. 9A and 9B show cavities similar to that of Fig. 8 except that ovoid members are employed in place of spherical members. Such arrangements are desirable to provide the desired lengths of electron paths in conjunction with a particular electrical efficiency.

In Fig. 9A ovoid member **90** encloses ovoid member **92**, the latter being maintained in position by insulating members **96**. Four orifices **94**, two in each member, are provided in alignment to furnish a rectilinear path through the shorter axis of the combination of members.

In Fig. 9B a combination essentially like that of Fig. 9A is shown except that a rectilinear path through the combination along the longer axis of the combination is provided by aligning orifices **95** thereon.

By choosing other rectilinear paths through the combinations of Figs. 9A and 9B and providing suitable additional orifices any one of a large number of different length paths may be obtained and a single pair of ovoid members can provide a plurality of paths of different length, it being desirable, however, that the total area of the orifices in each ovoid is small in comparison with the surface area of the ovoid as otherwise the efficiency of the device may be impaired. By simply turning the device to present a path of different length to an electron stream the timing of a velocity modulated system can be changed.

The above-described embodiments are illustrative of numerous other arrangements applying the principles of this invention, which will occur to those skilled in the art. No attempt has here been made to exhaustively cover such application. The scope of the invention is defined in the appended claims.

What is claimed is:

1. A tank circuit of conductive material for precisely fixing the frequency of a single frequency oscillatory system for high frequencies comprising a hemispheroidal member having a conductive base with a centrally positioned orifice in said base, a conical member enclosed within said first stated member, the apex of said conical member being coincident with the center point of the base of said hemispheroidal member, the axis of said conical member being normal to said base, the base of said conical member terminating in the spheroidal surface of said first member and a conductive member electrically connected to and extending from the apex of said conical member through the orifice in the base of the first-stated member but conductively insulated from said base.

2. A tank circuit of conductive material for precisely fixing the frequency of a single frequency oscillatory system for high frequencies comprising two coaxial conical members having a common apex point and extending in opposite directions from said apex point, a member having the form of a spherical segment enclosing said conical members, the center of said spherical member being coincident with the said apex point, a small portion of one of said conical members near said apex point being removed to form an orifice and a conductor electrically connected with the apex of the other of said conical members and extending through the orifice of the first-mentioned conical member but electrically insulated therefrom, the bases of said conical members being joined by said spherical segment.

3. The tank circuit of claim 1, the apex angle of said conical member being approximately 48 degrees.

4. A tank circuit of conductive material for precisely fixing the frequency of a single frequency oscillatory system for high frequencies comprising a pair of conical conducting members and a spherical segmental conducting member, the latter member joining the base peripheries of the said pair of conical members, the conical members being insulated from each other and said spherical member at all other points, one of said conical members having an orifice at its apex, and a conductor electrically connecting to the apex of the other conical member and passing through the orifice in the said one of the conical members but insulated therefrom.

5. A tank circuit of conductive material for precisely fixing the frequency of a single frequency oscillatory system for ultra-high frequencies comprising two hemispheroidal conducting members having conductive bases, one member enclosing the other, the bases of said members being adjacent, said members being insulated from each other, the base of the outer member having an aperture therein, and a conductor electrically connected to the base of the inner member and passing through the aperture in the base of the outer member but insulated therefrom.

6. The combination of claim 5, the said inner member thereof being so proportioned and situated within the said outer member thereof that the electrical capacity between the bases of said members is substantially greater than the electrical capacity between their respective spherical surfaces.

7. The tank circuit of claim 2, the conical members having vertex angles proportioned to present a predetermined input impedance.

8. The tank circuit of claim 1, said conical member having a vertex angle of approximately 18 degrees whereby the terminal impedance of said circuit is substantially a maximum.

9. The tank circuit of claim 2, said conical members each having vertex angles of approximately 67 degrees.

10. The tank circuit of claim 2, said conical members each having vertex angles of approximately 15 degrees.

11. A tank circuit of conductive material for precisely fixing the frequency of a single frequency oscillatory system for ultra-high frequencies comprising two coaxial conic members having a common apex point, their bases being joined by a section of a spherical surface, one of said conic members having a small aperture near its apex, the apex angles of said conic members being proportioned in accordance with the formula

$$Q=\frac{30\pi^2}{R}\;\frac{\sin\theta_1\sin\theta_2\log\left(\cot\frac{\theta_1}{2}\tan\frac{\theta_2}{2}\right)}{p(\sin\theta_1+\sin\theta_2)+\sin\theta_1\sin\theta_2\log\left(\cot\frac{\theta_1}{2}\tan\frac{\theta_2}{2}\right)}$$

to provide a predetermined electrical efficiency, a conductor extending through the aperture in one of the conical members but insulated therefrom, said conductor electrically connecting to the apex of the other of said conical members.

12. A tank circuit of conductive material for precisely fixing the frequency of a single frequency oscillatory system for high frequencies comprising two conical members, a spherical member enclosing the two first-stated members, the bases of said conical members being connected by said spherical member, the three members being otherwise insulated from each other, one of said conical members having an aperture near its apex, and a conductor extending through the said aperture of said one of said conical members but insulated therefrom, said conductor electrically connecting to the apex of the other conical member electrical connection to said conical members near their respective apex points.

13. An electrical resonator for high frequencies comprising a first member of conductive material in the form of a first surface of revolution generated by the rotation of a straight line about a point, the line being maintained during rotation at a predetermined angle with respect to an axis passing through said point, said first member having a small aperture therein, a second member of conductive material consisting of a second surface of revolution having the form of a spherical segment, the respective axes of symmetry of said first and said second surfaces of revolution being coincident, the periphery of said first surface of revolution being coincident with and conductively joined with a base periphery of said second surface of revolution, a third member of conductive material positioned within said second member and symmetrically coupled electrically with said second member and conductively insulated from said first member and a conductive member conductively connecting to said third member and extending through but conductively insulated from said first member.

14. An electrical resonator for high frequencies comprising a first member of conductive material in the form of a spherical segment of two bases,

a second member of conductive material in the form of a surface of revolution generated by the rotation of a straight line about a point, the line being maintained during rotation of a predetermined angle with respect to an axis passing through said point, the periphery of the said second member being coincident with and conductively connected with the periphery of one base of said first member, said second member having a small aperture therein, a third member of conductive material in the form of a cone, a portion at least of said cone being situated between the planes of the bases of said first member, the axis of said cone being coincident with the axis of symmetry of said first member, the base periphery of said cone being coincident with and conductively connected to the periphery of the second base of said first member, the third member being conductively insulated from the second member, and a conductive member conductively connecting to said third member and extending through but conductively insulated from said second member.

15. An electrical resonator for high frequencies comprising a first member of conductive material in the form of a spherical segment of two bases, a second and a third member, each being of conductive material and of conical form, said second and said third members being coaxial and having a common apex point, one of said conical members having a small aperture at its apex, the second and third members being insulated from each other, the base periphery of one conical member being coincident with and conductively connected to the periphery of one base of said first member, the base periphery of the other conical member being coincident with and conductively connected to the other base periphery of said first member and a conductor passing through the said aperture of said one of the conical members but conductively insulated therefrom, said conductor electrically connecting to the other of said conical members.

SERGEI A. SCHELKUNOFF.