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(54) **SYSTEM AND METHOD FOR DOMAIN STRETCHING FOR AN ADVANCED DUAL-REPRESENTATION POLYHEDRAL LOOP TRANSFORMATION FRAMEWORK**

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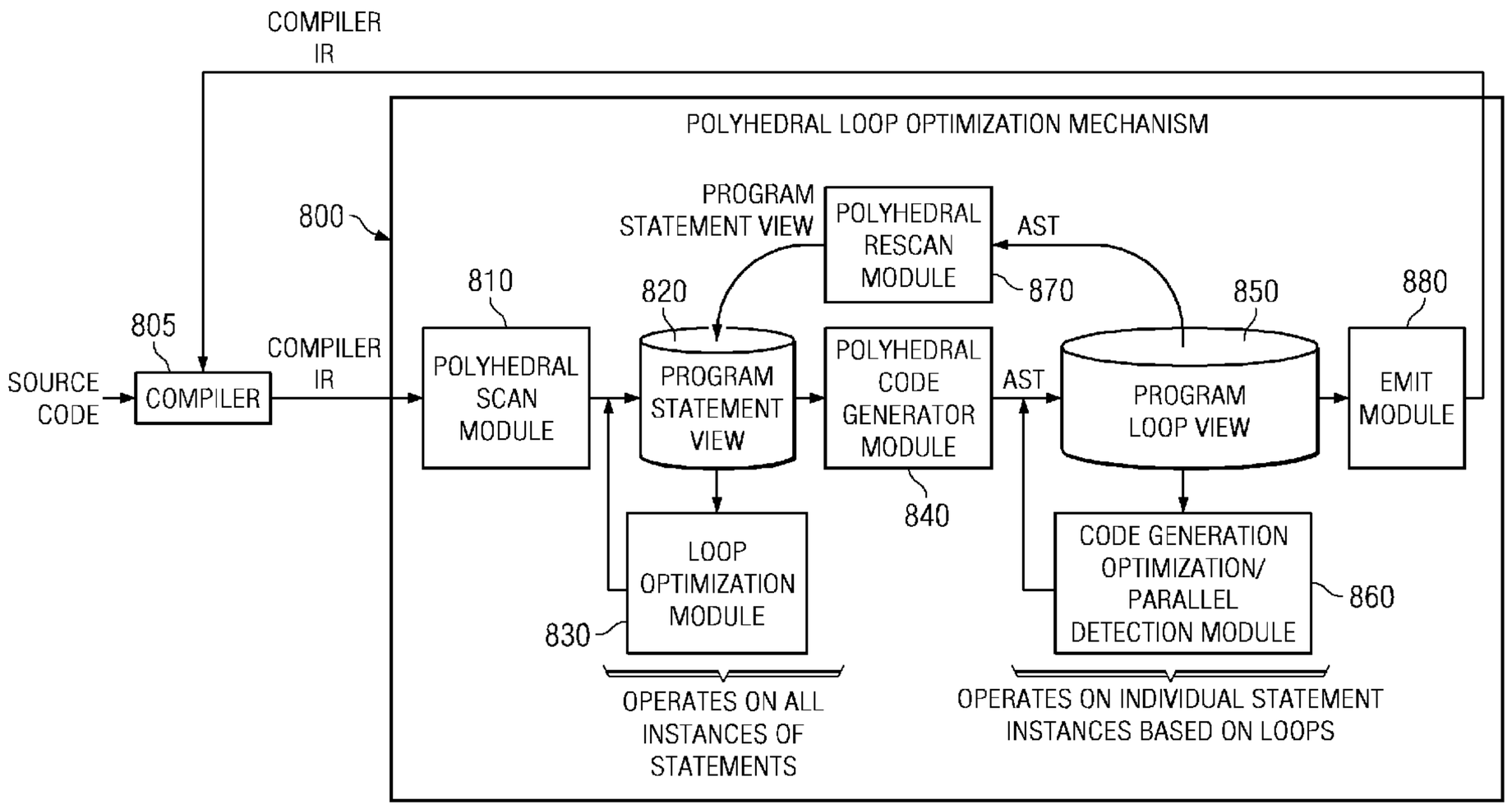
(57) **ABSTRACT**

A system and method for domain stretching for an advanced dual-representation polyhedral loop transformation framework are provided. The mechanisms of the illustrative embodiments address the weaknesses of the known polyhedral loop transformation based approaches by providing mechanisms for performing code generation transformations on individual statement instances in an intermediate representation generated by the polyhedral loop transformation optimization of the source code. These code generation transformations have the important property that they do not change program order of the statements in the intermediate representation. This property allows the result of the code generation transformations to be provided back to the polyhedral loop transformation mechanisms in a program statement view, via a new re-entrance path of the illustrative embodiments, for additional optimization. In addition, mechanisms are provided for stretching the domains of statements in a program loop view of the source code to thereby normalize the domains.



```
for(i=2; i<=n; i++)
  z[i] = 0;                          // S1
for(i=1; i<=n; i++)
  for(j=1; j<=n; j++)
    z[i+j] += x[i] * y[j]; // S2
```

FIG. 1A
(PRIOR ART)

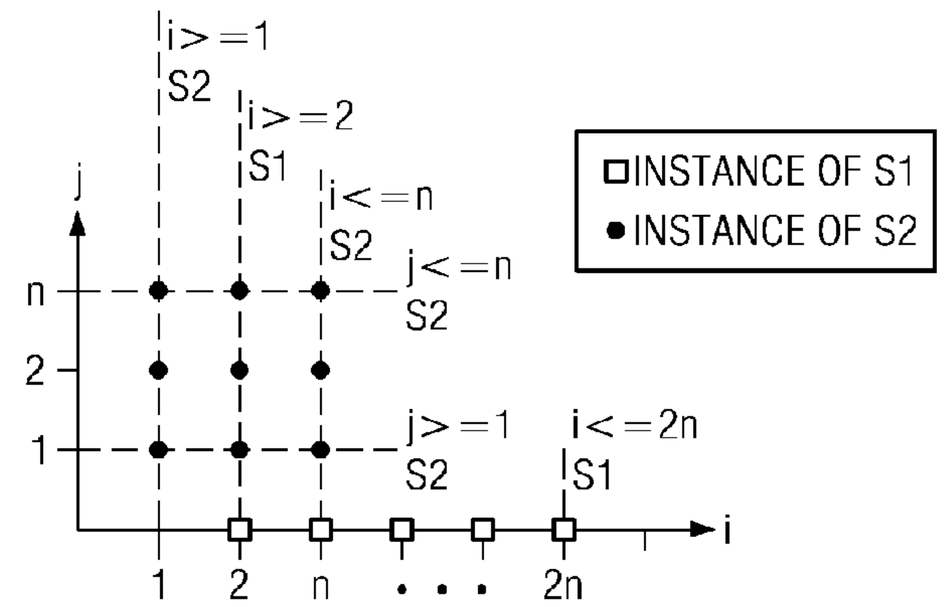


FIG. 1B
(PRIOR ART)

$$\left(\begin{array}{c|c} I & -T \\ \hline 0 & A \end{array}\right) \bullet \left(\frac{t}{x}\right) + \left(\frac{-Tp}{A_p}\right) \bullet p \quad \frac{-0}{\geq 0}$$

FIG. 1C
(PRIOR ART)

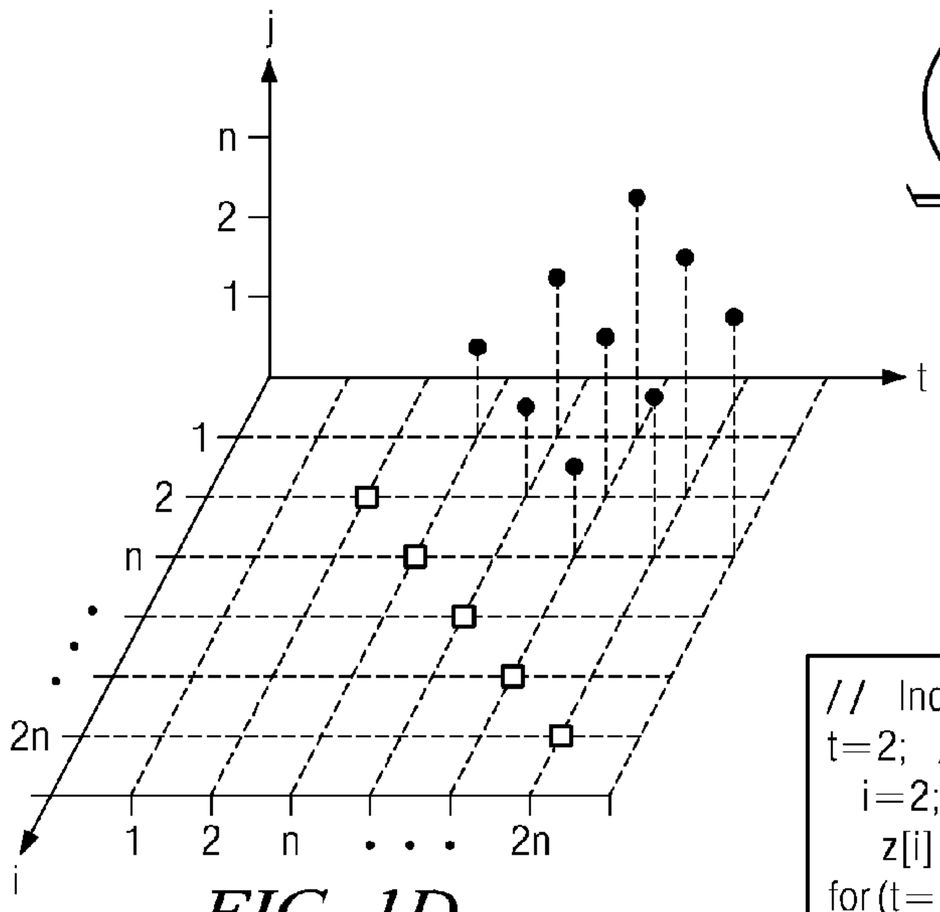


FIG. 1D
(PRIOR ART)

```
//  Indentation follows t,i,j domain dimensions
t=2;  // A loop running once
  i=2;
    z[i] − 0;                                  // S1
for (t=3;  t<=2*n;  t++)
  for (i=max(1,t-n-1) ; i<=min (t-2,n) ; i++)
    j  =  t-i-1;
      z[i+j]  +=  x[i]  *  y[j]                // S2
  i=t;
    z [i]  =  0;                               // S1
t=2*n+1;
  i=n;
    j=n;
      z[i+j]  +=  x[i]  *  y[j];               // S2
```

FIG. 2B
(PRIOR ART)

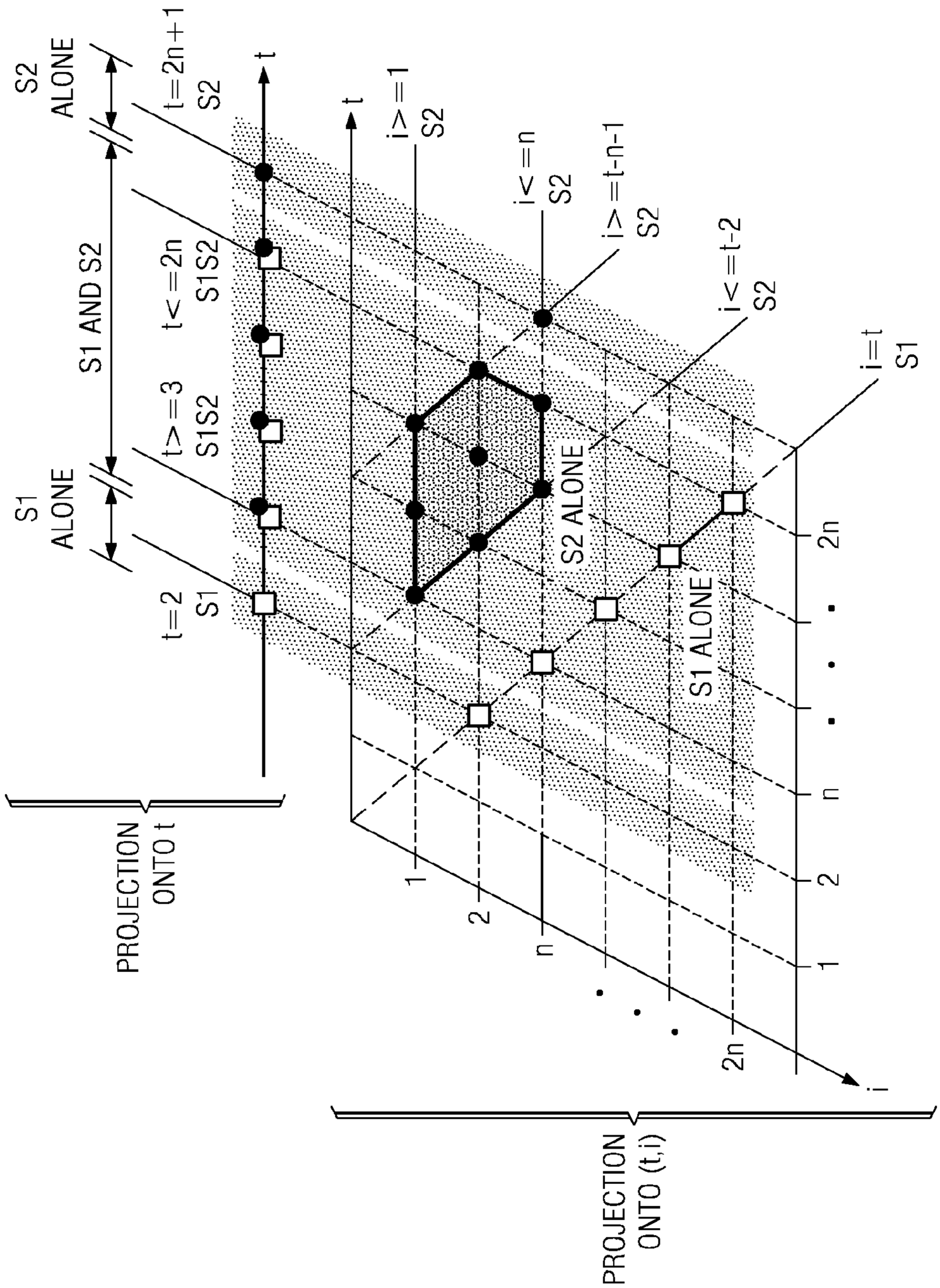


FIG. 2A
(PRIOR ART)

FIG. 3

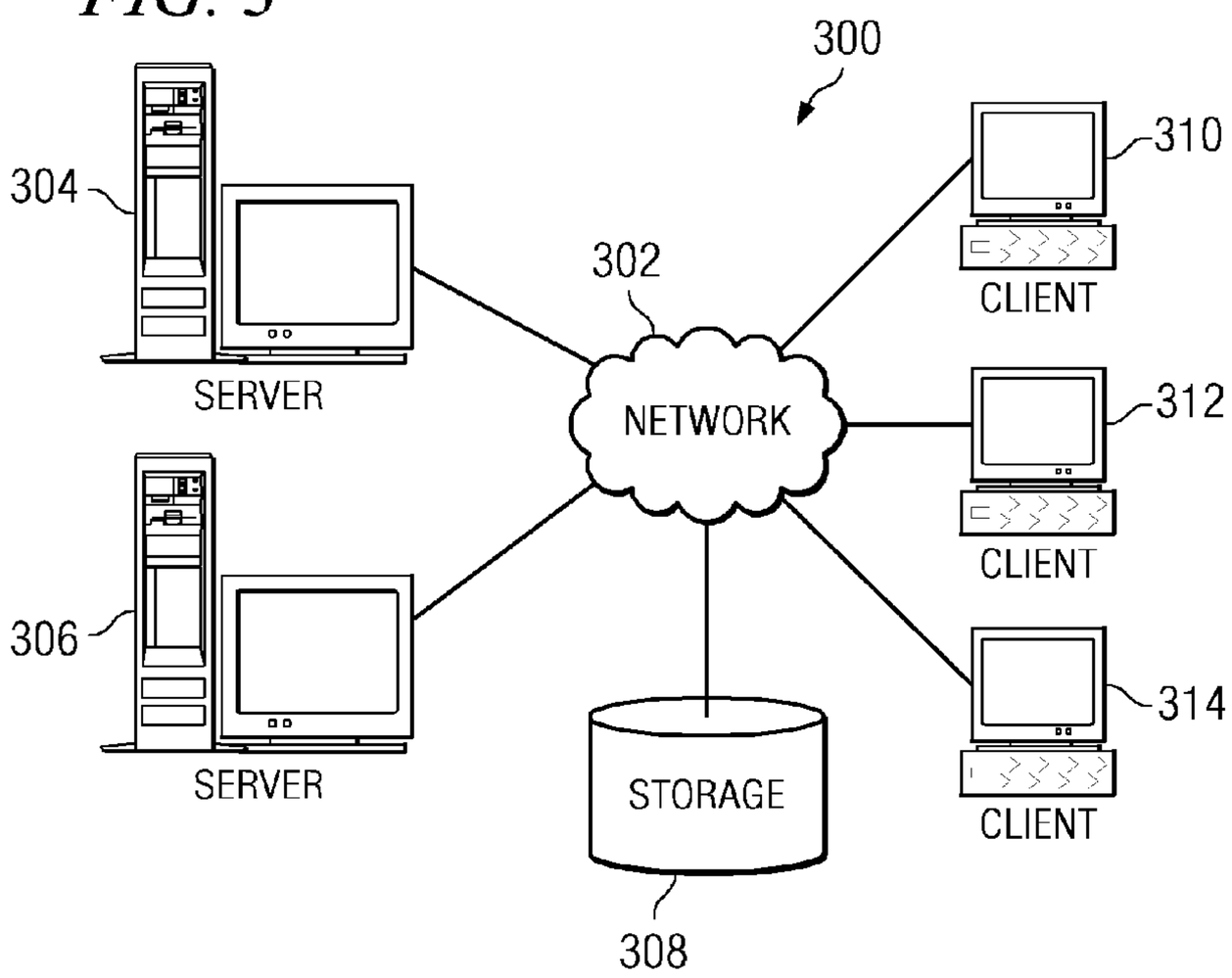
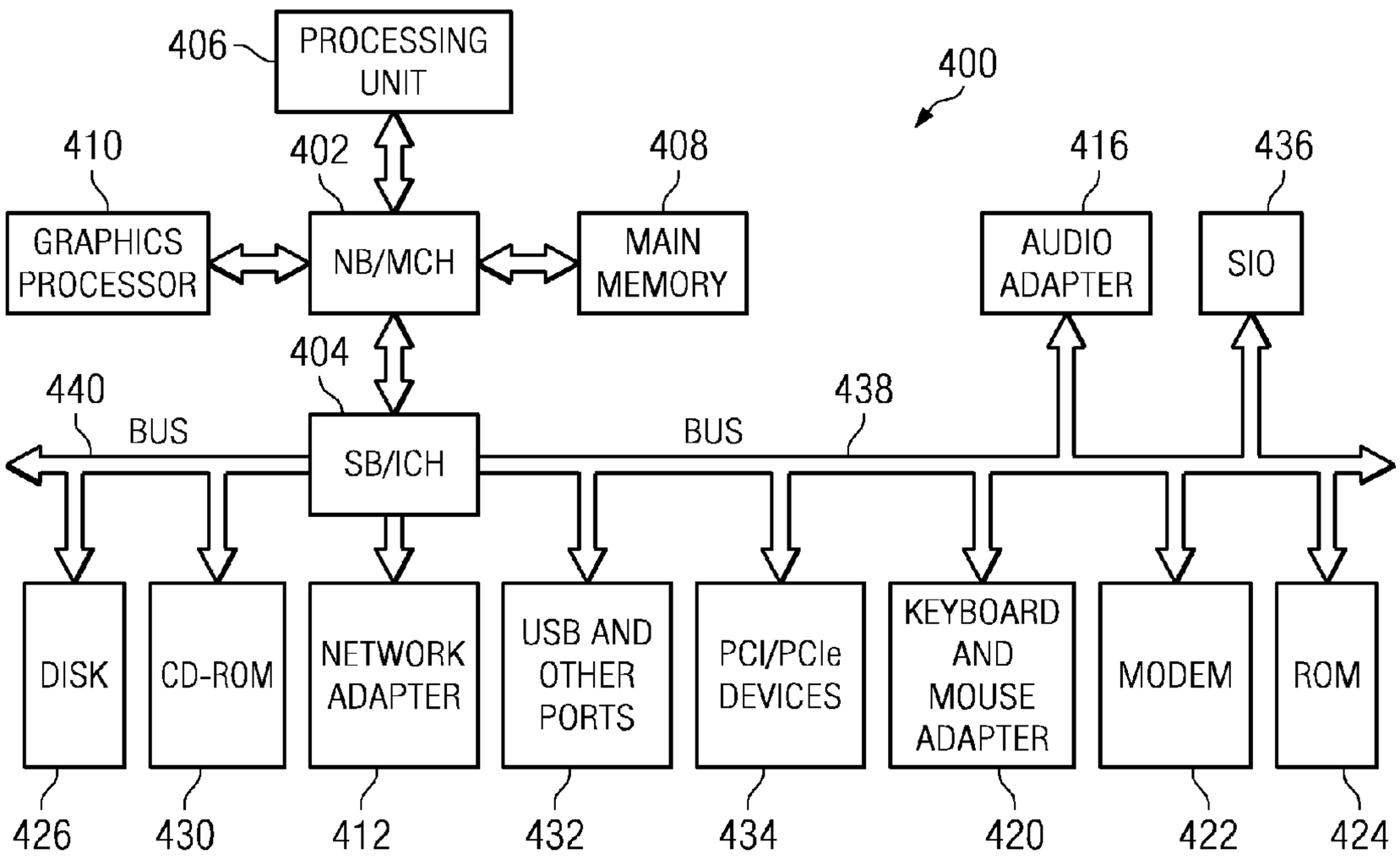

300
304
SERVER
302
NETWORK
310
CLIENT
312
CLIENT
306
SERVER
STORAGE
308
314
CLIENT

FIG. 4


406
PROCESSING UNIT
400
410
GRAPHICS PROCESSOR
402
NB/MCH
408
MAIN MEMORY
416
AUDIO ADAPTER
436
SIO
404
440
BUS
SB/ICH
438
BUS
DISK
CD-ROM
NETWORK ADAPTER
USB AND OTHER PORTS
PCI/PCIe DEVICES
KEYBOARD AND MOUSE ADAPTER
MODEM
ROM
426
430
412
432
434
420
422
424

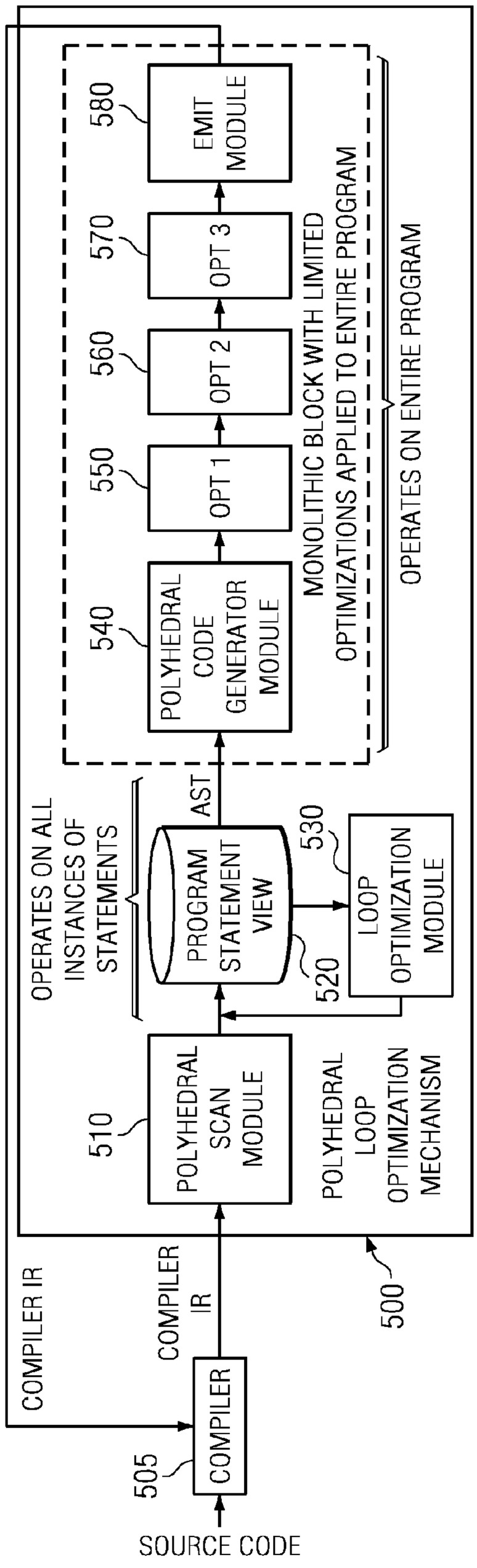


FIG. 5
(PRIOR ART)

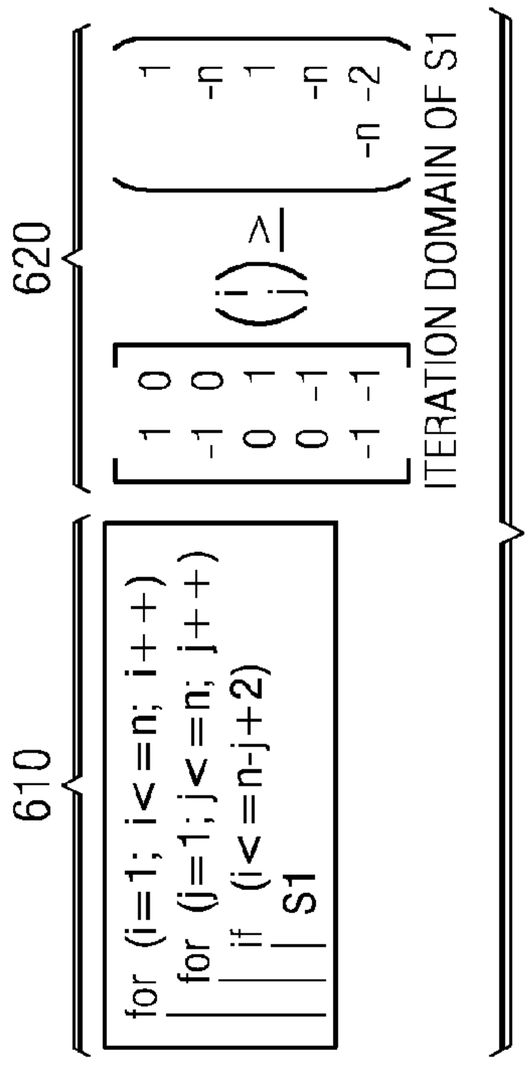


FIG. 6A
(PRIOR ART)

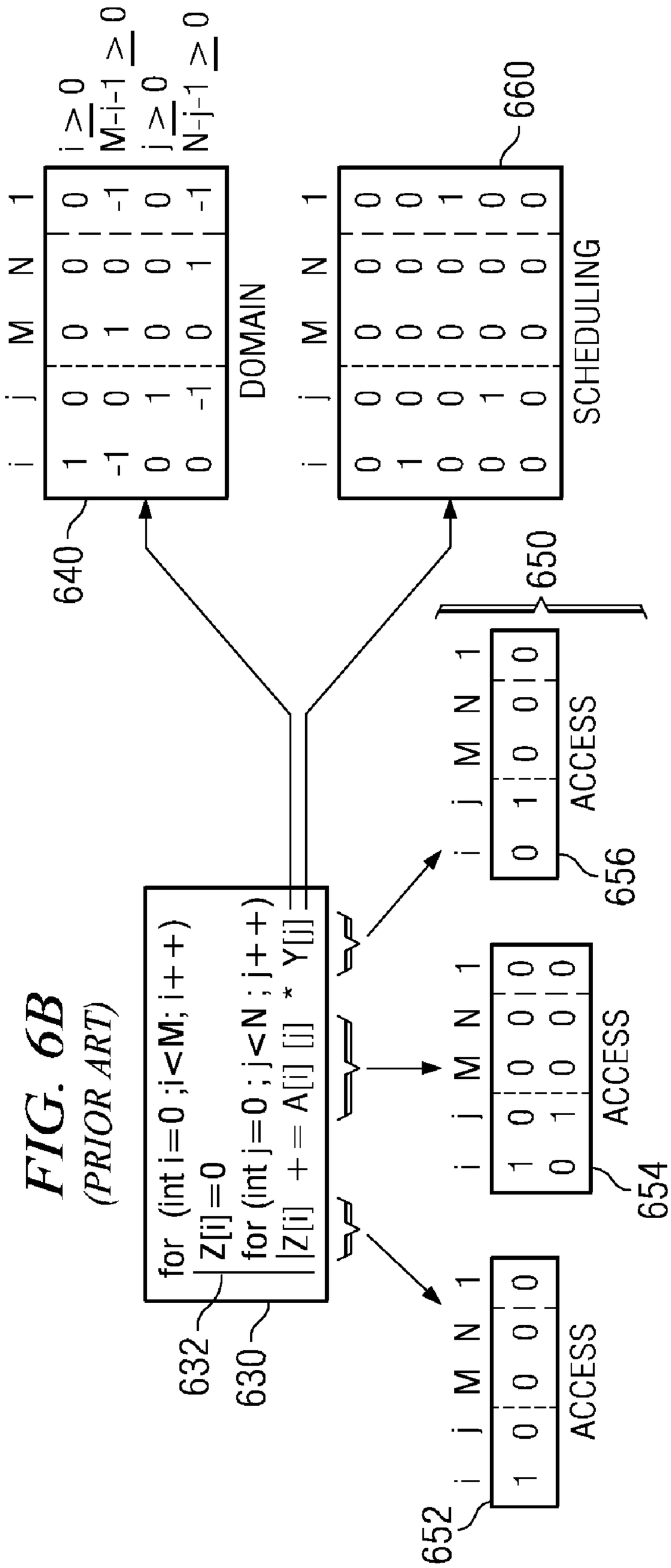


FIG. 6B
(PRIOR ART)
for (int i=0 ;i<M; i++)
Z[i]=0
for (int j=0 ; j<N ; j++)
Z[i] += A[i] [j] * Y[j]
632
630
640
DOMAIN
SCHEDULING
660
650
652
654
656
ACCESS

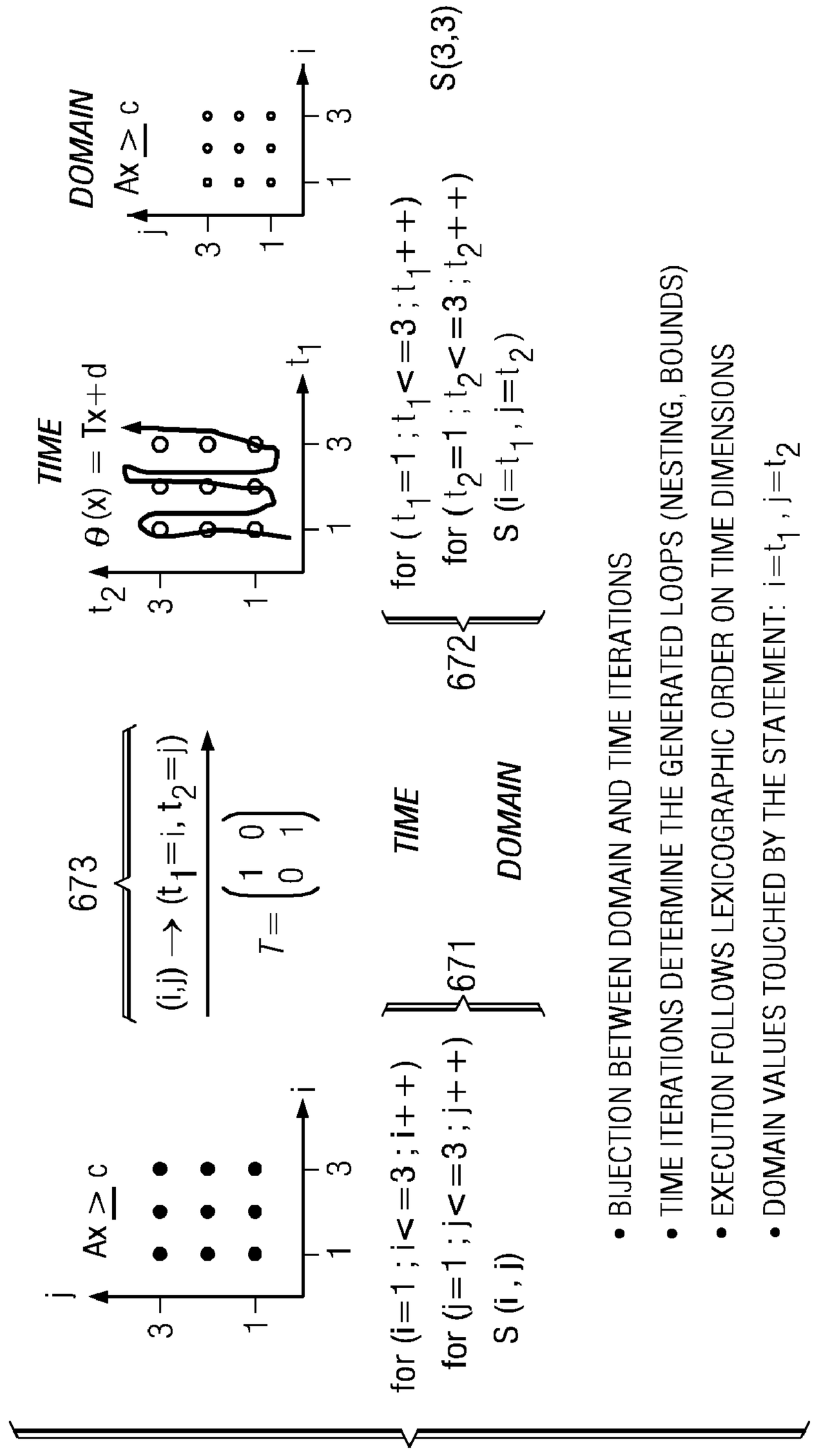


FIG. 6C
(PRIOR ART)

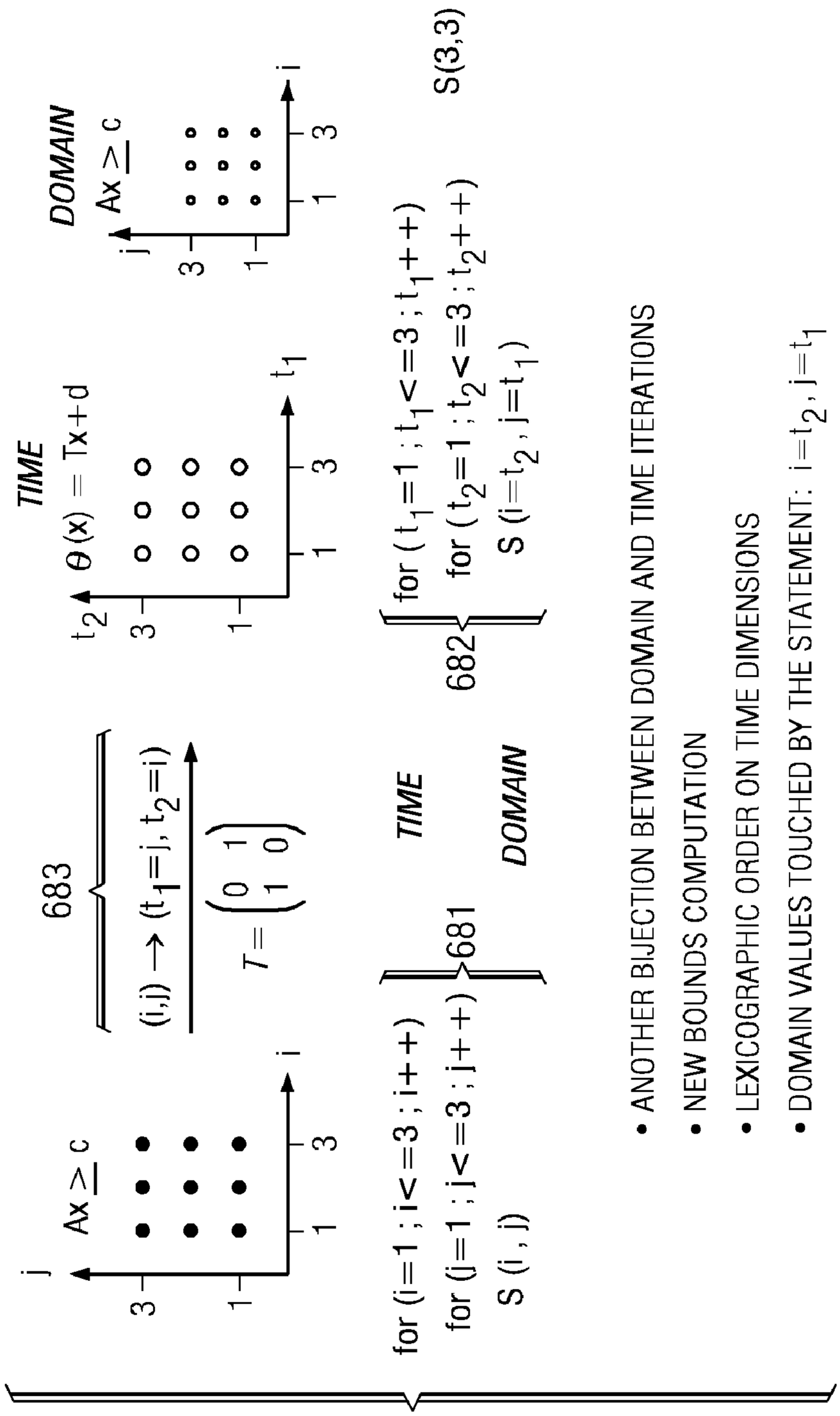


FIG. 6D
(PRIOR ART)

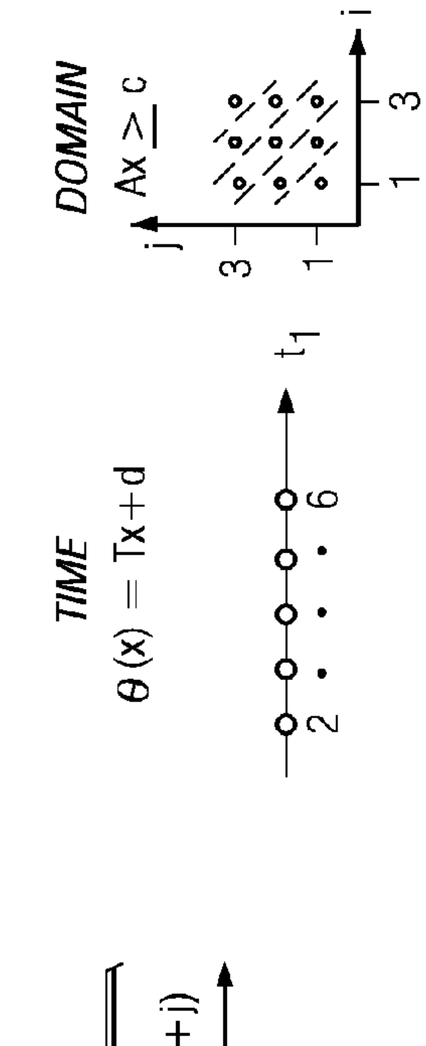


FIG. 6E
(PRIOR ART)

FIG. 7B

```
for  (i=1;i<=N;i++)
S1   |  S1 (i, i) ;
     |  for  (j=i;j<=N;j++)
S2   |  |  S2 (i, j)
S3   |  S3 (N, N) ;
```

FIG. 7A

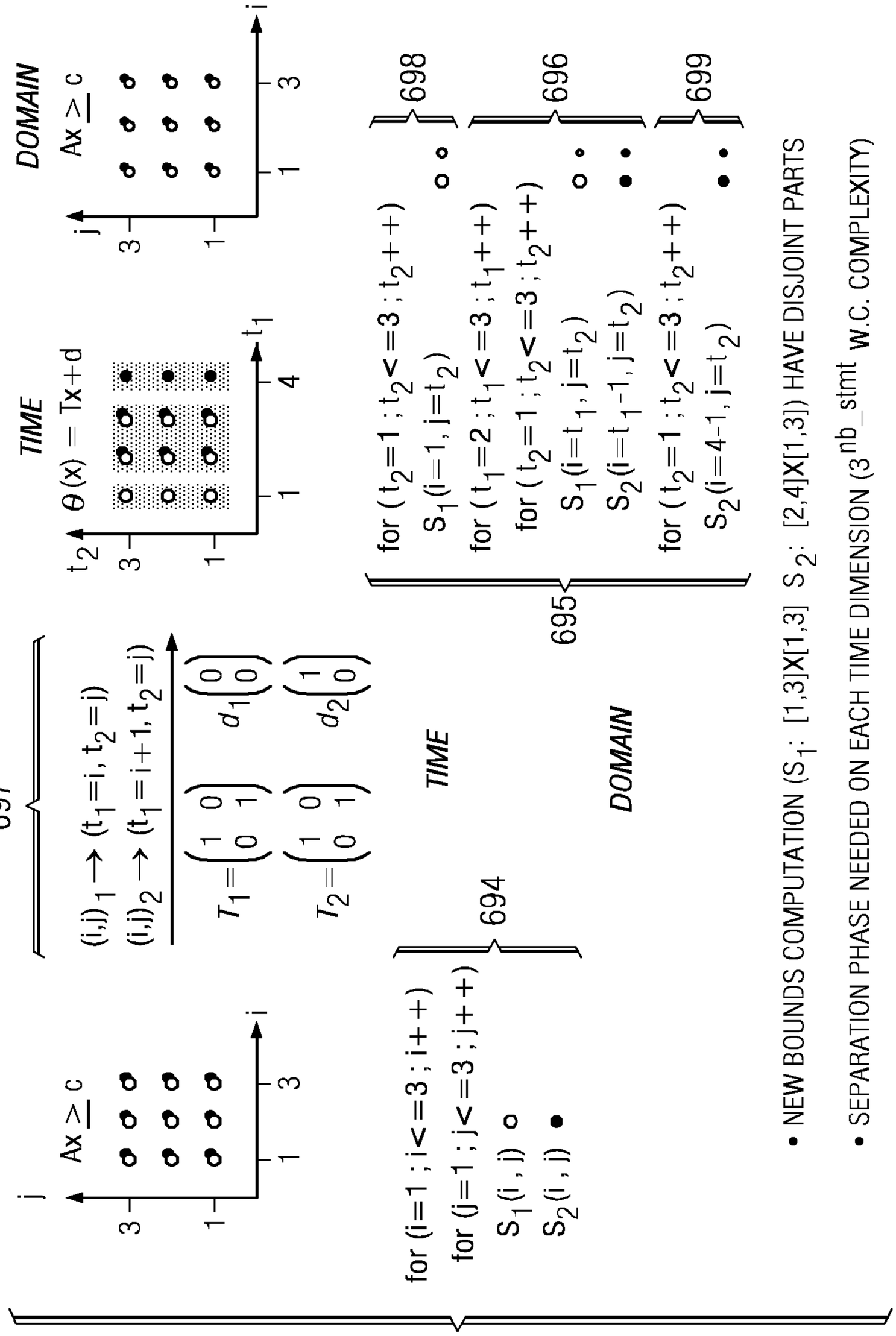
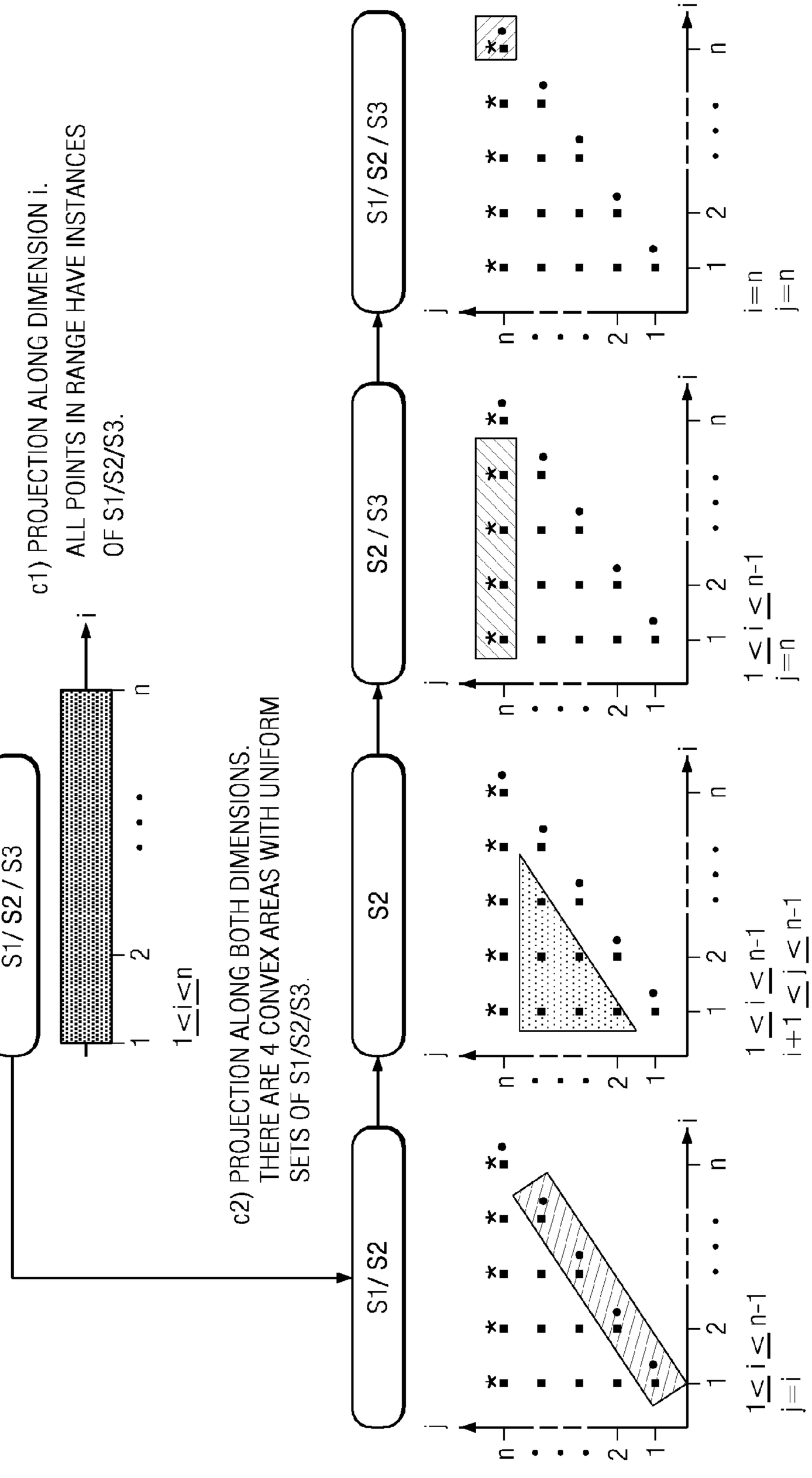


FIG. 6F
(PRIOR ART)

FIG. 7C
(PRIOR ART)



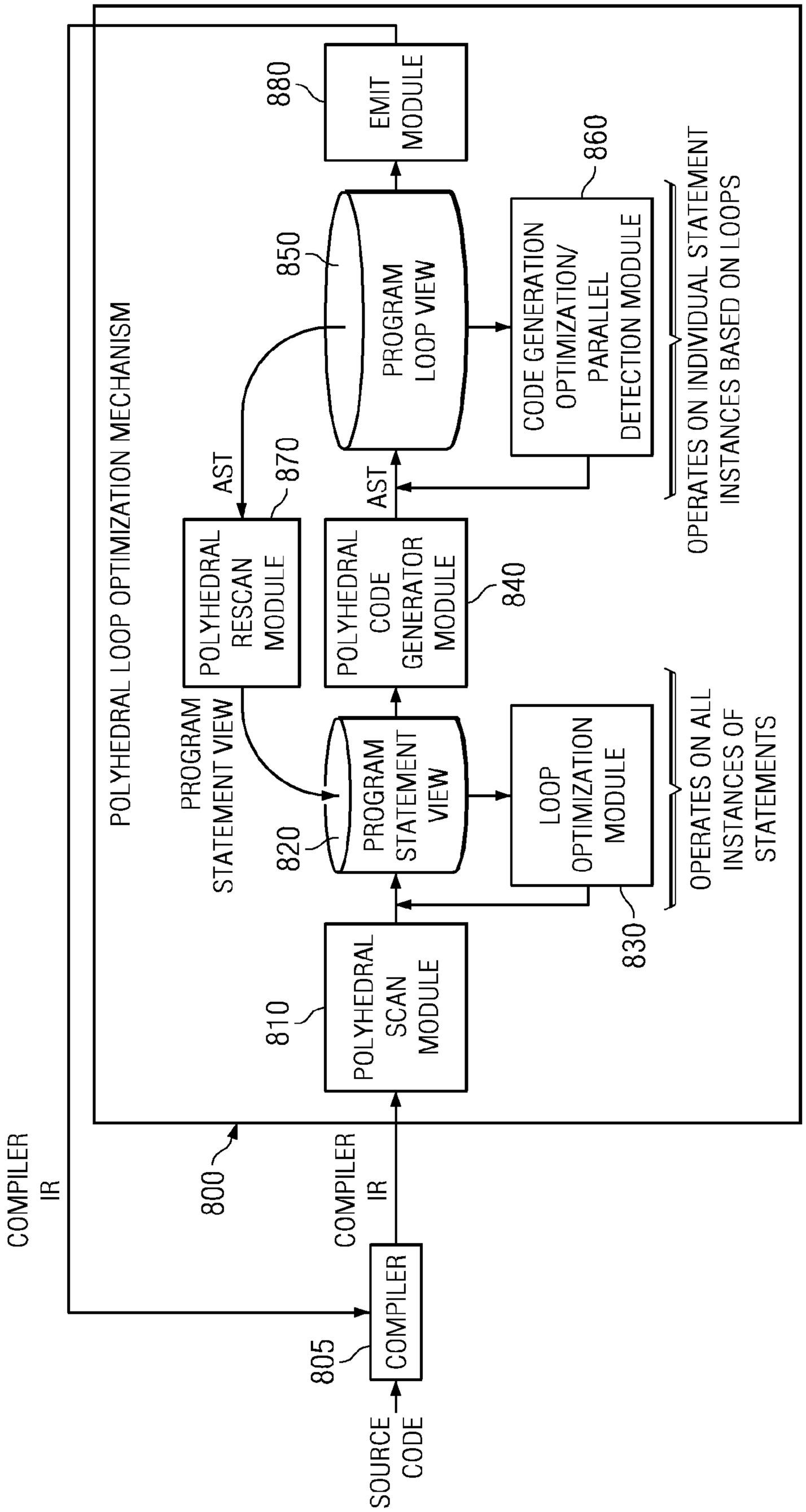

COMPILER IR
SOURCE CODE
COMPILER
805
800
COMPILER IR
POLYHEDRAL LOOP OPTIMIZATION MECHANISM
POLYHEDRAL SCAN MODULE
810
PROGRAM STATEMENT VIEW
820
LOOP OPTIMIZATION MODULE
830
OPERATES ON ALL INSTANCES OF STATEMENTS
POLYHEDRAL RESCAN MODULE
AST
870
POLYHEDRAL CODE GENERATOR MODULE
840
AST
PROGRAM LOOP VIEW
850
CODE GENERATION OPTIMIZATION/ PARALLEL DETECTION MODULE
860
OPERATES ON INDIVIDUAL STATEMENT INSTANCES BASED ON LOOPS
EMIT MODULE
880

FIG. 8

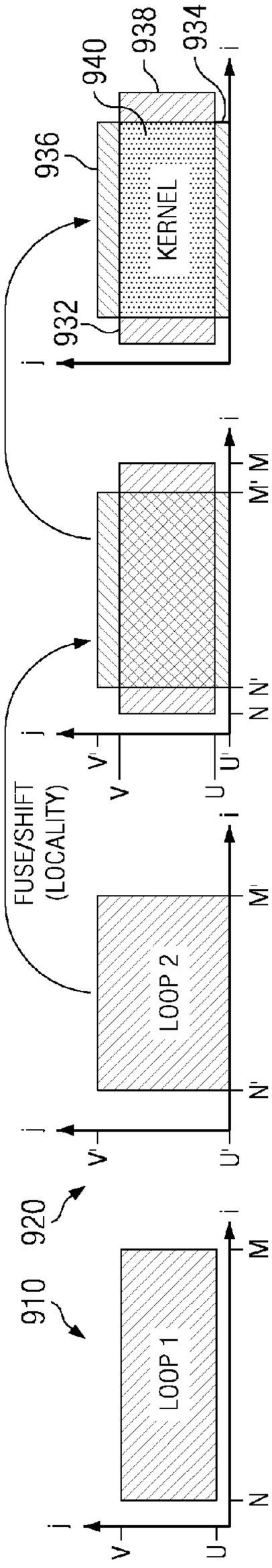


FIG. 9
(PRIOR ART)

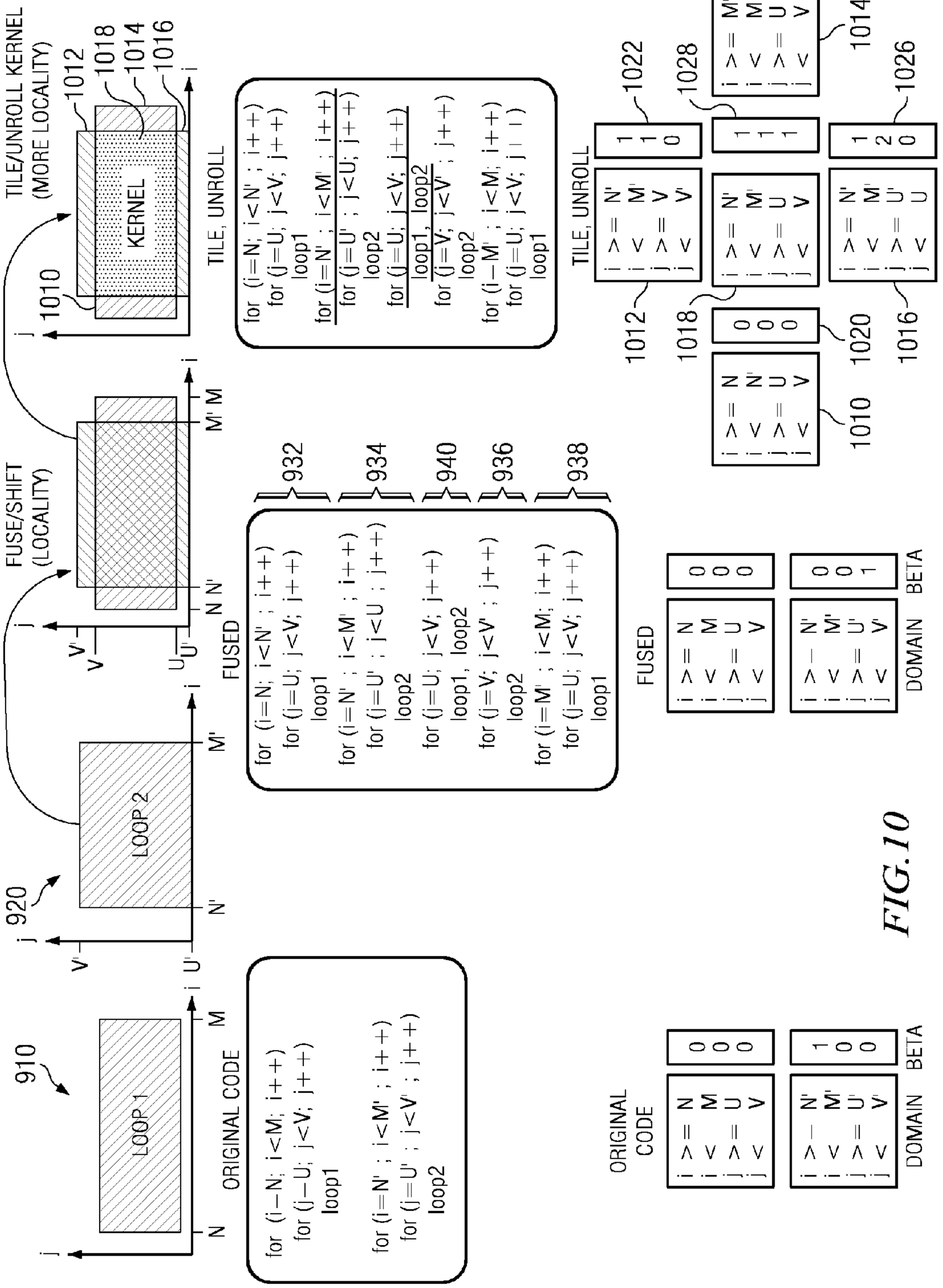

TILE/UNROLL KERNEL
(MORE LOCALITY)
1012
1018
1014
1016
KERNEL
1010
TILE, UNROLL
for (i=N; i<N'; i++)
for (j=U; j<V; j++)
loop1
for (i=N'; i<M'; i++)
for (j=U'; j<U; j++)
loop2
for (j=U; j<V; j++)
loop1, loop2
for (j=V; j<V'; j++)
loop2
for (i=M'; i<M; i++)
for (j=U; j<V; j++)
loop1
1022
1028
1014 1024
1026
1012
1018
1020
1016
1010
FUSED
932
934
940
936
938
FUSE/SHIFT
(LOCALITY)
FUSED
for (i=N; i<N'; i++)
for (j=U; j<V; j++)
loop1
for (i=N'; i<M'; i++)
for (j=U'; j<U; j++)
loop2
for (j=U; j<V; j++)
loop1, loop2
for (j=V; j<V'; j++)
loop2
for (i=M'; i<M; i++)
for (j=U; j<V; j++)
loop1
DOMAIN BETA
920
LOOP 2
910
LOOP 1
ORIGINAL CODE
for (i=N; i<M; i++)
for (j=U; j<V; j++)
loop1
for (i=N'; i<M'; i++)
for (j=U'; j<V'; j++)
loop2
ORIGINAL CODE
DOMAIN BETA

FIG. 10

```
ApplyCodegenTransformation: applies a transformation on P_IV
Input:
  node:A_ST root node
Output: P_IV after application of the transformation

  root ← node
  visitor ← new visitor(outer_visitor_type, node)
  while node has successors in traversal
    boolean changed ← false
    currentNode ← node.successor
    if currentNode is marked
1     core_node_list ← apply_core_function(currentNode, changed)
      if changed
        sort core_node_list under parent context
        foreach newNode in core_node_list
2         newNode.children ← currentNode.children
3         newNode ← propagate(newNode)
          if newNode.children size is 0
4           delete newNode

  return root
```

FIG. 11

```
          for (t1=1;t1 ≤ N ;t1++)
          |  if (t1≡N)
S11       |  |  S1(i=t1, j=N)
S21       |  |  S2(i=t1, j=N)
S31       |  |  S3(i=t1, j=N)
          |  if (t1≤ N-1)
S12       |  |  S1(i=t1, j=t1)
S22       |  |  S2(i=t1, j=t1)
          |  for(t2=t1+1;
          |  |   t2 ≤ N-1;t2++)
S23       |  |  S2(i=t1,j=t2)
          |  if (t1≤ N-1)
S24       |  |  S1(i=t1, j=N)
S32       |  |  S2(i=t1, j=N)
```

FIG. 12A

```
          for (t1=1;t1 ≤ N-1;t1++)
S11       |  S1(i=t1, j=t1)
S21       |  S2(i=t1, j=t1)
          |  for(t2=t1+1;
          |       t2 ≤ N-1;t2++)
S22       |  |  S2(i=t1, j=t2)
S23       |  S2(i−t1, j−N)
S31       |  S3(i=t1, j=N)
S12    S1(i=N, j=N)
S24    S2(i=N, j=N)
S32    S2(i=N, j=N)
```

FIG. 12B

```
        for (t1 = 1;t1 ≤ N - 2;t1++)
S1 1    | S1(i=t1, j=t1)
S2 1    | S2(i=t1, j=t1)
        | for(t2=t1+1;
        |     t2 ≤ N - 1;t2++)
S2 2    | | S2(i=t1,j = t2)
S2 3    | S2(i=t1,j = N)
S3 1    | S3(i=t1,j = N)
S1 2    S1(i=N - 1,j = N -1)
S2 4    S2(i=N - 1,j = N -1)
S2 5    S2(i=N - 1,j = N)
S3 2    S3(i=N - 1,j = N)
S1 3    S1(i=N,j = N)
S2 6    S2(i=N,j = N)
S3 3    S3(i=N,j = N)
```

FIG. 12C

ORIGINAL CODE

```
for (i=0;i ≤ N - 1;i++)
| for (j=0;i ≤ N - 1;j++)
| | S(i,j)
```

FIG. 13A

SKEWING ON FIRST DIMENSION

```
for (t1=0;t1≤ 2N - 2;t1++)
| for (t2=max(0,t1 - N + 1);
|     t2 ≤ min(t1,N - 1);t2++)
| | S(t1 - t2,t2)
```

FIG. 13B

AFTER OPTIMIZATION/TRANSFORMATION

```
for (t1=0;t1 ≤ N - 2;t1++)
| for (t2=0;t2 ≤ t1;t2++)
| | S(t1 -t2,t2)
for (t1 =N - 1;t1≤ 2*N - 2;t1++)
| for (t2=t1 - N + 1;t2 ≤ t1;t2++)
| | S(t1 -t2,t2)
```

FIG. 13C

NULL A

$$A^{S_1} = [\overset{i}{0}] \quad \beta^{S_1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$A^{S_2} = [\overset{i}{0}] \quad \beta^{S_2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

NULL A CODE

```
    t0 =0
    | #pragma omp parallel
    | for (d1 =1 ;d1 ≤ N ;d1++)
S1  | | S1(i = d1)
    | #pragma omp parallel
    | for (d1 =1 ;d1 ≤ N ;d1++)
S2  | | S2(i = d1)
```

FIG. 14A

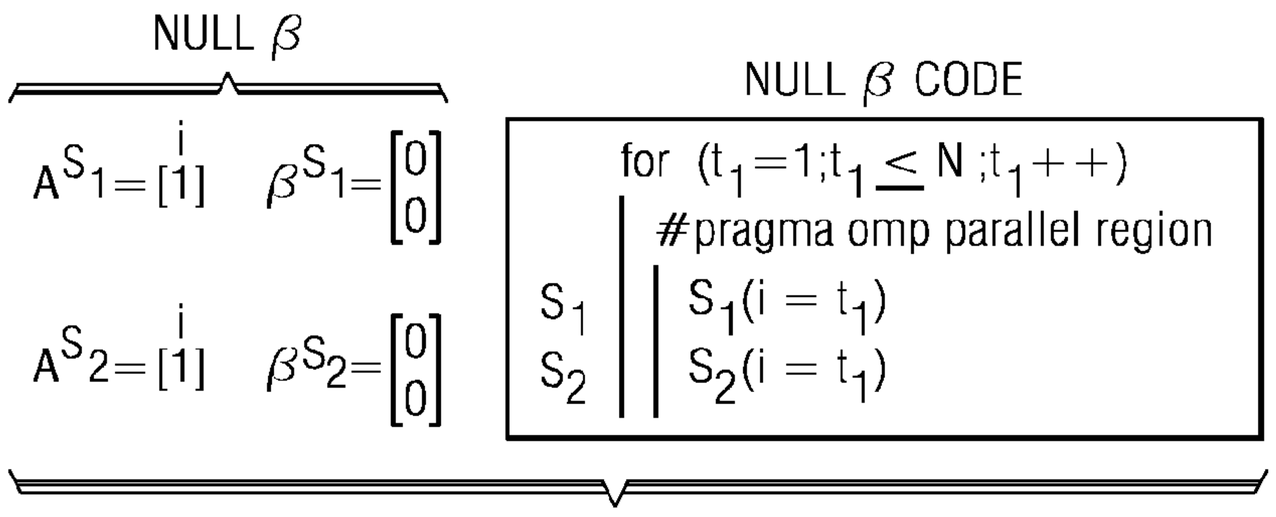


FIG. 14B

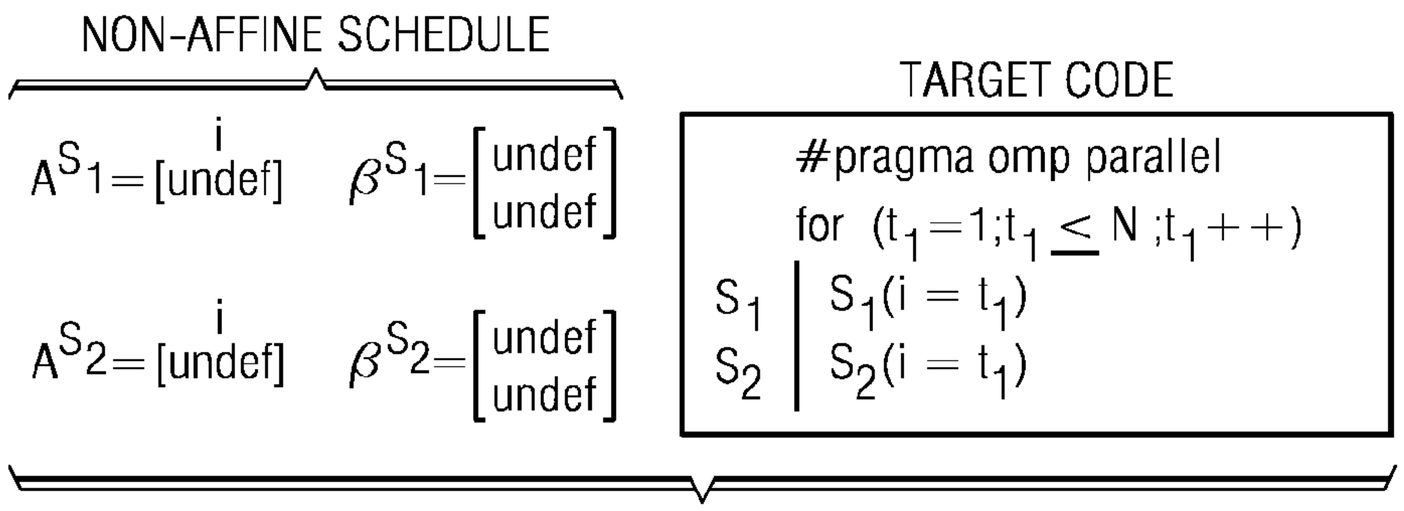


FIG. 14C

FIG. 15

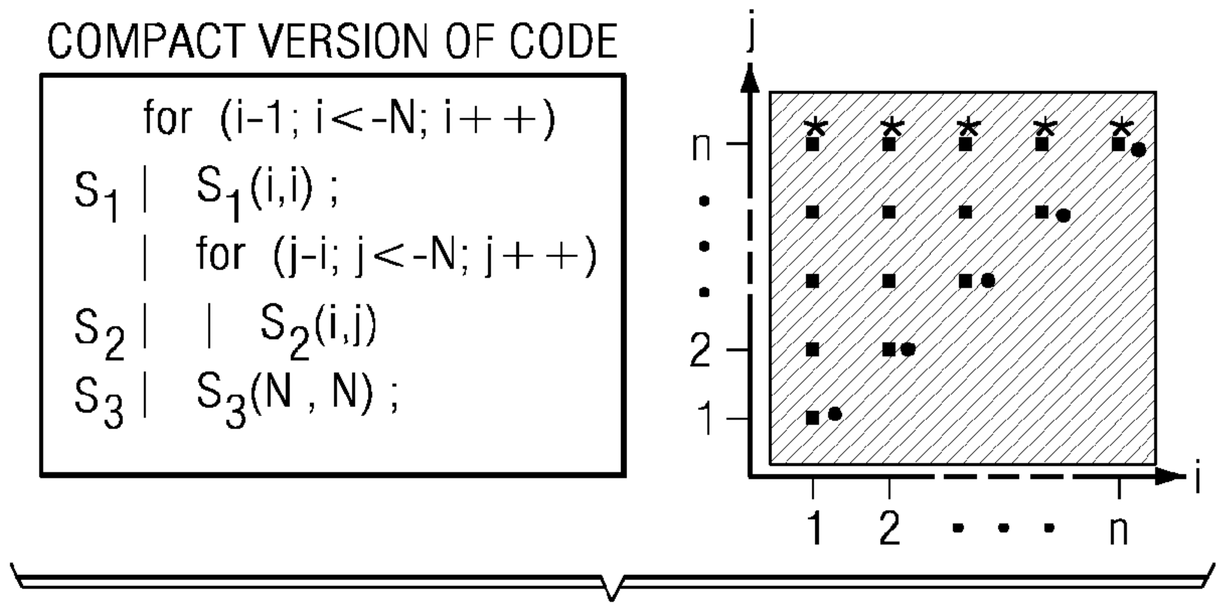


FIG. 16A

GUARDED VERSION OF CODE

```
      for  (i-1;i<-N;i++)
      |    if   (i<=N-1)
S1.1  |    |    S1 (i,i);
S2.1  |    |    S2 (i,i)
      |    |    for  (j-i+1;j<-N-1;j++)
S2.2  |    |    |   S2  (i,j )
S2.3  |    |    S2 (i,N);
S3.1  |    |    S3 (i,N);
      |    if   (i==N)
S1.2  |    |    S1 (i,N)
S2.4  |    |    S2 (i,N)
S3.2  |    |    S3 (i,N)
```

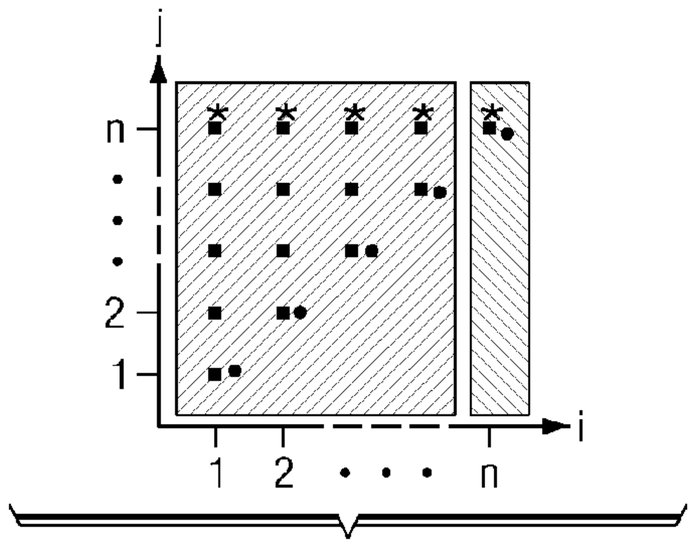


FIG. 16B

RESCANNED VERSION OF CODE

```
      for  (i-1;i<-N-2;i++)
S1.1  |    S1 (i,i);
S2.1  |    S2 (i,i)
      |    for  (j=i+1;j<=N-1;j++)
S2.2  |    |   S2 (i,j )
S2.3  |    S2 (i,N);
S3.1  |    S3 (i,N);
      if   (1<=N-1)
S1.2  |    S1 (i,N-1)
S2.4  |    S2 (i,N-1)
S2.5  |    S2 (i,N-1)
S3.2  |    S3 (i,N-1)
      if   (1<=N)
S1.3  |    S1 (i,N)
S2.6  |    S2 (i,N)
S3.3  |    S3 (i,N)
```

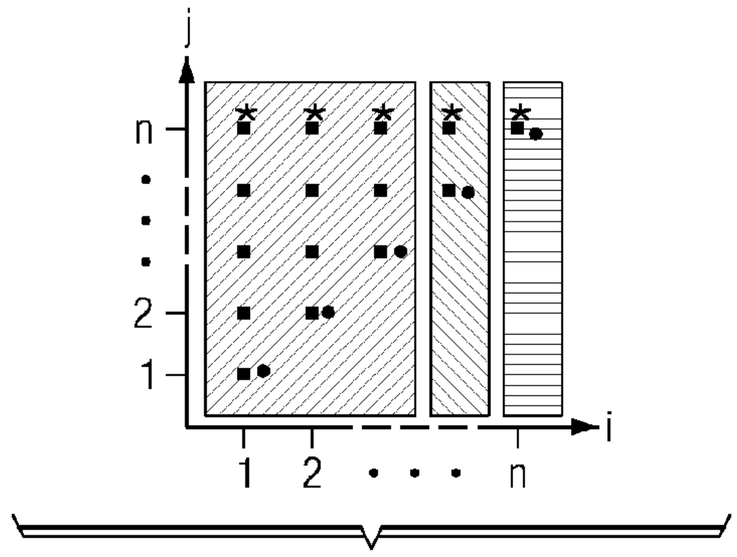


FIG. 16C

ORIGINAL COMPLEX SCHEDULE

```
      for  (t1=1;t1≤ 3M - 7;t1+=1)
      |    for  (t2=(t1 + 4) /3;t2≤ (t1 + 6) /3;t2+=1)
      |    ...
      |    |    if (t1 % 3 ≡ 0)
S1    |    |    |  S1 (t2,  t1/3);
      |    |    if (t1 % 3 ≡ 2)
S1    |    |    |  S2((t1 +1) /3,  t2);
      |    ...
```

FIG. 17A

AFTER REGENERATION WITHOUT STRETCHING

```
                for (t1=5;t1 ≤ 3M - 9;t1+=1)
                |  ...
                |  if(-t1 + 3M ≡ 9 && M ≥ 5)
                |  |  if (t1 % 3 ≡ 0)
S11             |  |  |  S1 (M - 2, t1/3);
                |  ...
                |  |  if (t1 % 3 ≡ 0)
                |  |  |  t2=(t1 + 3)/3;
                |  |  |  if (t2 ≥ 3 && t2 ≤ M - 3)
                |  |  |  |  if (t1 % 3 ≡ 0)
S12             |  |  |  |  |  S1 (t2, t1/3);
                |  if (t1 ≤ 3M - 10)
                |  |  for (t2=(t1 + 4) /3;t2 ≤ (t1 + 6) /3;t2+=1)
                |  |  |  if (t1 % 3 ≡ 0)
S13             |  |  |  |  S1 (t2, t1/3);
                |  |  |  if (t1 % 3 ≡ 2)
S21             |  |  |  |  S2((t1 +1) /3, t2);
                |  ...
                |  if (-t1 + 3M ≡ 9 && M ≥ 5)
                |  |  for (t2=M - 1;t2 ≤ M ;t2+=1)
                |  |  |  if (t1 % 3 ≡ 0)
S14             |  |  |  |  S1 (t2, t1/3);
                |  |  |  if (t1 % 3 ≡ 2)
                |  ...
S22             |  |  |  |  S2((t1 +1) /3, t2);
```

FIG. 17B

AFTER REGENERATION WITH STRETCHING

```
                for (t1=1;t1 ≤ 3M - 7;t1+=1)
                |  for (t2=(t1 + 4) /3;t2 ≤ (t1 + 6) /3;t2+=1)
                |  ...
                |  |  if (t1 % 3 ≡ 0)
S1              |  |  |  S1 (t2, t1/3);
                |  |  if (t1 % 3 ≡ 2)
S1              |  |  |  S2((t1 +1) /3, t2);
                |  ...
```

FIG. 17C

ORIGINAL PROGRAM $\mathcal{P}$

$$A^{S_1} = [\overset{i}{3}] \quad \beta^{S_1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \Gamma^{S_1} = \left[\begin{array}{c|c} \overset{N}{\cdot} & \overset{1}{\cdot} \end{array}\right] \quad \mathcal{D}^{S_1} = \left[\begin{array}{c|c|c} \overset{i}{1} & \overset{N}{\cdot} & \overset{1}{-1} \\ -1 & 1 & \cdot \end{array}\right]$$

$$A^{S_2} = [\overset{i}{3}] \quad \beta^{S_2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \Gamma^{S_2} = \left[\begin{array}{c|c} \overset{N}{\cdot} & \overset{1}{5} \end{array}\right] \quad \mathcal{D}^{S_2} = \left[\begin{array}{c|c|c} \overset{i}{1} & \overset{N}{\cdot} & \overset{1}{-1} \\ -1 & 1 & \cdot \end{array}\right]$$

FIG. 18A

SCATTER AT FIRST GENERATION

$$\hat{\mathcal{D}}^{S_1} = \left[\begin{array}{c|c|c} \overset{t_1}{1} & \overset{N}{\cdot} & \overset{1}{-3} \\ -1 & 3 & \cdot \end{array}\right]$$

$$\hat{\mathcal{D}}^{S_2} = \left[\begin{array}{c|c|c} \overset{t_1}{1} & \overset{N}{\cdot} & \overset{1}{-8} \\ -1 & 3 & 5 \end{array}\right]$$

FIG. 18B

$A^{\mathcal{P}}_{ST}$ AFTER FIRST GENERATION

```
        for (t1 = 3; t1 ≤ 7; t1++)
        |  if(t1 %3 ≡ 0)
S11     |  |  S1 (i = t1/3)
        for (t1 = 8; t1 ≤ 3N; t1++)
        |  if(t1 %3 ≡ 0)
S12     |  |  S1 (i = t1/3)
        |  if(t1 %3 ≡ 2)
S21     |  |  S2 (i = (t1 - 5)/3)
        for (t1 = 3N + 1; t1 ≤ 3N + 5; t1++)
        |  if(t1 %3 ≡ 2)
S22     |  |  S2 (i = (t1 - 5)/3)
```

FIG. 18C

REGENERATED PROGRAM $\mathcal{P}'$

$$A^{S1_1} = [\overset{i}{3}] \quad \beta^{S1_1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \Gamma^{S1_1} = \left[\begin{array}{c|c} \overset{N}{\cdot} & \overset{1}{\cdot} \end{array}\right] \quad \mathcal{D}^{S1_1} = \left[\begin{array}{c|c|c} \overset{i}{1} & \overset{N}{\cdot} & \overset{1}{-1} \\ -1 & \cdot & 2 \end{array}\right]$$

$$A^{S1_2} = [\overset{i}{3}] \quad \beta^{S1_2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \Gamma^{S1_2} = \left[\begin{array}{c|c} \overset{N}{\cdot} & \overset{1}{\cdot} \end{array}\right] \quad \mathcal{D}^{S1_2} = \left[\begin{array}{c|c|c} \overset{i}{1} & \overset{N}{\cdot} & \overset{1}{3} \\ -1 & 1 & \cdot \end{array}\right]$$

$$A^{S2_1} = [\overset{i}{3}] \quad \beta^{S2_1} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \Gamma^{S2_1} = \left[\begin{array}{c|c} \overset{N}{\cdot} & \overset{1}{5} \end{array}\right] \quad \mathcal{D}^{S2_1} = \left[\begin{array}{c|c|c} \overset{i}{1} & \overset{N}{\cdot} & \overset{1}{-1} \\ -1 & 1 & -2 \end{array}\right]$$

$$A^{S2_2} = [\overset{i}{3}] \quad \beta^{S2_2} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \quad \Gamma^{S2_2} = \left[\begin{array}{c|c} \overset{N}{\cdot} & \overset{1}{5} \end{array}\right] \quad \mathcal{D}^{S2_2} = \left[\begin{array}{c|c|c} \overset{i}{1} & \overset{N}{1} & \overset{1}{-1} \\ -1 & 1 & \cdot \end{array}\right]$$

FIG. 18D

SCATTER AT SECOND GENERATION WITHOUT STRETCHING

$$\tilde{\mathcal{G}}^{S1_2} = \begin{array}{c|c|c} t_1 & N & 1 \\ \hline 1 & \cdot & -9 \\ -1 & 3 & \cdot \end{array}$$

$$\tilde{\mathcal{G}}^{S2_1} = \begin{array}{c|c|c} t_1 & N & 1 \\ \hline 1 & \cdot & -8 \\ -1 & 3 & -1 \end{array}$$

FIG. 18E

UNSTABLE $A_{ST}^{\mathcal{P}'}$ AT SECOND GENERATION WITHOUT STRETCHING

```
                for (t1=3;t1≤6;t1++)
                    if(t1 %3≡0)
S11             |   S1(i =t1/3)
                t1=8;
                    if(t1 %3≡2)
S21             |   S2(i = (t1 - 5)/3)
                for (t1=9;t1≤3N - 1;t1++)
                    if(t1 %3≡0)
S12             |   S1(i = t1/3)
                    if(t1 %3≡2)
S22             |   S2(i = (t1 - 5)/3)
                t1=3N;
                    if(t1 %3≡0)
S13             |   S1(i = t1/3)
                for (t1=3N + 2;t1≤3N + 5;t1++)
                    if(t1 %3≡2)
S23             |   S2(i = (t1 - 5)/3)
```

FIG. 18F

SCATTER AT SECOND GENERATION WITH STRETCHING

$$\tilde{\mathcal{G}}^{S1_2} = \begin{array}{c|c|c} t_1 & N & 1 \\ \hline 1 & \cdot & -6 \\ -1 & 3 & 2 \end{array}$$

$$\tilde{\mathcal{G}}^{S2_1} = \begin{array}{c|c|c} t_1 & N & 1 \\ \hline 1 & \cdot & -6 \\ -1 & 3 & 2 \end{array}$$

FIG. 19A

STABLE $A_{ST}^{\mathcal{P}'}$ AT SECOND GENERATION WITH STRETCHING

```
                for (t1=1;t1≤5;t1++)
                    if(t1%3≡0)
S11             |   S1(i =t1/3)
                for (t1=6;t1≤3N + 2;t1++)
                    if(t1%3≡0)
S12             |   S1(i =t1/3)
                    if(t1%3 ≡ 2)
S22             |   S2(i = (t1 - 5)/3)
                for (t1=3N + 3;t1≤3N + 7;t1++)
                    if(t1%3≡2)
S23             |   S2(i = (t1 - 5)/3)
```

FIG. 19B

ORIGINAL, ILL BEHAVED PROGRAM $\mathcal{P}$

$$A^{S_1} = [3]^{i} \quad \beta^{S_1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \Gamma^{S_1} = \left[\begin{array}{c|c} N & 1 \\ 1 & -2 \end{array}\right] \quad \mathfrak{D}^{S_1}_{om} = \left[\begin{array}{c|c|c} i & N & 1 \\ 1 & \cdot & -1 \\ -1 & 1 & \cdot \end{array}\right]$$

$$A^{S_2} = [3]^{i} \quad \beta^{S_2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \Gamma^{S_2} = \left[\begin{array}{c|c} N & 1 \\ 2 & 5 \end{array}\right] \quad \mathfrak{D}^{S_2}_{om} = \left[\begin{array}{c|c|c} i & N & 1 \\ 1 & \cdot & -1 \\ -1 & 1 & \cdot \end{array}\right]$$

FIG. 20A

$A^{\mathcal{P}}_{ST}$ AFTER FIRST GENERATION

```
S1 1 | for (t1 = N ; t1 ≤ 2N + 7; t1++)
     |   if((t1 - N)%3 ≡ 1)
     |     S1(i = (t1 - N + 2)/3)
     | for (t1 = 2N + 8; t1 ≤ 4N - 2; t1++)
S1 2 |   if((t1 - N)%3 ≡ 1)
     |     S1(i = (t1 - N + 2)/3)
     |   if((t1 - 2N)%3 ≡ 1)
S2 1 |     S2(i = (t1 - 2N - 5)/3)
     | for (t1 = 4N - 1; t1 ≤ 5N + 5; t1++)
     |   if((t1 - 2N)%3 ≡ 1)
S2 2 |     S2(i = (t1 - 2N - 5)/3)
```

FIG. 20B

$A^{\mathcal{P}'}_{ST}$ AFTER SECOND GENERATION

```
S1 1 | for (t1 = N ; t1 ≤ 2N + 7; t1++)
     |   if((t1 - N)%3 ≡ 1)
     |     S1(i = (t1 - N + 2)/3)
     | for (t1 = 2N + 8; t1 ≤ 4N - 2; t1++)
S1 2 |   if((t1 - N)%3 ≡ 1)
     |     S1(i = (t1 - N + 2)/3)
     |   if((t1 - 2N)%3 ≡ 1)
S2 1 |     S2(i = (t1 - 2N - 5)/3)
     | for (t1 = 4N - 1; t1 ≤ 5N + 5; t1++)
     |   if((t1 - 2N)%3 ≡ 1)
S2 2 |     S2(i = (t1 - 2N - 5)/3)
```

FIG. 20C

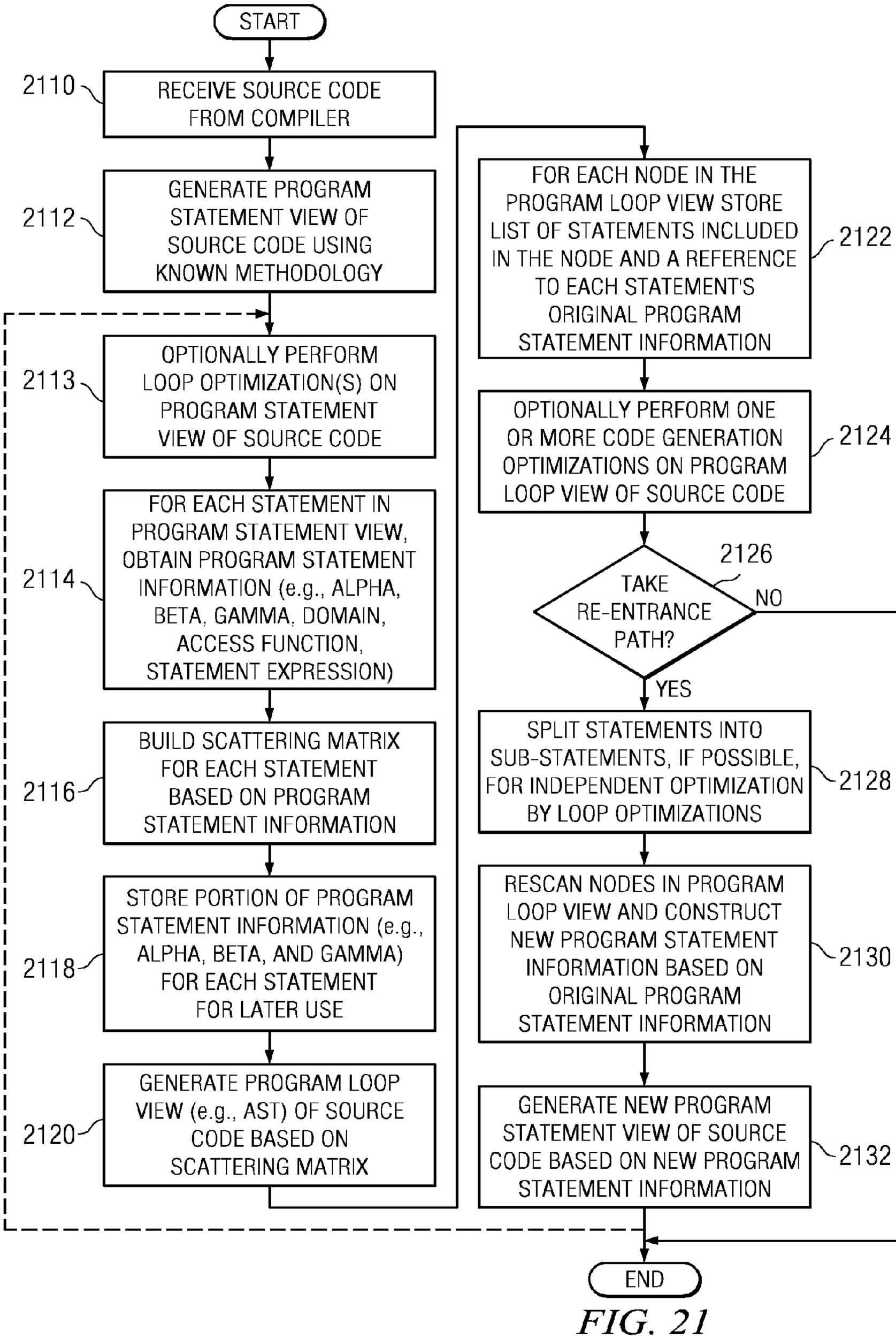

START
2110
RECEIVE SOURCE CODE FROM COMPILER
2112
GENERATE PROGRAM STATEMENT VIEW OF SOURCE CODE USING KNOWN METHODOLOGY
2113
OPTIONALLY PERFORM LOOP OPTIMIZATION(S) ON PROGRAM STATEMENT VIEW OF SOURCE CODE
2114
FOR EACH STATEMENT IN PROGRAM STATEMENT VIEW, OBTAIN PROGRAM STATEMENT INFORMATION (e.g., ALPHA, BETA, GAMMA, DOMAIN, ACCESS FUNCTION, STATEMENT EXPRESSION)
2116
BUILD SCATTERING MATRIX FOR EACH STATEMENT BASED ON PROGRAM STATEMENT INFORMATION
2118
STORE PORTION OF PROGRAM STATEMENT INFORMATION (e.g., ALPHA, BETA, AND GAMMA) FOR EACH STATEMENT FOR LATER USE
2120
GENERATE PROGRAM LOOP VIEW (e.g., AST) OF SOURCE CODE BASED ON SCATTERING MATRIX
2122
FOR EACH NODE IN THE PROGRAM LOOP VIEW STORE LIST OF STATEMENTS INCLUDED IN THE NODE AND A REFERENCE TO EACH STATEMENT'S ORIGINAL PROGRAM STATEMENT INFORMATION
2124
OPTIONALLY PERFORM ONE OR MORE CODE GENERATION OPTIMIZATIONS ON PROGRAM LOOP VIEW OF SOURCE CODE
2126
TAKE RE-ENTRANCE PATH?
NO
YES
2128
SPLIT STATEMENTS INTO SUB-STATEMENTS, IF POSSIBLE, FOR INDEPENDENT OPTIMIZATION BY LOOP OPTIMIZATIONS
2130
RESCAN NODES IN PROGRAM LOOP VIEW AND CONSTRUCT NEW PROGRAM STATEMENT INFORMATION BASED ON ORIGINAL PROGRAM STATEMENT INFORMATION
2132
GENERATE NEW PROGRAM STATEMENT VIEW OF SOURCE CODE BASED ON NEW PROGRAM STATEMENT INFORMATION
END

FIG. 21

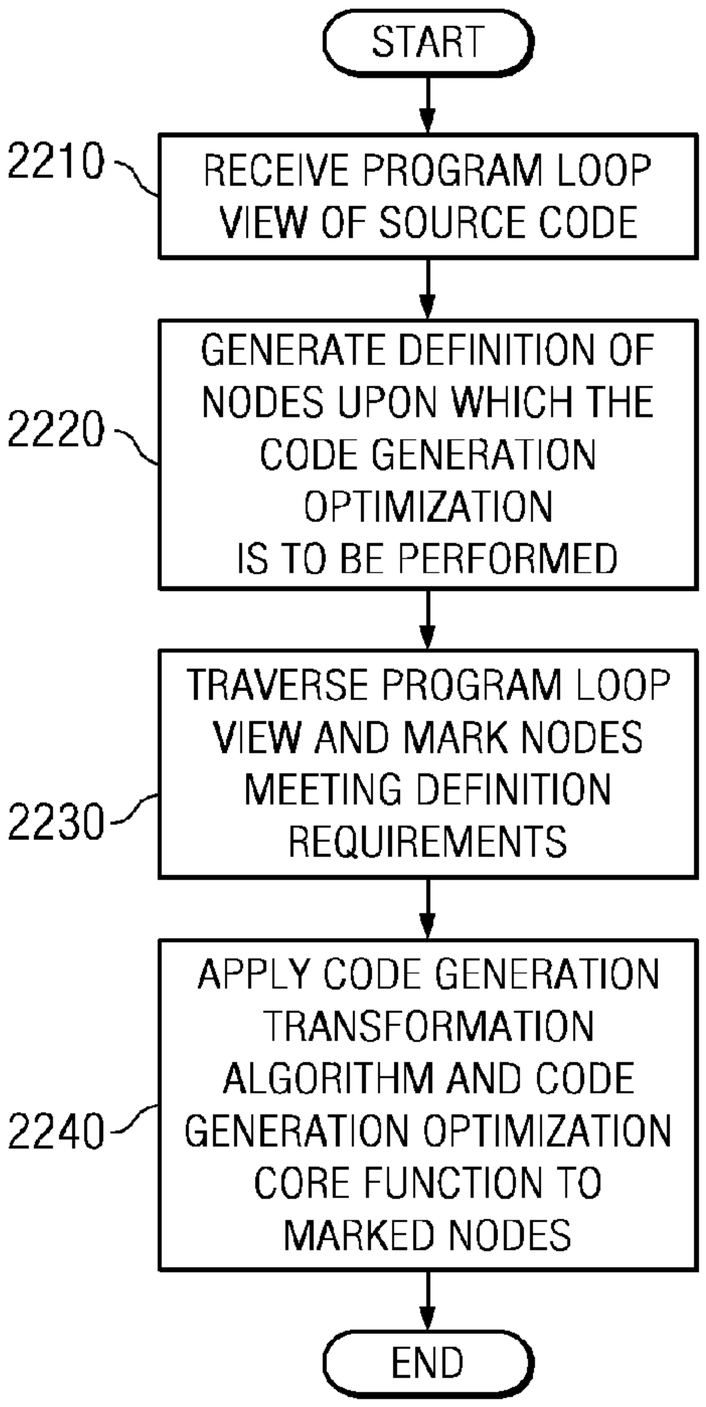

START
2210
RECEIVE PROGRAM LOOP VIEW OF SOURCE CODE
2220
GENERATE DEFINITION OF NODES UPON WHICH THE CODE GENERATION OPTIMIZATION IS TO BE PERFORMED
2230
TRAVERSE PROGRAM LOOP VIEW AND MARK NODES MEETING DEFINITION REQUIREMENTS
2240
APPLY CODE GENERATION TRANSFORMATION ALGORITHM AND CODE GENERATION OPTIMIZATION CORE FUNCTION TO MARKED NODES
END

FIG. 22

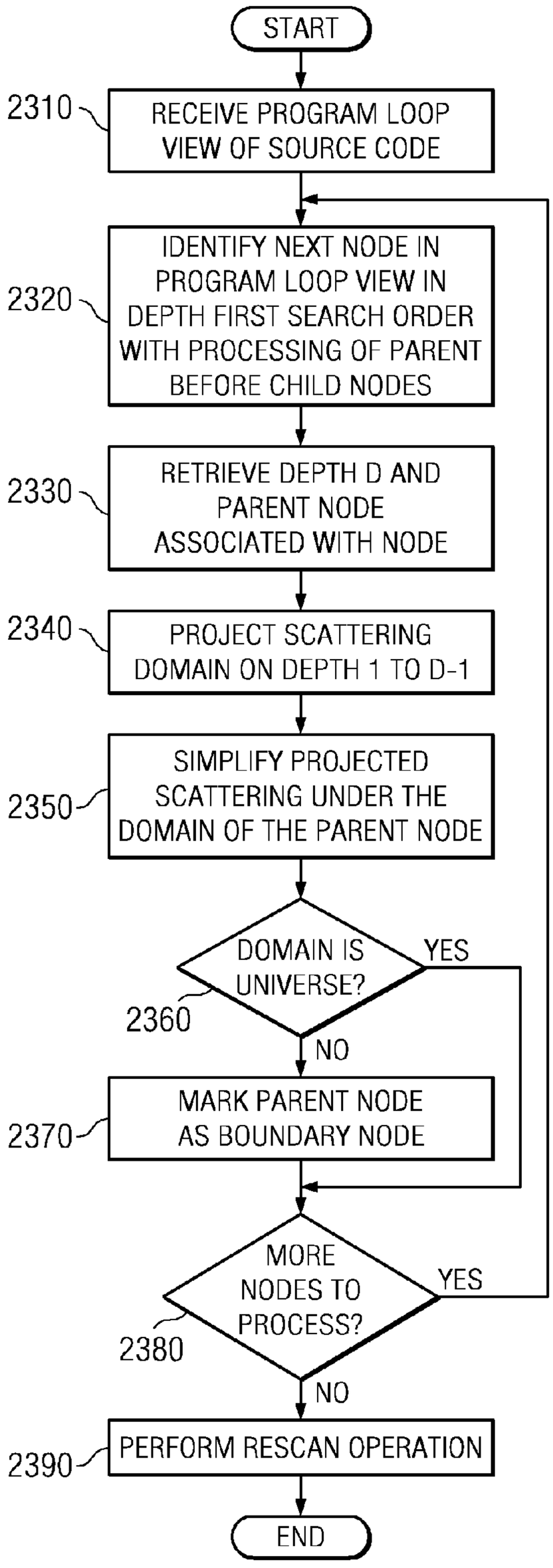

START
2310
RECEIVE PROGRAM LOOP VIEW OF SOURCE CODE
2320
IDENTIFY NEXT NODE IN PROGRAM LOOP VIEW IN DEPTH FIRST SEARCH ORDER WITH PROCESSING OF PARENT BEFORE CHILD NODES
2330
RETRIEVE DEPTH D AND PARENT NODE ASSOCIATED WITH NODE
2340
PROJECT SCATTERING DOMAIN ON DEPTH 1 TO D-1
2350
SIMPLIFY PROJECTED SCATTERING UNDER THE DOMAIN OF THE PARENT NODE
DOMAIN IS UNIVERSE?
2360
YES
NO
2370
MARK PARENT NODE AS BOUNDARY NODE
MORE NODES TO PROCESS?
2380
YES
NO
2390
PERFORM RESCAN OPERATION
END

FIG. 23

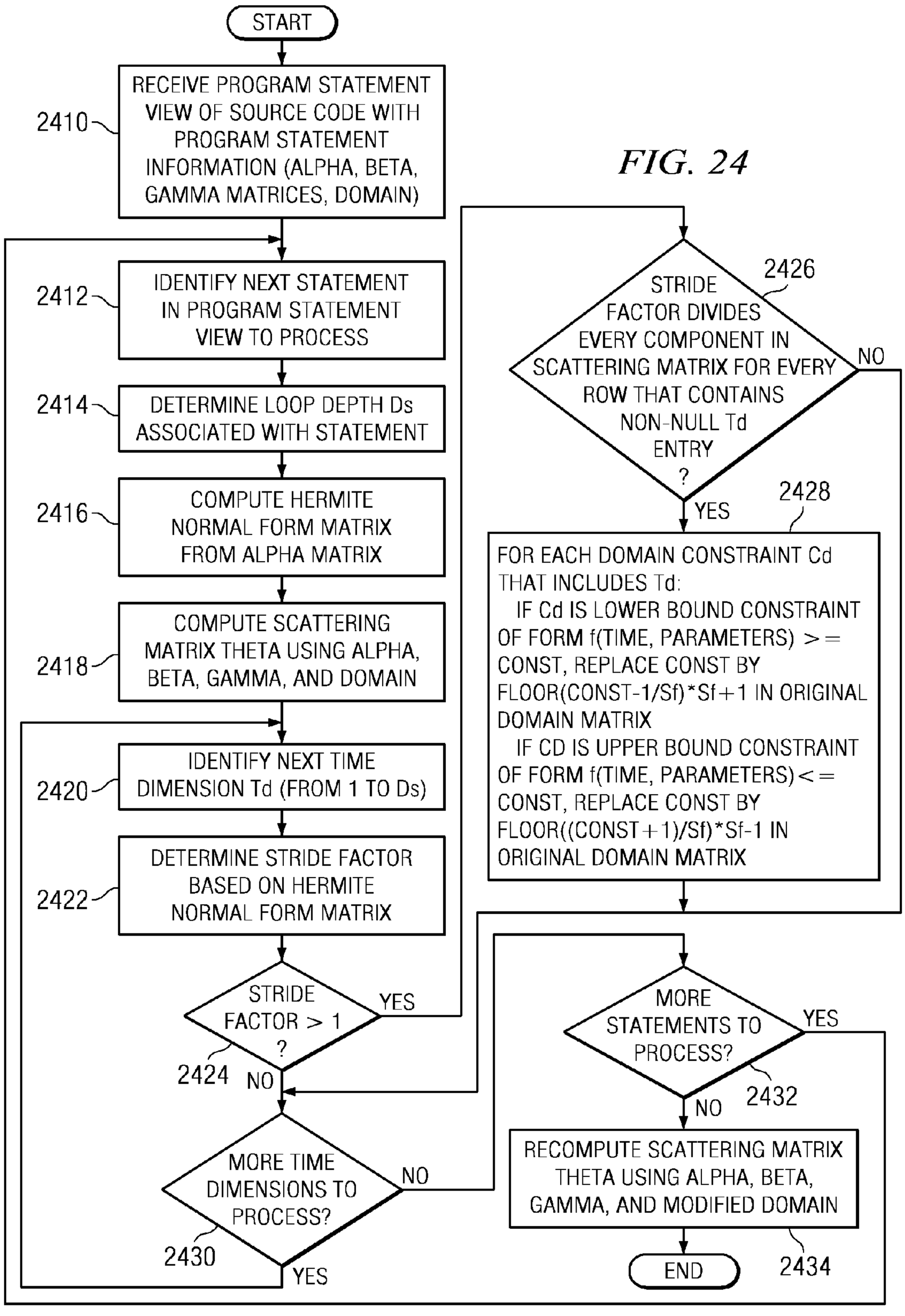

START
2410
RECEIVE PROGRAM STATEMENT VIEW OF SOURCE CODE WITH PROGRAM STATEMENT INFORMATION (ALPHA, BETA, GAMMA MATRICES, DOMAIN)
2412
IDENTIFY NEXT STATEMENT IN PROGRAM STATEMENT VIEW TO PROCESS
2414
DETERMINE LOOP DEPTH Ds ASSOCIATED WITH STATEMENT
2416
COMPUTE HERMITE NORMAL FORM MATRIX FROM ALPHA MATRIX
2418
COMPUTE SCATTERING MATRIX THETA USING ALPHA, BETA, GAMMA, AND DOMAIN
2420
IDENTIFY NEXT TIME DIMENSION Td (FROM 1 TO Ds)
2422
DETERMINE STRIDE FACTOR BASED ON HERMITE NORMAL FORM MATRIX
STRIDE FACTOR > 1 ?
YES
2424
NO
MORE TIME DIMENSIONS TO PROCESS?
NO
2430
YES
2426
STRIDE FACTOR DIVIDES EVERY COMPONENT IN SCATTERING MATRIX FOR EVERY ROW THAT CONTAINS NON-NULL Td ENTRY ?
NO
YES
2428
FOR EACH DOMAIN CONSTRAINT Cd THAT INCLUDES Td:
IF Cd IS LOWER BOUND CONSTRAINT OF FORM f(TIME, PARAMETERS) >= CONST, REPLACE CONST BY FLOOR(CONST-1/Sf)*Sf+1 IN ORIGINAL DOMAIN MATRIX
IF CD IS UPPER BOUND CONSTRAINT OF FORM f(TIME, PARAMETERS)<= CONST, REPLACE CONST BY FLOOR((CONST+1)/Sf)*Sf-1 IN ORIGINAL DOMAIN MATRIX
MORE STATEMENTS TO PROCESS?
YES
2432
NO
RECOMPUTE SCATTERING MATRIX THETA USING ALPHA, BETA, GAMMA, AND MODIFIED DOMAIN
END
2434

FIG. 24

SYSTEM AND METHOD FOR DOMAIN STRETCHING FOR AN ADVANCED DUAL-REPRESENTATION POLYHEDRAL LOOP TRANSFORMATION FRAMEWORK

GOVERNMENT RIGHTS

[0001] This invention was made with Government support under the Defense Advanced Research Projects Agency (DARPA), HR0011-07-9-0002. THE GOVERNMENT HAS CERTAIN RIGHTS IN THIS INVENTION.

BACKGROUND

[0002] 1. Technical Field

[0003] The present application relates generally to an improved data processing system and method. More specifically, the present application is directed to a system and method for domain stretching for an advanced dual representation polyhedral loop transformation framework.

[0004] 2. Description of Related Art

[0005] Generating computer code that is efficiently processed (i.e., "optimized") is one of the most important goals in software design and execution. Computer code which performs the desired function accurately and reliably but too slowly (i.e., code which is not optimized) is often discarded or unused by computer users.

[0006] As those of ordinary skill in the art are aware, most source code (i.e., that code which is a human readable form) is typically converted into object code, and thereafter an executable application, by use of a compiler and a linker. The executable application is in a form and language that is machine readable (i.e., capable of being interpreted and executed by a computer). Other languages, such as Java available from Sun Microsystems, Inc. of California, USA, may be in source code form that is, on execution, transformed into a form understood by a computer system which then executes the transformed instructions. In any case, the source code, when transformed into a form capable of being understood and executed by a computer system, is frequently optimized. That is, a transformation is performed such that the instructions are performed more efficiently (i.e., optimized) and, hopefully, without any undue delay.

[0007] One common structure found in source code that is optimized, during the compilation process to transform source code into executable code, is the loop. Loops are used to repeat one or more operations or instructions. Loops may be provided as single, non-nested loops, or nested loops, i.e. loops within loops. For example, an array may be used to store the purchase price of individual articles (e.g., where the i^{th} element in the array A is denoted, in Fortran, as A(i)—other similar notations are used in other languages) generate a single instruction to add each of the purchase prices together (e.g., sum=A(1)+A(2)+ . . . +A(n)). This however would take the programmer some time to code and is not easily adapted to the situation where the computer programmer does not know, at development time, the number of articles in the array. That is, when the number of elements in the array can only be determined at run time (i.e., during execution). Accordingly, the loop was developed to repeat an operation (e.g., sum=sum+A(i))) where the induction variable, i, is changed for each iteration. Other forms of loops are known and are equally applicable.

[0008] However, when the instructions of a loop are transformed into machine readable code (e.g., executable code), the executed instructions may not be processed efficiently. For the example above, some computer systems may require that the processor fetch from memory, rather than from a register or cache memory, the various elements of the array "A". Fetching data from memory requires the processor to wait while the data is retrieved thereby increasing the latency of the program execution. Also, while loops may be an efficient way to write certain repetitive source code operations, a loop does insert additional operations that would not be present if the repetitive operations were replicated. These additional operations (e.g., branching operations) are considered to be the loop "overhead".

[0009] To address some of the inefficiencies in processing loops, various optimization techniques have been created and applied. Examples of these various optimization techniques include loop inversion, loop skewing, loop tiling, unrolling and jamming, and the like. For example, with unrolling and jamming (hereinafter "unrolling") a portion of the loop is replicated, or "unrolled," and the replicated portions are inserted, or "jammed," into the code. Typically, when the unroll and jam loop transformation technique is applied to the outer loop of a nested loop pair, the outer loop's induction variable (e.g., "i") is advanced only a few times (the number of times being governed by a parameter referred to as the unroll factor—UF) rather than completely during the unrolling portion of this optimization technique. During the jamming portion of this technique, the inner loop would be replicated "UF" times. Persons of ordinary skill in the art will appreciate that the replicated loop bodies are not identical but only similar. In the replicated loop bodies, portions of the loop bodies which use the induction of the outer loop will be advanced as required (e.g., if the loop body included reference to array element A(i), where "i" is the outer loop induction variable, a replicated loop body would include reference to the next required array element—A(i+1)). The unroll and jam technique effectively reorders the calculations being performed in the nested loop.

[0010] Typically, such optimizations are performed with regard to a compiler's intermediate representation of the source code, e.g., an abstract syntax tree. The abstract syntax tree is a finite, labeled, directed tree, where the internal nodes are labeled by operators, and the leaf nodes represent the operands of the operators. The abstract syntax tree (AST) is used in a parser as an intermediate between a parse tree and a data structure, the latter of which is often used as a compiler or interpreter's internal representation of a computer program while it is being optimized and from which code generation is performed. ASTs are usually not appropriate for complex program restructuring since, while simple optimizations such as constant folding or scalar replacement may be achieved without hard modifications of the data structures, more complex transformations such as loop inversion, skewing, tiling, etc., modify the execution order, which is far away from the syntax. See Cedric Bastoul, "Code Generation in the Polyhedral Model is Easier Than You Think," PACT'13 IEEE International Conference on Parallel Architecture and Compilation Techniques, pages 7-18, Juan-les-Pins, September 2004, which is hereby incorporated by reference.

[0011] The polyhedral model, which is based on a linear algebraic representation of programs and transformations, was developed to address this issue. See Bastoul et al. "Putting Polyhedral Loop Transformations to Work," LCPC'16 International Workshop on Languages and Compilers for Parallel Computers, LNCS 2958, pages 209-225, College Sta-

tion, October 2003, which is hereby incorporated by reference. The polyhedral model is basically a plugin to the conventional compilation process. It starts from the AST by translating the program parts that fit the model into a linear-algebraic representation. A new execution order is then selected by using a reordering function, e.g., using a schedule, placement or chunking function. Then, in a code generation step, an AST or new source code is returned that implements the execution order implied by the reordering function.

[0012] As an example of the polyhedral transformation consider the syntactic form of a polynomial multiplication kernel as represented in FIG. **1**A. See Vasilache et al., "Polyhedral Code Generation in the Real World," INRIA, 2006, available at http://hal.inria.fr/inria-00001106/en/. This example is concerned only with the control aspects of the program source code with the two computational statements (array assignments) being referred to herein by their names S**1** and S**2**. The polyhedral transformation model considers statement instances. For each statement, the iteration domain where every statement instance belongs is considered. The iteration domains are described using affine constraints that can be extracted from the program control. For example, the iteration domain of statement S**1**, referred to as D_{S1}, is the set of values (i) such that $2 \leqq i \leqq n$. As shown in FIG. **1**B, a matrix representation is used to represent such constraints: A*x+Ap*p≧0, where A is the iteration matrix, x is the iteration vector (composed of the loop counters), Ap is the parameter matrix and p is the parameter vector (composed of the unknown constants and the scalar **1**). Thus, in the example of FIGS. **1**A and **1**B, D_{S1} is characterized by:

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix} \cdot (i) + \begin{bmatrix} 0 & -2 \\ 1 & 0 \end{bmatrix} \cdot \begin{pmatrix} n \\ 1 \end{pmatrix} \geq 0.$$

[0013] In this framework, a transformation is a set of affine scheduling functions written θ(x)=T*x+Tp*p. Each statement has its own scheduling function which maps each runtime statement instance to a logical execution time. In the polynomial multiplication example of FIGS. **1**A and **1**B, an optimizer may notice a locality problem and discover a good data reuse potential over array z, then suggest $\theta_{S1}(i)=(i)$ and

$$\theta_{S2}\begin{pmatrix} i \\ j \end{pmatrix} = (i + j + 1)$$

to achieve better locality. See Bastoul et al., "Improving Data Locality by Chunking," CC'12 Intl. Conf. on Compiler Construction, LNCS 2622, pages 320-335, Warsaw, April 2003, which is hereby incorporated by reference, for a method to compute such functions. The intuition behind such transformation is to execute consecutively the instances of S**2** having the same i+j value (thus accessing the same array element of z) and to ensure that the initialization of each element is executed by S**1** just before the first instance of S**2** referring to this element. A transformation is applied in the polyhedral model by using the transformation formula shown in FIG. **1**C, where t is the time-vector, i.e. the vector of the scheduling dimensions. The resulting polyhedra, for the example, is shown in FIG. **1**D with the additional dimension t.

[0014] Once the transformation has been applied in the polyhedral model, one needs to generate the target code. A syntax tree construction scheme, which may consist of a recursive application of domain projections and separations, such as described in Bastoul "Code Generation in the Polyhedral Model is Easier Than You Think" and Quillere et al., "Generation of Efficient Nested Loops from Polyhedra," International Journal of Parallel Programming, 28(5):469-496, October 2000, is applied to the transformation. The final code is deduced from the set of constraints describing the polyhedra attached to each node in the AST.

[0015] In the above example, the first step is a projection onto the first dimension t, followed by a separation into disjoint polyhedra as shown on the top of FIG. **2**A. This builds the first loop level of the target code (the loops with iterator t shown in FIG. **2**B). The same process is applied onto the first two dimensions (on the bottom of FIG. **2**A) to build the second loop level, and so on. The final code is shown in FIG. **2**B. Note that the separation step for two polyhedra needs three operations: $D_{S1}-D_{S2}$, $D_{S2}-D_{S1}$, and $D_{S2} \cap D_{S1}$, thus for n statements, the worst case complexity is 3^n.

[0016] The polyhedral loop transformation-based approach to compiler optimization addresses several weaknesses of the traditional loop-based approaches to source code optimization. The polyhedral loop transformation approach addresses non-perfectly nested loops, has a precise instant-wise representation of data dependencies, and naturally supports compositions of complex transformations. As a result, it can detect more parallelism and exploit more data locality for more complex loop nests than the traditional loop-based approaches.

[0017] However, while the polyhedral loop transformation-based approach provides improved optimization of source code during the compilation process, it is not more widely used because of two main drawbacks. First, the code that is generated from the polyhedral representation is not always optimal with regard to some optimization criteria. This means that code that has excellent properties, such as data-parallelism (meaning that the work within a given loop or set of loops is data parallel and thus can be computed in parallel by possibly multiple threads on possibly multiple processors) and data locality (meaning the data needed to compute a specific amount of work generated by a given loop or set of loops often reuses the same set of data or a set of data that is collocated in memory) may be slowed down because of sub-par scalar performance (meaning that the generated code has high overhead due to unnecessary checks, branch, loop bound computations, and/or any other overheads) and/or unnecessary code bloat, i.e. an increase in the size of the code due to compiler optimizations being run on the source code. Second, transformations applied to a statement by current polyhedral loop transformation approaches necessarily touch all instances of a given statement. This means that, for example, it is hard to express parallelism for a statement that is partially parallel, i.e. a statement that is parallel in all but a few boundary instances. Similarly, for data locality enhancement, requiring that tiling must be performed on all instances of a statement, including the rarely executed boundary conditions, results in unnecessary code bloat as well as increased loop overhead. Tiling is a loop optimization that aims at increasing the data locality of a computation by cutting a large set of computation, e.g. a 2 dimensional computation iterating over 0-1023 times 0-1023 by a smaller set of computation on a smaller tile, e.g. 0-63×0-63, where once the first tile is completed, one may then iterate over the second tile, e.g. 0-63×64-127, with

this operation repeating with subsequent tiles until all of the original computation is completed.

SUMMARY

[0018] The illustrative embodiments provide a system and method for advanced polyhedral loop transformations of source code in a compiler. The mechanisms of the illustrative embodiments address the weaknesses of the known polyhedral loop transformation based approaches by providing mechanisms for performing code generation transformations on the intermediate representation (IR), e.g., an abstract syntax tree (AST), generated by the polyhedral loop transformation optimization of the source code. These code generation transformations have the important property that they do not change program order of the statements in the intermediate representation. This property allows the result of the code generation transformations, i.e. a new AST, to be provided back to the polyhedral loop transformation mechanisms in a program statement view, via a new re-entrance path of the illustrative embodiments, for additional optimization.

[0019] Such code generation transformations may induce statement splitting or aggregation, may modify domain and schedule components, and the like. However, they do so in a transparent manner ensuring strict equivalence of the relative orders induced by the new schedules for all instances of all statements. This strict equivalence involves program equivalence and schedule equivalence, i.e. only relative execution order of all instances of statements is required and thus, is ensured via strict equivalence. Thus, the AST generated by the polyhedral loop transformation optimizations on the program statement view will be equivalent to the new AST generated by the code generation optimizations applied to this AST from a program and schedule equivalence standpoint.

[0020] Code generation transformations may include, for example, conditional hoisting, kernel extraction, parallelism detection, modulo copy propagation. Each of these code generation transformations involves taking two arguments, i.e. a list of AST nodes referred to by prefix vectors in a loop-centric view of the program (the prefix vector list) and a propagation mode that can be "any" (all the nodes in the AST are visited), "prefix" (all the children of a given node are visited), or "exact" (only the specified node is visited). Based on the prefix vector list and the kind of propagation, a first pass of the AST is performed to flag the nodes that need to be processed. Thereafter, visitors are instantiated and used to apply core functions of the code generation transformations. The result of the code generation transformations is a modified or new AST that has lower control flow overhead. The code generation transformations do not modify the program semantics in any way although they may result in different equivalent schedules after regeneration.

[0021] The generated modified or new AST may then undergo program regeneration, which along with the code generation transformations makes the polyhedral framework of the illustrative embodiments fully iterative. Program regeneration involves transforming the modified or new AST into a stable program with respect to code generation. In order to generate a stable program, each statement in the new stable program needs to have its own domain that does not overlap with other instances of the same original statement. Each schedule must enforce the same relative order with respect to all other instances of any other statement. Furthermore, subsequent call to a separation algorithm in the program statement view optimizations of the compiler should result in the same AST as originally presented to the code generation transformations. In order to achieve all of these goals, schedule reconstruction, domain reconstruction, and domain stretching transformations are performed to generate a new stable program. This new stable program may be fed back to the program statement view stage of the compiler for further optimizations by the program statement view optimizations.

[0022] In one illustrative embodiment, a method is provided for optimizing program code. The method may comprise receiving source code for a program in a compiler, transforming the source code into a program statement view of the source code, and transforming the program statement view of the source code into a program loop view of the source code. The method may further comprise applying one or more code generation optimizations to the program loop view of the source code to generate optimized code and outputting the optimized code to a compiler for use in generating executable code for execution on a computing device. Transforming the program statement view of the source code into a program loop view of the source code may comprise applying a domain stretching operation to domains of statements in the program statement view of the source code to normalize the domains by stretching each domain to its largest possible value without adding execution points.

[0023] Applying a domain stretching operation to domains of statements in the program statement view may comprise, for each statement and each time dimension, statically determining a factor f that is greater than one and will divide the time dimension t from a remapping matrix, checking the factor f to ensure that the factor f divides every factor on all time, domain, and parametric dimensions, and stretching a constraint associated with the domain of the statement based on the factor. Stretching the constraint may comprise stretching the constraint to a closest multiple of the factor f minus 1 if the constraint is a lower bound on the time dimension t. Moreover, stretching the constraint may comprise stretching the constraint to a closest multiple of the factor f plus 1 if the constraint is not a lower bound on the time dimension t.

[0024] Applying a domain stretching operation to domains of statements in the program statement view may comprise, for each statement and each time dimension, receiving an Alpha matrix, Beta matrix, and Gamma matrix associated with the statement, receiving a domain for the statement, applying a scatter domain with stretching transformation to the statement based on the Alpha, Beta, and Gamma matrices and the domain, and receiving as output of the scatter domain with stretching transformation, a first scattering matrix. The Alpha matrix represents a speed at which an associated statement is performed along a given time dimension. The Beta matrix represents a sequential interleaving of the associated statement along different loop depths. The Gamma matrix represents a constant parametric shifting of the associated statement along each time dimension.

[0025] Applying the scatter domain with stretching transformation may comprise determining a depth of the statement within the program statement view, computing a second scattering matrix based on the Alpha, Beta, and Gamma matrices and the domain of the statement, determining a modified domain for the statement based on the Alpha matrix, and generating the first scattering matrix based on the Alpha, Beta, and Gamma matrices and the modified domain. Determining a modified domain for the statement may comprise computing a Hermite Normal Form matrix based on the Alpha matrix, determining a stride factor based on the Her-

mite Normal Form matrix, and modifying constraints associated with the domain of the statement based on the stride factor.

[0026] In other illustrative embodiments, a computer program product comprising a computer useable medium having a computer readable program is provided. The computer readable program, when executed on a computing device, causes the computing device to perform various ones, and combinations of, the operations outlined above with regard to the method illustrative embodiment.

[0027] In yet another illustrative embodiment, a system is provided. The system may comprise a processor and a memory coupled to the processor. The memory may comprise instructions which, when executed by the processor, cause the processor to perform various ones, and combinations of, the operations outlined above with regard to the method illustrative embodiment.

[0028] These and other features and advantages of the present invention will be described in, or will become apparent to those of ordinary skill in the art in view of, the following detailed description of the exemplary embodiments of the present invention.

BRIEF DESCRIPTION OF THE DRAWINGS

[0029] The invention, as well as a preferred mode of use and further objectives and advantages thereof, will best be understood by reference to the following detailed description of illustrative embodiments when read in conjunction with the accompanying drawings, wherein:

[0030] FIG. **1**A is a syntactic form of a polynomial multiplication kernel;

[0031] FIG. **1**B is a matrix representation of the polynomial multiplication kernel of FIG. **1**A;

[0032] FIG. **1**C is a transformation formula that is applied in the polyhedral model where t is the time-vector;

[0033] FIG. **1**D is a polyhedra resulting from the application of the transformation formula of FIG. **1**C to the matrix representation in FIG. **1**B with the additional dimension t;

[0034] FIG. **2**A illustrates a separation into disjoint polyhedra for code generation in accordance with a known operation;

[0035] FIG. **2**B illustrates example final code obtained by using the separation operation of FIG. **2**A;

[0036] FIG. **3** is an exemplary representation of an exemplary distributed data processing system in which aspects of the illustrative embodiments may be implemented;

[0037] FIG. **4** is a block diagram of an exemplary data processing system in which aspects of the illustrative embodiments may be implemented;

[0038] FIG. **5** is an exemplary diagram illustrating a traditional or conventional polyhedral approach to source code optimization during compilation;

[0039] FIGS. **6**A-**6**F are diagrams illustrating the program statement view and examples of the loop optimizer **530** transformations that may be performed on the program statement view;

[0040] FIGS. **7**A-**7**C illustrate a mapping using Quillere's projection and decomposition technique;

[0041] FIG. **8** is an exemplary block diagram of an advanced polyhedral loop transformation mechanism in accordance with one illustrative embodiment;

[0042] FIG. **9** illustrates an example of a polyhedral loop transformation fusion operation;

[0043] FIG. **10** illustrates the separation of fused loops into separate representations for each portion of the fused loops such that the representations are not shared;

[0044] FIG. **11** is an example of pseudocode of an algorithm for applying code generation optimizations/transformations;

[0045] FIG. **12**A represents an example of original code for illustrating a conditional hoisting operation in accordance with one illustrative embodiment;

[0046] FIG. **12**B represents the same code as in FIG. **12**A with a gentle, or least aggressive, mode of conditional hoisting having been performed in accordance with one illustrative embodiment;

[0047] FIG. **12**C represents the same code as in FIG. **12**A with an aggressive mode of conditional hoisting having been applied in accordance with one illustrative embodiment;

[0048] FIG. **13**A represents an example of original code for illustrating a kernel extraction operation in accordance with one illustrative embodiment;

[0049] FIG. **13**B illustrates skewing of the original code of FIG. **13**A along a first dimension in accordance with one illustrative embodiment;

[0050] FIG. **13**C illustrates a result of the kernel extraction code generation optimization/transformation in accordance with one illustrative embodiment;

[0051] FIGS. **14**A-**14**C are exemplary diagrams illustrating a manner for expressing parallelism in source code in accordance with one illustrative embodiment;

[0052] FIG. **15** is an exemplary diagram illustrating a transition graph for iterative polyhedral loop transformation optimizations in accordance with one illustrative embodiment;

[0053] FIGS. **16**A-**16**C are exemplary diagrams illustrate a large code growth that may occur as a byproduct of successive polyhedral rescan operations by a polyhedral rescan module and a successive operation of the code generation optimization/parallelism detection module;

[0054] FIGS. **17**A-**17**C are exemplary diagrams illustrating an example of code optimization where two statements have had their speed accelerated by a factor of 3 in accordance with one illustrative embodiment;

[0055] FIGS. **18**A-**18**F are exemplary diagrams illustrating code instability that may be introduced by a re-entrance path in a polyhedral loop transformation mechanism;

[0056] FIGS. **19**A-**19**B are exemplary diagrams illustrating scattering domains for statements S**1** and S**2** and a resulting stable $ASTP^{P'}$ obtained using the scatter domain with stretching transformation of one illustrative embodiment;

[0057] FIGS. **20**A-**20**C are exemplary diagrams illustrating an example of domain stretching under re-entrance in accordance with one illustrative embodiment;

[0058] FIG. **21** is a flowchart outlining an exemplary operation for utilizing a re-entrance path to obtain further optimization of code in accordance with one illustrative embodiment;

[0059] FIG. **22** is a flowchart outlining an exemplary operation for applying a code generation transformation algorithm in accordance with one illustrative embodiment;

[0060] FIG. **23** is a flowchart outlining an exemplary operation for preserving stability of code in the presence of conditionals for re-entrance in accordance with one illustrative embodiment; and

[0061] FIG. **24** is a flowchart outlining an exemplary operation for performing scatter domain stretching in accordance with one illustrative embodiment.

DETAILED DESCRIPTION OF THE ILLUSTRATIVE EMBODIMENTS

[0062] The illustrative embodiments provide a system and method for advanced polyhedral loop transformations of source code in a compiler. The mechanisms of the illustrative embodiments address the weaknesses of the known polyhedral loop transformation based approaches by providing mechanisms for performing code generation transformations on the intermediate representation (IR), e.g., an abstract syntax tree (AST), generated by the polyhedral loop transformation optimization of the source code. These code generation transformations have the important property that they do not change program order of the statements in the intermediate representation. This property allows the result of the code generation transformations, i.e. a new AST, to be provided back to the polyhedral loop transformation mechanisms in a program statement view, via a new re-entrance path of the illustrative embodiments, for additional optimization. As such, the mechanisms of the illustrative embodiments may be implemented in a stand-alone or distributed data processing system in which a compiler is utilized to compile source code into executable code for execution on one or more data processing devices.

[0063] For example, in a distributed data processing system, the source code may be provided, such as by a client data processing device, to a server or other data processing device that runs a compiler for compiling the source code into executable code. The server or other data processing device may implement the mechanisms of the illustrative embodiments to perform polyhedral loop transformation optimizations on an intermediate representation of the source code during such compilation. Alternative, the mechanisms of the illustrative embodiments may be implemented in the same data processing device in which the source code is generated and/or originally provided. The following FIGS. **3-4** are provided as examples of a distributed and/or stand-alone data processing system which may be used to implement the mechanisms of the illustrative embodiments.

[0064] With reference now to the figures and in particular with reference to FIGS. **3-4**, exemplary diagrams of data processing environments are provided in which illustrative embodiments of the present invention may be implemented. It should be appreciated that FIGS. **3-4** are only exemplary and are not intended to assert or imply any limitation with regard to the environments in which aspects or embodiments of the present invention may be implemented. Many modifications to the depicted environments may be made without departing from the spirit and scope of the present invention.

[0065] With reference now to the figures, FIG. **3** depicts a pictorial representation of an exemplary distributed data processing system in which aspects of the illustrative embodiments may be implemented. Distributed data processing system **300** may include a network of computers in which aspects of the illustrative embodiments may be implemented. The distributed data processing system **300** contains at least one network **302**, which is the medium used to provide communication links between various devices and computers connected together within distributed data processing system **300**. The network **302** may include connections, such as wire, wireless communication links, or fiber optic cables.

[0066] In the depicted example, server **304** and server **306** are connected to network **302** along with storage unit **108**. In addition, clients **310**, **312**, and **314** are also connected to network **302**. These clients **310**, **312**, and **314** may be, for example, personal computers, network computers, or the like. In the depicted example, server **304** provides data, such as boot files, operating system images, and applications to the clients **310**, **312**, and **314**. Clients **310**, **312**, and **314** are clients to server **304** in the depicted example. Distributed data processing system **300** may include additional servers, clients, and other devices not shown.

[0067] In the depicted example, distributed data processing system **300** is the Internet with network **302** representing a worldwide collection of networks and gateways that use the Transmission Control Protocol/Internet Protocol (TCP/IP) suite of protocols to communicate with one another. At the heart of the Internet is a backbone of high-speed data communication lines between major nodes or host computers, consisting of thousands of commercial, governmental, educational and other computer systems that route data and messages. Of course, the distributed data processing system **300** may also be implemented to include a number of different types of networks, such as for example, an intranet, a local area network (LAN), a wide area network (WAN), or the like. As stated above, FIG. **3** is intended as an example, not as an architectural limitation for different embodiments of the present invention, and therefore, the particular elements shown in FIG. **3** should not be considered limiting with regard to the environments in which the illustrative embodiments of the present invention may be implemented.

[0068] With reference now to FIG. **4**, a block diagram of an exemplary data processing system is shown in which aspects of the illustrative embodiments may be implemented. Data processing system **400** is an example of a computer, such as hosts **310** in FIG. **3**, in which computer usable code or instructions implementing the processes for illustrative embodiments of the present invention may be located.

[0069] In the depicted example, data processing system **400** employs a hub architecture including north bridge and memory controller hub (NB/MCH) **402** and south bridge and input/output (I/O) controller hub (SB/ICH) **404**. Processing unit **406**, main memory **408**, and graphics processor **410** are connected to NB/MCH **402**. Graphics processor **410** may be connected to NB/MCH **402** through an accelerated graphics port (AGP).

[0070] In the depicted example, local area network (LAN) adapter **412** connects to SB/ICH **404**. Audio adapter **416**, keyboard and mouse adapter **420**, modem **422**, read only memory (ROM) **424**, hard disk drive (HDD) **426**, CD-ROM drive **430**, universal serial bus (USB) ports and other communication ports **432**, and PCI/PCIe devices **434** connect to SB/ICH **404** through bus **438** and bus **440**. PCI/PCIe devices may include, for example, Ethernet adapters, add-in cards, and PC cards for notebook computers. PCI uses a card bus controller, while PCIe does not. ROM **424** may be, for example, a flash binary input/output system (BIOS).

[0071] HDD **426** and CD-ROM drive **430** connect to SB/ICH **404** through bus **440**. HDD **426** and CD-ROM drive **430** may use, for example, an integrated drive electronics (IDE) or serial advanced technology attachment (SATA) interface. Super I/O (SIO) device **436** may be connected to SB/ICH **404**.

[0072] An operating system runs on processing unit **406**. The operating system coordinates and provides control of

various components within the data processing system **400** in FIG. **4**. As a client, the operating system may be a commercially available operating system such as Microsoft® Windows® XP (Microsoft and Windows are trademarks of Microsoft Corporation in the United States, other countries, or both). An object-oriented programming system, such as the Java™ programming system, may run in conjunction with the operating system and provides calls to the operating system from Java™ programs or applications executing on data processing system **400** (Java is a trademark of Sun Microsystems, Inc. in the United States, other countries, or both).

[0073] As a server, data processing system **400** may be, for example, an IBM® eServer™ pSeries® computer system, running the Advanced Interactive Executive (AIX®) operating system or the LINUX® operating system (eServer, pSeries and AIX are trademarks of International Business Machines Corporation in the United States, other countries, or both while LINUX is a trademark of Linus Torvalds in the United States, other countries, or both). Data processing system **400** may be a symmetric multiprocessor (SMP) system including a plurality of processors in processing unit **406**. Alternatively, a single processor system may be employed.

[0074] Instructions for the operating system, the object-oriented programming system, and applications or programs are located on storage devices, such as HDD **426**, and may be loaded into main memory **408** for execution by processing unit **406**. The processes for illustrative embodiments of the present invention may be performed by processing unit **406** using computer usable program code, which may be located in a memory such as, for example, main memory **408**, ROM **424**, or in one or more peripheral devices **426** and **430**, for example.

[0075] A bus system, such as bus **438** or bus **440** as shown in FIG. **4**, may be comprised of one or more buses. Of course, the bus system may be implemented using any type of communication fabric or architecture that provides for a transfer of data between different components or devices attached to the fabric or architecture. A communication unit, such as modem **422** or network adapter **412** of FIG. **4**, may include one or more devices used to transmit and receive data. A memory may be, for example, main memory **408**, ROM **424**, or a cache such as found in NB/MCH **402** in FIG. **4**.

[0076] Those of ordinary skill in the art will appreciate that the hardware in FIGS. **3-4** may vary depending on the implementation. Other internal hardware or peripheral devices, such as flash memory, equivalent non-volatile memory, or optical disk drives and the like, may be used in addition to or in place of the hardware depicted in FIGS. **3-4**. Also, the processes of the illustrative embodiments may be applied to a multiprocessor data processing system, other than the SMP system mentioned previously, without departing from the spirit and scope of the present invention.

[0077] Moreover, the data processing system **400** may take the form of any of a number of different data processing systems including client computing devices, server computing devices, a tablet computer, laptop computer, telephone or other communication device, a personal digital assistant (PDA), or the like. In some illustrative examples, data processing system **400** may be a portable computing device which is configured with flash memory to provide non-volatile memory for storing operating system files and/or user-generated data, for example. Essentially, data processing system **400** may be any known or later developed data processing system without architectural limitation.

[0078] With the data processing systems of FIGS. **3-4**, assuming a distributed data processing system implementation, source code may be provided to a data processing system **400**, such as server **304** in FIG. **3**, from a client data processing device, such as client **310** in FIG. **3**, or the like, for compilation into an executable program. The mechanisms of the illustrative embodiments improve upon known compiler techniques that utilize polyhedral loop transformations to optimize the source code during compilation. The mechanisms of the illustrative embodiments convert a traditional program statement view of the source code into a program loop view of the source code such that each individual statement in the source code may be operated on rather than the program as a whole. Thereafter, the loops in the program loop view are optimized using code generation transformations that improve the control flow overhead of the program without modifying the program order of the statements in the program statement view of the program. As a result, after the optimizations, the resulting modified program loop view of the program may be converted back to a program statement view of the program for further optimization.

[0079] FIG. **5** is an exemplary diagram illustrating a traditional or conventional polyhedral approach to source code optimization during compilation. The polyhedral loop optimization mechanisms are part of one of many compiler phases. The input and output of the polyhedral loop optimization mechanism **500** are given in a compiler internal representation (IR) of statements, conditionals, and loops received from a compiler **505**. As shown in FIG. **5**, the polyhedral loop optimization proceeds from left to right in FIG. **5** starting with a polyhedral scan by a polyhedral scan module **510** of the original program from the compiler's IR into a polyhedral representation, referred to as the program statement view **520**. In this representation, each statement in the source code is associated with a polyhedron describing its domain (how many times it iterates in each of its loop dimensions) as well as its schedule (when it is executed with respect to all other statements). The schedule may be represented as a structured matrix having three sub-matrices: (1) the Alpha matrix, which represents the speed at which statements are fired along a given time dimension; (2) the Beta matrix, which represents the sequential interleaving of statements along the different loop depths; and (3) the Gamma matrix, which represents the constant parametric shifting along each time dimension. See Girbal et al. "Semi-Automatic Composition of Loop Transformations for Deep Parallelism and Memory Hierarchies, IJPP 2006, which is hereby incorporated by reference. The generation of a program statement view using a polyhedral representation is generally known in the art and thus, a detailed explanation of the mechanisms for representing a program in a program statement view using a polyhedral transformation will not be provided herein.

[0080] In the program statement view of the program, a loop optimizer module **530** is used to perform transformations on the program statement view to optimize the code. Examples of transformations performed by the loop optimizer module **530** include loop interchange, parallel wavefront, and statement shifting loop transformations, discussed in more detail hereafter. The transformations performed by the loop optimizer module **530** serve to modify the schedule of each individual statement in the program statement view to achieve better data parallelism and/or data locality of the execution of the program. The transformations performed by the loop optimizer module **530** affect all runtime instances of

a given statement. Thus, it is possible to skew the execution of one statement in a loop with respect to another, or to pull a statement out of one loop and put it before/after a statement in another loop, for better data locality/parallelism.

[0081] The resulting transformed program schedule and its corresponding domain are provided to a polyhedral code generator **540** which operates on the entire program as represented by the modified IR generated by the loop optimizer module **530**, based on the program statement view **520** of the program output by the polyhedral scan module **510**. The polyhedral code generator **540** generates an abstract syntax tree (AST) representation of the program based on the modified IR. Some limited optimizations **550-560** may be applied to the entire program as represented by the AST. These optimizations are limited in two ways. First they apply to all nodes in the AST regardless of needs or benefits. Some known implementations apply a transformation to all nodes skipping the top X levels of the AST tree (e.g. X=2 skipping the root and the next level down but then applying the transformation to all the next levels below). Second, the list of optimization used is fairly limited to aggressive if-hoisting (meaning removal of if-conditions at the expense of sometime out-of-control code replication) and some modulo guard removal (meaning simplification of complex modulo calculus present in bound computation or if-conditional computations).

[0082] Note also that code optimizations are very different from loop optimizations in that loop optimizations typically change the structure of the computations by changing the order in which statements are executed with respect to each others and/or by adding/removing loops altogether. Code optimization, like the ones described here, however, typically do not transform the order in which statements are executed with respect to each other but simply attempt to reduce overhead generated by a overly simplistic code generation scheme.

[0083] In essence, the module **540** is designed to generate valid code, possibly with overhead due to extra bound computation, if conditional, modulo calculus in bounds and/or conditional computations. It is then the responsibility of optimizations like **550**, **560**, and **570** to clean up some of the introduced inefficiencies as best as possible. The resulting optimized AST is provided to a code emitter **570** which generates code from the AST in the compiler's internal representation (IR) by simply converting the internal AST and stripping it of its polyhedral information and generating an equivalent structure that is familiar and recognized by the traditional compiler.

[0084] The polyhedral code generator **540**, code optimizations **550-570**, and code emitter **580** operate as a monolithic block. Moreover, any, all, or any subset of the optimizations **550-570** may be bypassed if desired, as represented by the dashed curved lines, such that the particular optimizations bypassed are not applied to the AST.

[0085] Again, it is important to note that in the known mechanism shown in FIG. **5**, at the program statement view **520**, optimizations are performed with regard to all runtime instances of a given statement in the code. Moreover, the code optimizations **550-560** are performed with respect to the entire program. At no time are optimizations made possible with regard to individual instances of statements in a program or even sub-parts of a statement. The mechanisms of the illustrative embodiments, as discussed hereafter, provide such optimization abilities.

[0086] FIGS. **6**A-**6**F are diagrams illustrating the program statement view and examples of the loop optimizer **530** transformations that may be performed on the program statement view. FIG. **6**A provides some general notations for explaining the program statement view and these transformations. As shown in FIG. **6**A, a statement S**1** in source code **610** may be represented as an array-based inequality **620** defining the iteration domain of the statement S**1**. That is, each statement control can be captured through parameterized affine inequalities: $Ax \geqq c$ where A is a n times m element matrix of integer numbers, x is a m element vector representing each of the iteration variables, and where c is a n element vector of integer numbers or symbolic parameters. A maximal set of such consecutive statements is referred to as a static control part (SCoP) in the polyhedral loop transformation literature. For each statement, an affine function $\theta(x)=Tx+d$ (where T is a n' times m element matrix of integer numbers, x is a m element vector representing each of the iteration variables, and where d is a n' element vector of integer numbers or symbolic parameters) assigns logical dates, e.g., time steps starting to zero and monotonically increasing, to iterations of $Ax \geqq c$.

[0087] As shown in FIG. **6**B, the polyhedral scan module **510** extracts an iteration domain **640**, access functions **650**, and schedule **660** of each statement in the source code **630**. The iteration domain **640** of a statement is a set of integer values taken by the multidimensional iteration i. The iteration domain **640** may be defined as a set of linear inequalities, e.g., $i \geqq 0, M-i-1 \geqq 0, j \geqq 0, N-j-1 \geqq 0$ in FIG. **6**B, forming a convex polyhedron. The access functions **650** correspond to the polyhedral representation of which specific memory location is accessed for a given statement. For example, in FIG. **6**B, the second statement **632** is controlled by the index variables i and j for, respectively, the outermost and the innermost loop. An access function for memory reference "Z[i]" in statement **632** will indicate, to the internal representation, which specific memory location will be written into when computing the data associated with that statement for a given instance of i and j. The access function is a matrix with one row per dimension of the array (Z[i] is a one dimensional array) and one column for each of the index variables (i and j here), parameters (M and N here) plus a constant integer. Thus the access function for Z[i] is [1 0 0 0 0] as shown in **652**, as it is only a function of the index variable i.

[0088] For the "a[i][j]" reference, the access function **654** is a two dimensional array and, as a result, the access function **654** is a 2×5 element matrix. The first row corresponds to the access function for the first dimension of the array A, solely a function of index variable i here. The second row corresponds to the access function for the second dimension of the array A, solely a function of index variable j here. For Y[j], the access function **656** is again a one dimensional array that is solely a function of the index variable j.

[0089] The schedule is a linear function assigned to a statement that precisely determines a logical timestamp for the execution of each instance of a statement. These logical timestamps express a partial order between instances of statements. As with the domain **640** and the access functions **650**, the schedule **660** is a linear function of the domain iterators, e.g., i and j, and global parameters, M and N. The extraction of iteration domain **640**, access functions **650**, and schedule **660** is generally known in the art and thus, a more detailed explanation is not provided herein.

[0090] Having extracted the iteration domain **640**, access functions **650**, and schedule **660**, to generate a program statement view **520** of the source code, the loop optimizer **530** may perform transformations on the schedule to achieve better parallelism/locality. FIGS. **6**C-**6**F illustrate various types of transformations that may be performed on the program statement view **520**.

[0091] In FIG. **6**C, the original code **671** is scanned through the polyhedral representation and a null-transformation is applied, depicted as element **673** in FIG. **6**C. Namely, the outer loop in the original code **671**, i.e. the loop iterating over index i, is mapped to the first time dimension t**1** and the inner loop in the original code **671**, i.e. the loop iterating over index j, is mapped to the second time dimension t**2**. The resulting code is shown in FIG. **6**C as element **672**. Note that in the statement S in the resulting code **672**, the original index i is set to the same value as t**1**, and the original index j is set to the same value as t**2**. Those of ordinary skill in the art will notice that the original code **671** and the resulting code **672** will execute the same statements exactly in the same order.

[0092] In FIG. **6**D, the original code **681** is scanned through a polyhedral representation and a loop interchange transformation **683** is applied. Namely, the outer loop in the original code **681**, i.e. the loop iterating over index i, is mapped to the second time dimension t**2** and the inner loop in the original code **681**, i.e. the loop iterating over index j, is mapped to the first time dimension t**1**. The resulting code is shown as element **682** in FIG. **6**D. Note that in the statement S of the resulting code **682**, the original index i is set to same value as t**2**, and the original index j is set to the same value as t**1**. One of ordinary skill in the art will notice that the original code **681** executes the statements in which, for a given value of i, all the values of j will be visited before visiting the next value of i. However, in the resulting code **682**, the code executes the statements in a different order. Namely, for a given value of j, all the values of i will be visited before visiting the next value of j. This transformation is referred to as an interchange of the loop i and j, precisely because of this change in ordering.

[0093] In FIG. **6**E, the original code **691** is scanned through the polyhedral representation and a loop skewing and parallelization transformation **693** is applied. In this transformation **693**, the two i and j indices are projected to a single time dimension t**1**=i+j. Thus, at the logical time date t**1**=3, the original iteration (i=1,j=2) and (i−2, j=1) are logically executed. This is illustrated by the DOALL loop in the resulting code **692**. A DOALL loop is a parallel execution of a loop, where logically all the iterations can be executed in parallel.

[0094] In FIG. **6**F, the original code **694** is scanned through the polyhedral representation and a loop shift transformation **697** is applied where the two matrices T**1** and T**2** are unchanged but where the vector D**2** is (1, 0) instead of the original default value of (0, 0). This means that the second statement S**2** will start executing one full outer-loop iteration step after the first statement S**1**. The resulting code is shown as element **695**. Note that in the first portion **698**, the statement S**1** executes alone for the full t**2**=1 to 3 range of iterations, before statements S**1** and S**2** start to jointly operate in the portion **696**. At the end of the execution of the doubly nested loop in portion **696**, one full outer-loop iteration step will remain to be computed for the second statement S**2**. This remaining outer-loop iteration step is computed in portion **699**.

[0095] With reference again to FIG. **5**, the optimized program statement view **520** generated through operation of the loop optimizer mechanism **530** is mapped, by the polyhedral code generator **540** to a hierarchical set of loop structures referred to as an abstract syntax tree (AST). The mapping is performed by using a Quillere's projection and decomposition technique, which is generally known in the art. See Quillere et al., "Generation of Efficient Nested Loops From Polyhedra," International Journal of Parallel Programming, 28(5):469-498, October 2000. The AST represents the imperative execution of the program statements in the source code. More precisely, each intermediate node of the program in the AST bears a polyhedron whose size is related to the depth d of the node. Each such polyhedron defines the non-redundant set of constraints needed to scan all the points in the corresponding transformed loop of depth d. The nesting of these polyhedra is directly translated into the nesting of the resulting loop nest. The leaves of the tree represent the polyhedral statements.

[0096] FIGS. **7**A-**7**C illustrate a mapping using Quillere's projection and decomposition technique. Given the code fragment shown in FIG. **7**A, three statements S**1**, S**2**, and S**3** are present and the subject of the polyhedral loop transformation. A set of points **710** iterated by the three statements is generated and the set of points **710** is separated into convex areas of uniform statement sets **712**-**718**. The abstract syntax tree (AST) is formulated by projecting along each of the dimensions using the above mentioned Quillere's projection and decomposition technique. Usually, the most straightforward way to generate the resulting AST is to apply the following simplified steps: 1) create the scattering polyhedron for each statement by extending the iteration domain with the equalities linking time iterations to domain iterations; 2) recursively project the previous polyhedron on the outermost time dimensions to determine the span of each statement along every transformed time dimension; 3) recursively perform the intersection, difference, and ordering of the previously projected scattering polyhedra for all statements to distribute their iterations along the new time loops; and 4) recursively sort the disjoint union of polyhedra along each time dimension. For more detailed explanations of these steps, reference is made to the work of Quiller'e et al. previously mentioned above.

[0097] The resulting AST is hierarchical, with the top node representing an outermost loop. This node corresponds to a single interval parallel to the i-axis after projecting away the j-dimension. Since, in the depicted example, all 3 statements have the same interval i=1 . . . n in this projected one-dimensional space, all 3 statements belong to this single node. Thus, there are 4 nodes **720**-**750**, one for each distinct area in FIG. **7**B. A domain is associated with each node. Domains are shown next to each node **720**-**750** in FIG. **7**C.

[0098] It is important to note that nodes sharing a single parent are ordered. This order must represent a correct sequence with respect to the original order in the original program, or at least, if the original order was modified by loop transformation, must not violate any dependence present in the original program.

[0099] As discussed above, minor optimizations may be made to the AST generated through the operations above which are applied to the program as a whole. The resulting AST is then used to generate a compiler internal representation (IR) and is provided back to the compiler.

[0100] Moreover, as discussed above, the polyhedron-based approach described above has some significant drawbacks in that (1) it may not always generate optimal code; and

(2) the transformations performed necessarily touch all instances of a given statement, rather than individual instances. This means that the code may suffer from sub-par scalar performance, unnecessary code bloat, and parallelism may be difficult to express for statements that are partially parallel, i.e. a statement that is parallel in all but a few boundary instances. Moreover, for data locality enhancement, requiring that tiling must be performed on all instances of a statement, including the rarely executed boundary conditions, results in unnecessary code bloat as well as increased loop overhead. Thus, it would be beneficial to have a mechanism for allowing optimizations to be performed on individual instances of statements.

[0101] FIG. **8** is an exemplary block diagram of an advanced polyhedral loop transformation mechanism in accordance with one illustrative embodiment. As shown in FIG. **8**, with the advanced polyhedral loop transformation mechanism **800**, a compiler internal representation (IR) of statements, conditionals, and loops is provided from the compiler **805** to the polyhedral scan module **810** which operates in much the same manner as the known polyhedral scan module **510** in FIG. **5** to generate a program statement view **820** of the source code. Loop optimizer module **830** may apply various loop optimizations on the statements in the program statement view **820** in much the same manner as the loop optimizer module **530** in FIG. **5**. The resulting optimized program statement view of the source code is provided to the polyhedral code generator **840**. This is where the mechanisms of the illustrative embodiments depart from the known polyhedral loop transformation mechanisms and provide advanced optimization mechanisms not previously known.

[0102] There are two main directions in which the mechanisms of the illustrative embodiments improve upon known compiler mechanisms. First, the mechanisms of the illustrative embodiments provide a re-entrance path (shown as the arc passing through the polyhedral rescan module **870**). In order to make this re-entrance path workable in the illustrative embodiments, data about the way in which the polyhedral code generation is performed, is maintained for use by the polyhedral rescan module **870** to convert the AST, or program loop view, **850** back into a program statement view **820**. This data may include, for example, the Alpha, Beta, and Gamma matrices for each of the statements in the AST **850** along with a remapping matrix that identifies how a transformation back to a program statement view from the AST, or program loop view, **850** may be performed.

[0103] In the second major direction of improvement, the mechanisms of the illustrative embodiment provide an improved manner by which code generation optimizations may be applied by the code generation optimization/parallel detection module **860**. These code generation optimizations may be applied with greater flexibility than known mechanisms since there is no fixed sequence of code generation optimizations, there are a greater number of code generation optimizations, and the code generation optimizations may be applied to specific sub-trees of a given AST **850**.

[0104] In addition to the improvements above, the mechanisms of the illustrative embodiments provide functionality for determining when to reunite statements, upon re-entrance, that were split by the polyhedral code generator module **840**. That is, when the polyhedral code generator module **840** operates on the program statement view **820**, sometimes the Quillere projection discussed above may result in statements being split into multiple nodes of the resulting AST or program loop view **850**. For example, in FIGS. **7**A-**7**C, statement S**1** appears in two of the leaf nodes of the resulting AST and S**2** appears in four of the leaf nodes due to such splitting of statements.

[0105] Sometimes it is desirable to keep the statement separated when doing re-entrance and in other situations it is more desirable to reunite the separated statement into a single statement in the resulting program statement view of the re-entrance. Thus, in the polyhedral rescan module **870**, a determination may be made whether to keep the separated statement separate or to reunite the separated statement. This will be described in greater detail hereafter with regard to FIG. **10**, for example. As will be described later, it is desirable, for example, to keep a statement separated in order to perform optimizations on the kernel only, while other times, reuniting the statement is proper. This decision may be made automatically by the polyhedral rescan module **870**, or in response to user commands, such as through directives/scripts, or the like. The details of the various improvements provided by the illustrative embodiments will be described in greater detail hereafter.

[0106] As shown in FIG. **8**, the polyhedral code generator **840** of the illustrative embodiments converts the program statement view **820** of the source code into a program loop view or AST **850**. In this view of the source code, each loop is associated with a set of statements that iterates over the same number of iteration points as experienced at a given depth level in the loop nest hierarchy. An example of the program statement view **820** is the data gathered in FIG. **6**B. An example of the program loop view or AST **850** is shown in FIG. **7**C. The major difference between the program statement view **820** and the program loop view or AST **850** is that in a program loop based representation, two or more statements can be assigned to the same logical loop even though the statements do not have exactly the same domain. For example, in FIG. **9**, discussed hereafter, loop**1** and loop **2** can be fused even though the domains do not strictly overlap. When generating the AST **850**, each of the nodes (which correspond to a loop, but for the leaf nodes that correspond to a statement) correspond to strictly one loop in which all the statements inside have exactly the same sub-domain. This is why one statement in the program statement view **820** may become many nodes in the program loop view or AST **850**, e.g. statements in loop**1** in FIG. **10**, hereafter, become 3 statements and statements in loop**2** also become 3 statements.

[0107] Thus, in the program loop representation, an original statement in the program statement view **820** may be split among several loops in the program loop view **850**. For example, if two statements S**1** and S**2** were fused into a single loop but S**1** iterates from 0 to 100 whereas S**2** iterates only from 0 to 80, then the statement S**1** may be split into two loops in the program loop view **850**, one from 0 to 80 when it co-executes with S**2** and one from 81 to 100 where it executes by itself.

[0108] That is, one type of loop optimization that is often performed, such as by loop optimizer module **830**, in polyhedral loop transformations is "fusion." Fusion attempts to improve data locality by fusing two loops that iterate over similar ranges into a single loop. FIG. **9** illustrates an example of a polyhedral loop transformation fusion operation. As discussed above, with polyhedral loop transformations, statements are represented as polyhedrons representing the iteration space of the loops associated with the statement. Thus, for example, as shown in FIG. **9**, a first statement S**1** is

associated with two loops with bounds i=N, i<M, j=U, j<V. These bounds define a rectangle as illustrated in the first graph **910**. Similarly, the second statement S**2** is associated with two loops whose bounds i=N', i<M', j=U', j<V', overlap the area of the first statement S**1**. This second statement S**2** is represented by the rectangle shown in the second graph **920**.

[0109] Through fusion, the two statements are integrated with the result being as shown in the graph **930**. As shown in FIG. **9**, the corresponding code includes loops **932-938** for representing the non-overlapping portions of the rectangles **910** and **920** and a loop **940** that represents the fused "kernel" of the statements S**1** and S**2** which represents the majority of instances of execution of the statements S**1** and S**2**. Fusion thus, increases the speed of execution of the original code by causing the majority of area where the two statements S**1** and S**2** execute to be executed together. However, it can be seen from FIG. **9** that fusion requires a large amount of replication of code.

[0110] With reference again to FIG. **5**, in generating the program loop view **550** from the loop optimized program statement view **520**, the polyhedral code generator **540**, separates the kernels, i.e. the common area or range of iterations, of fused statements such that separate representations of the statements are provided. This is because, in the program loop view or AST **550**, output by the polyhedral code generator **540**, each loop can only contain a single set of instructions. However, since there is no reentrance in the mechanism shown in FIG. **5**, the framework of FIG. **5** cannot apply specific loop optimizations implemented in the loop optimization module **530** only to the statements that are part of the kernel (e.g., the section of code fragment **940** in FIG. **9** where both statements S**1** and S**2** are being jointly executed). To the contrary, as mentioned above, in the mechanism of FIG. **5**, loop optimizations can only be applied to the program as a whole.

[0111] In other words, the original representation **520** includes statements S**1** and S**2** for their entire domain on which optimizations are being applied. The polyhedral code generator **540** discovers the actual loops that the fused statements S**1** and S**2** will execute in, but by then, i.e. after the polyhedral code generator **540** has operated, it is too late to apply new loop optimizations defined in the loop optimization module **530** because the optimizations in **550**, **560**, and **570** operate on a AST representation that is not amendable to optimizations such as in loop optimization module **530**. As a result, for example, it is not possible in the framework described in FIG. **5** to perform such loop optimizations as a unrolling optimization of the kernel using the powerful loop optimization module **530** operating over the program statement view **520**.

[0112] The mechanisms of the illustrative embodiments provide functionality for separating out the kernels of fused loops such that separate representations of statements are obtained upon which optimizations may be performed. FIG. **10** illustrates the separation of fused loops into separate representations for each portion of the fused loops such that the representations are not shared. With reference again to FIG. **8**, in generating the program loop view or AST **850** from the loop optimized program statement view **820**, the polyhedral code generator **840**, separates the kernels of fused statements such that separate representations of the statements are provided. This is again because in the program statement view or AST **850** representation output by the polyhedral code generator **840**, each loop can only contain a single set of instructions.

[0113] However, with the mechanisms of the illustrative embodiments, the reentrance path (depicted as the arc from element **850** through element **870** to element **820**) may be activated. With this reentrance path, each of the instances of statement S**1** and S**2** may be considered as separate. In other words, the domain of statement S**1** may be split into the sub domains corresponding to the area/code fragment **1010**, **1018**, and **1014** in FIG. **10** and the domain of statement S**2** may be split into the sub domains corresponding to the area/code fragment **1012**, **1018**, and **1016**. As a result of S**1** and S**2** now not sharing a common representation, loop transformations may now be applied, e.g., by the loop optimization module **830** in FIG. **8**, on the separate representations. For example, the loop containing both instances of statements S**1** and S**2**, e.g., the area/code fragment corresponding to **1018** in FIG. **10**, may be unrolled without impact on the other instances of statements S**1** and S**2**, e.g., in the areas/code fragments corresponding to area **1010**, **1012**, **1016**, and **1014**.

[0114] As shown in FIG. **10**, for a fusion of two statements S**1** and S**2**, when the statements are separated out, 5 different domains **1010-1018** and corresponding schedules **1020-1028** are generated by extracting the kernel in the representation of the fused statements. This is done by providing a separate domain and schedule for each of the boundary portions **1030-1036** of the fused statements where only one statement applies, and a separate domain and schedule for the kernel **1040** where both statements apply.

[0115] As a result, the program loop view of the source code includes separate representations for each statement, as well as the kernel, upon which code generation optimizations may be applied by the code generation optimizer/parallel detection module **860**. For example, code generation optimizations such as simplification and unstretching, if hoisting, substitute modulo, loop unrolling, etc. may be applied to the program loop view to obtain lower control overhead of the code. The program loop view may then be rescanned and converted back to a program statement view via the reentrance path after having undergone code generation optimizations by the. The result of the reentrance path is a program statement view of the code generation optimized program loop view that may be operated upon to provide even further optimization through an iterative process.

[0116] The optimizations that may be performed on the program loop view of the program, i.e. the "code generation optimizations," may be applied by code generation optimization/parallel detection module **860** in FIG. **8** to the program loop view **850** of the program. These code generation optimizations are performed on the polyhedral abstract syntax tree (AST), or the program loop view, and generate a new polyhedral AST, i.e. a new program loop view **850** for reentrance to the program statement view **820** and/or emission back to the compiler **805**. The code generation optimizations represent a set of transformations that are performed in an iterative, modular, and flexible manner to help generate the code with the least impeding control overhead as possible. An important property of the code generation optimizations is that they do not change the program order in the original polyhedral AST of the program loop view. They may induce statement splitting or aggregation and may even modify domain and schedule components. However, they do so in a

transparent manner ensuring the strict equivalence of the relative orders induced by the new schedules for all instances of all statements.

[0117] When considering program correctness, it is straightforward to realize that different scheduling functions may produce the exact same execution. Indeed, only the relative execution order of all instances of statements is required. Therefore, a simple transformation like the shifting of all the statements by the same amount does not change any relative order of any statement instance and is said to produce equivalent schedules. Equivalence is a relation between two programs P and P' with respective global schedules S_{ch} and S_{ch}'. When a schedule transformation is applied to P, the resulting program P' bears the exact same statements. Thus, program schedule equivalence is implied by program equivalence. This notion must be preserved by any code generation optimization or transformation performed by the code generation optimization/parallelism detection module **860** in FIG. **8**.

[0118] With this requirement in mind, each code generation optimization or transformation executed by the code generation optimization/parallelism detection module **860** takes two arguments: (1) a list of nodes in the program loop view referred to by prefix vectors in the program loop view of the program; and (2) a propagation mode that can be, but is not limited to, "any" (all the nodes in the AST are visited), "prefix" (all the children of the given node are visited), or "exact" (only the specified node is visited). The list of nodes, i.e. the prefix vector list, is made up of prefix vectors for the nodes that are to be optimized by the particular code generation optimization selected. The nodes of the AST of the program loop view, e.g., the nodes **720-750** in FIG. **7**C, may be characterized as a vector of numbers which indicate its path from the root (top-most) node. This vector of numbers is referred to as the prefix vector for the node.

[0119] The list of nodes may be made up of a listing of such prefix vectors. The prefix vector list defines the scope of the code generation optimization in that it indicates where the code generation optimization is allowed to modify nodes. Traversals of this list of nodes may be performed, for example, by a depth-first-search listing a parent before any of its children, a depth-first-search listing a parent after each of its children, a depth-first-search listing leaves only, or the like.

[0120] A code generation optimization may be called by the mechanisms of the illustrative embodiments, such as by the code generation optimization/parallelism detection module **860** in FIG. **8** either automatically or in response to a user request for a particular code generation optimization, to operate on individual statement instances, and/or sub-statement instances, in the program loop view **850** using a call such as "CG_codegenopt(optName, P, preftype), where optName is the name of the code generation optimization that is to be applied, P is the prefix vector list that will serve as a basis for flagging nodes in the AST to which optimizations are to be applied, and preftype specifies the prefixes found in P are to be treated as exact filters or prefix filters, i.e. "any," "pref," or "exact." For example, the call "codegenopt(simplify, {{1}, {2,0}, {3,3,3}}, BMT_exact)" will try to simplify exactly the nodes at the specified list of vectors in the AST of the program loop view.

[0121] The types of code generation optimizations that may be applied to the AST of the program loop view are varied and evolving. A current listing of code generation optimizations includes, but is not limited to, simplify, simplify-unstretch, simplify-trivial-modulo-remapping, extract-kernel, if-hoist/if-hoist-gentle, if-hoist-brutal, substitute-modulo, and loop-unroll. The simplify code generation optimization is a basic simplification under context, normally called from inside the code generation optimization module. The simplify-unstretch code generation optimization is a more elaborate simplification that also reverts any "domain stretching" phase that prevents over-separation in the code generation optimization phase when non-unimodular schedules are present. The simplify-trivial-modulo-remapping code generation optimization is a basic simplification plus explicit instantiation of equalities propagated to the leaves which results in either disproving or simplifying modulo conditionals. This can be viewed as a constant propagation for modulo guards that may also disprove some statements when the modulo guards cannot be met.

[0122] The extract-kernel code generation optimization computes and extracts a fully unrollable kernel from a loop with complex bounds (min, max, floor, and ceiling). This usually results in 0+ prologues, 1 kernel and 0+ prologues and may yield code bloat if not done carefully. The if-host/if-hoist-gentle code generation optimization walks the children of the given node and finds conditions on the current loop's depth and hoists them. The if-hoist-brutal code generation optimization walks the leaf nodes, finds any condition on any depth smaller than the current loop's depth and brutally hoists everything. The substitute-modulo code generation optimization simplifies modulos aggressively without taking care of compatibility within different statements. When all statements in a loop have the same modulo substitutions, this is a powerful tool to embed the modulos into the enclosing loops' bounds. The loop-unroll code generation optimization performs a full unroll of a loop with static constant bounds difference. This code generation optimization should usually be preceded by an extract-kernel and a if-hoist-gentle code generation optimization if the bounds are complex (min, max, floor, ceiling) otherwise many inner conditionals may be generated. These are only examples of currently known code generation optimizations and not intended to be limiting in any way. Other code generation optimizations may be used in addition to, or replacement of, the listed code generation optimizations without departing from the spirit and scope of the present invention.

[0123] When applying the code generation optimizations using the code generation optimization/parallelism detection module **860** in FIG. **8**, based on the prefix vector list and the propagation mode, a first pass is performed on the program loop view, e.g., the AST of the program loop view, to flag the nodes that need to be processed. The reason why a separate pass is used to mark the nodes is that some optimizations/transformations may duplicate nodes while some others may delete nodes, making the prefix vectors obsolete very quickly. Thus, prior to any code generation optimization/transformation being performed, the nodes that are to be processed are first flagged.

[0124] After having flagged the nodes to be "visited" by a code generation optimization/transformation, a code generation optimization/transformation application algorithm is executed by the code generation optimization/parallelism detection module **860** in FIG. **8** to thereby apply the called code generation optimization/transformation. FIG. **11** is an example of pseudocode of an algorithm for applying code generation optimizations/transformations. All code generation optimizations/transformations share the same template implementation based on visitors. Visitors are instantiated at

3 different points in the algorithm. First, a visitor is used to perform an outermost scan of the nodes in the AST of the program loop view. Each code generation optimization scans the nodes that have been marked by the filtering pass. For each marked node N, the code generation optimization applies its core function which determines if the node's domain is modified and returns the list of new domains to replace the obsolete ones. This list is sorted with respect to the current time dimension under the parent polyhedral context with the same algorithm proposed by Quillere, referenced above.

[0125] The core function is the second point where an inner visitor is instantiated. Each new domain in the list then generates a new node N' and its corresponding subtree, which is a copy of the subtree rooted at N and simplified in the context of N'. Once the new subtree list is attached in place of the original node N, the propagation function is called along each path to every new leaf. Such propagations are performed by a third inner visitor that may void nodes in the new subtrees which need to be removed recursively in a bottom-up order. To avoid interfering with the outer visitor traversal, special care is taken. Therefore, the node removal function is implemented with a boundary node argument and is only allowed to delete descendents of that node. This guarantees that the outer application visitor and the inner propagation visitor are always operating on non-conflicting regions of the AST.

[0126] Some of the above example code generation optimizations that may be implemented using the mechanisms of the illustrative embodiments will now be described in greater detail. It should be appreciated that while specific code generation optimizations are described herein, the illustrative embodiments are not limited to these code generation optimizations and may operate to implement other code generation optimizations in addition to, or in replacement of, one or more of the herein described code generation optimizations.

[0127] As mentioned above, one of the code generation optimizations that may be performed by the code generation optimization mechanism of the illustrative embodiments is the conditional hoisting, or if-hoisting, code generation optimization. Conditional hoisting performs a controlled tradeoff between code size growth and spurious inner conditionals removal. The core function determines all spurious conditionals for a marked node and factorizes them.

[0128] Two application modes are possible when processing a node N of depth d. In the least aggressive mode, the visitor traverses all the children of node N. The visitor looks for conditionals directly expressed as a function of $(t_i)_{i\in[1,d]}$ and constants only. Such constraints do not concern the time iterators at depth d'>d and are thus, affine guards that can be hoisted. In the aggressive mode, the visitor traverses only the leaf nodes under node N and performs a polyhedron projection of each separate statements' domain on the vector space $(t_1, \ldots, t_d, N)$. A subsequent simplification in the context of the parent node yields the new conditionals.

[0129] In both modes, the non-redundant list of conditionals is maintained. Eventually, the difference is computed with the reference node's domain, yielding the core list of conditionals representing all possible case distinctions. In each of these cases, a single condition holds. As an application, consider the following variants with the different conditional hoisting modes as shown in FIGS. **12**A-**12**C. FIG. **12**A represents the original code. FIG. **12**B represents the same code with a gentle, or least aggressive, mode of conditional hoisting having been performed. FIG. **12**C represents the same code with an aggressive mode of conditional hoisting having been applied.

[0130] As one can see, FIG. **12**A includes several conditional statements, i.e. the three "if" clauses in FIG. **12**A, that will execute at each iteration of the outermost t**1** loop, even though the condition associated with the conditional statement will evaluate to true for only one or two iterations of the entire t**1** loop iteration. This represents a significant overhead, which can be removed the conditional hoisting optimization. While in general the aggressive technique can get rid of more conditional statements at the cost of more replication, one can see that in this case, the gentle approach (result shown in FIG. **12**B) was sufficient to remove all conditionals. The aggressive technique (results shown in FIG. **12**C) also resulted in all of the conditionals being removed, but resulted in more code than the gentle approach.

[0131] In most cases, the gentle mode is enough and yields potentially much smaller code. In special cases, however, the more aggressive, or "brutal," mode is needed to perform more advanced conditional hoisting, such as in the case of loop unrolling after tiling. To see that program equivalence is preserved is rather straightforward. Conditional hoisting is actually a domain splitting on the time dimensions. Suppose I and I' are ordered instances of two statements that execute respectively at time t and t' such that $t \leqq t'$. Two cases arise: (1) both instances belong to the same new split domain after transformation and their order is enforced by the schedule; and (2) each instance belongs to a different sub-domain, in which case the relative order is enforced by the disjunction and the subsequent ordering. Lastly, since the difference is computed with the reference node's domain, no iteration is lost.

[0132] Another code generation optimization/transformation that may be applied to the program loop view **850** in FIG. **8** by the code generation optimization/parallelism detection module **860** is the kernel extraction code generation optimization/transformation. When loop optimizations/transformations such as skewing or strip-mining are applied, the generated loops exhibit complex bounds which can degrade performance or prevent further desired loop unrolling. Kernel extraction is a transformation that enforces the separation of such complex bounds in different versions of the loops. This transformation has three versions: (1) the unrollable kernel extraction detects pairs of lower/upper bounds that exhibit a static constant difference; (2) the lower bounds kernel extraction creates a list of conditionals where every lower bound is minimal exactly once. If at depth d, the scattered, separated domain exhibits $t_d \geqq (1_i)_{i\in[1,k]}$, the resulting conditionals are a list of k elements such that $t_d \geqq (1_i)_{i\in[1,k]-\{j\}} > 1_j$; and (3) the upper bounds kernel extraction creates the same list of conditionals with the upper bounds, i.e. $t_d \geqq (u_i)_{i\in[1,k]} => t_d \geqq (u_i)_{i\in[1,k]-\{j\}} > u_j$.

[0133] The result of a simple example is shown in FIGS. **13**A-**13**C. FIG. **13**A represents the original code of the depicted example. FIG. **13**B illustrates skewing of the original code along a first dimension. FIG. **13**C illustrates a result of the kernel extraction code generation optimization/transformation. in the kernel extraction code generation optimization/transformation, the program equivalence property is obtained the same way as in the conditional hoisting code generation optimization/transformation. Kernel extraction may be followed by a subsequent loop unrolling pass. This transformation (loop-unroll) performs full unrolling of the

iterations of a given node. Given its natural code expansion characteristics, is applied at the innermost depths.

[0134] As shown in the example of FIG. **13**A, there is a simple statement inside a doubly nested loop. FIG. **13**B shows this same statement after loop skewing optimization was applied. Loop skewing is most often used to parallelize code by modifying the order in which statements are executed so that the dependencies present in the original code do not hinder parallelization of the inner loop(s). The salient observation is that in FIG. **13**B, there are minimum (min) and maximum (max) functions in the loop bound computation of the innermost loops. These min/max functions were introduced by the loop skewing optimization, and their cost may significantly impact on the execution time of the loop. Using the kernel extraction of the illustrative embodiments, the domain of execution of the outer loop may be split so that the min(X,Y) function always takes its X value in one of the copy of the outermost loop, or Y value in the second copy of the outermost loop, in FIG. **13**C. Because of this property, the min function may be safely removed from the code as it is statically known that the smallest number will come from the X value in the first instance of the loop and the Y value in the second instance of the loop.

[0135] In addition to code generation optimizations, the code generation optimization/parallelism detection module **860** in FIG. **8** further detects parallelism in the program loop view **850**. When specifying parallelism at the schedule level only, there is a semantic gap between the syntactic loop model and the schedules that are expressed with the polyhedral model. One such example, is given in FIGS. **14**A-**14**C for 2 statements with the same domain Dom={i in [1,N]}, with only the A and Beta parts shown. A and Beta are two components of the schedule associated with each of the statements. The schedule essentially associates a time stamp, e.g., date, time, etc., with each instance of the statements. These time stamps indicate which of the instances of a statement comes first in the execution of the code. Having a smaller time stamp number means that that instance of a statement comes prior to another instance that has a higher time stamp number. The A matrix indicates how each of the iterations surrounding a statement are taken into account to compute that statement's time stamp. The Beta part indicates the order in which a statement is to be found in the code. For example, a Beta of [0, 0] associated with statement S**1** in FIG. **14**A states that S**1** is the first loop and is the first statement inside that first loop.

[0136] Suppose that the target version shown in FIG. **14**C is to be generated and the dependencies between statement S**1** and S**2** allow it to be generated. However, if parallelism is to be expressed in the schedule directly, the code in FIG. **14**B is obtained where the parallel loops undergo a fission optimization. In other words, to express parallelism in the way proposed in FIG. **14**A, a loop fission operation, i.e. statements inside a single loop nest are separated into two or more loops each containing a subset of the statements inside the original loop, is applied. This loop fission will have significant impact on the performance of the code.

[0137] On the other hand, enforcing the fusion of the loops yields the code in FIG. **14**B where only the body of the loops is parallel. The parallel fused version requires the schedule to specify the same arbitrary order on the outermost time iterator for the two statements while still enforcing sequentiality between S**1** and S**2** at the loop independent level. Eventually, the constraints borne by the syntactical parallel version are not expressible with a polyhedral schedule only, as shown by the "undef" labels inside of the A and beta representation in FIG. **14**C, which represent the desired code for which the above scheme to express parallelism within the polyhedral A and Beta scheduling matrices cannot be used.

[0138] This expressivity issue is alleviated by the illustrative embodiments by means of a code generation optimization/transformation to detect parallelism. When it is applied to a node N of depth d, it creates a list of all statement leaves with their restricted domains after separation. This list is used to filter the dependence graph and to check if node N defines a loop which does not bear any dependence. Each dependence is intersected with the current schedule of its source and target statements but also with their restricted domains. If the resulting polyhedron is not empty, the algorithm stops when a parallelism preventing dependence on depth d is found. On the other hand, if all resulting dependences are empty, the loop is marked parallel and an OpenMP directive along with the shared and private variables information are generated. OpenMP used here is an exemplary compiler and runtime support system that enable parallelism to be expressed. OpenMP uses directives (generated either by the application user and/or the compiler) that state which loop/region can safely be executed in parallel. OpenMP is used herein as only an example and is not limiting to the mechanisms of the illustrative embodiments in any way. To the contrary, in one illustrative embodiment, OpenMP is only used as one way to convey parallelism information to the remainder of the compiler/runtime system of the illustrative embodiment. Any other parallel compiler/runtime system may be used without departing from the spirit and scope of the illustrative embodiments.

[0139] All this processing is performed by the core function of the code generation optimization/parallelism detection engine **860** in FIG. **8** which never modifies the constraints on the domain of node N, but rather sets a parallel bit to 1 in the internal representation associated with the loop. When it becomes time to emit the code, e.g., from emit module **880** in FIG. **8**, a parallel bit will be read to determine, for each loop, whether this loop is parallel or not. Upon a determination that the loop is parallel, the emit module **880** will generate the appropriate construct to inform the rest of the compiler/runtime system that this loop is a parallel loop. In one illustrative embodiment where OpenMP is used, this consists of emitting a pragma directive just prior to the loop.

[0140] Returning again to FIG. **8**, as shown, the result of the code generation optimizations and parallelism detection is a modified program loop view **850** that is then either output to the compiler **805** or sent along a re-entrance path to the polyhedral rescan module **870** for conversion back into a program statement view **820** for further optimization. Determination on whether to go along the re-entrance path or not depends on various factors. First, it may be desirable to transform the code to the program loop view **850** representation before completing all optimizations in the program statement view **820** representation in order to gather some knowledge about the code. For example it may be desirable to use the program loop view **850** to determine which loops are parallel, to see if kernels need to be extracted, to evaluate the complexity of the current code, and/or any other qualitative information that may be gathered from the program loop view **850** representation. Once this information is gathered, it is desirable to go back to the program statement view **820** representation to exploit this additional knowledge for further optimi-

zation of the code upon determination, based on this additional knowledge, that particular optimizations are advantageous.

[0141] Second, it may be desirable to apply all optimizations in the program statement view **820** representation at once. In this framework, some loop optimizations from the loop optimization module **830** may be applied and then the program statement view **820** representation may be converted to the program loop view or AST **850** representation. Specific code optimizations may be applied by the code generation optimization/parallel detection module **860**, such as kernel extraction and/or other code generation optimizations, and then the re-entrance path may be traversed to go back to the program statement view **820**. Further optimizations of specific aspects of the program loop view or AST **850** representations (after being modified by the code generation optimizations) may then be performed. Both approaches above are not exclusive and may be jointly applied or applied repetitively in some alternating fashion.

[0142] With regard to the re-entrance path, the modified program loop view **850** is parsed by the polyhedral rescan module **870** and data structures expected in the program statement view **820** are recreated from the modified program loop view **850**. In this way, iterative calls to the loop optimizer module **830** and the code generation optimization/parallelism detection module **860** may be performed successively until a desired level of optimization is achieved at which time the optimized code may be output back to the compiler **805**. This is contrary to known mechanisms in which a single pass of the loop optimizer module **530** in FIG. **5** is performed with some minor optimizations being performed on the entire program thereafter just prior to the emitting of the code back to the compiler. No successive iterations are possible in known mechanisms.

[0143] In order to perform successive (iterative) calls to the optimizing framework comprising the loop optimizer module **830** and the code generation optimization/parallelism detection module **860**, the output of a given polyhedral optimization must be fed to the next phase without disrupting the properties of the optimization found so far. In particular, if no further optimization is performed in the latter phase, one expects the result to exhibit the same properties, i.e. parallelism, memory locality, code size, control flow overhead, etc., as have been observed in the previous phase. That is, for example, the output of the code generation optimization/parallelism detection module **860** should have the same properties as the input to the code generation optimization/parallelism detection module **860**. In other words, the re-entrance process must be stable by imposing the following constraints: (1) the code size must not increase; (2) the amount and granularity of parallelism must not be modified; and (3) the relative execution order of all statements in the program must be preserved. Memory, or data, locality and reuse are strongly tied to the scheduling of the program and thus, no particular concern occurs with respect to these features. On the other hand, code size and control flow overhead are very dependent on the code generation optimizations and the aggressiveness of the transformations, such as conditional hoisting or modulo guard removal. Furthermore, when parallelism is directly expressed in the schedule, such as via the parallelism detection mechanisms of the code generation optimization/parallelism detection module **860**, it may be hard to exploit properly at the syntax tree level and even harder to reparse properly.

[0144] In practice, the mechanisms of the illustrative embodiments apply transformations that will change the representations **820** and **850**. However, from an implementation perspective, even if no transformations are applied, the quality of the representation is not degraded by the reentrance path. In other words, if the reentrance path is followed to apply a specific sequence of loop and code generation optimizations, which will result in a faster running code, but in the process degrade the representation which happens to slow the resulting code, then such degradation must be weighed against the benefit (better optimization). However, by practically ensuring that there is no degradation of representation while exercising the re-entrance path, as in the mechanisms of the illustrative embodiments, the cost of implementing the mechanisms of the illustrative embodiments with regard to code performance is approximately zero. Thus, the re-entrance path of the illustrative embodiments should always be exercised if the compiler/application writer can determine a beneficial loop/code generation optimization sequence.

[0145] With the mechanisms of the illustrative embodiments, the polyhedral rescan module **870** performs program regeneration in order to generate the program statement view **820** from the program loop view **850**. Program regeneration involves transforming the modified or new AST of the program loop view **850** into a stable program with respect to code generation. In order to generate a stable program, each statement in the new stable program needs to have its own domain that does not overlap with other instances of the same original statement. Each schedule must enforce the same relative order with respect to all other instances of any other statement. Furthermore, subsequent call to a separation algorithm in the program statement view optimizations of the compiler should result in the same AST as originally presented to the code generation transformations. In order to achieve all of these goals, schedule reconstruction, domain reconstruction, and domain stretching transformations are performed to generate a new stable program. This new stable program may be fed back to the program statement view stage of the compiler for further optimizations by the program statement view optimizations.

[0146] As discussed above, the loop optimizer **830** applies polyhedral transformations to the program statement view **820** of the program or source code. The code generation optimizations, on the other hand, have the sole purpose of reshaping the AST of the program loop view **850** for lower control flow overhead. These latter transformations do not modify the program semantics in any way although they may result in different equivalent schedules after regeneration via the re-entrance path. Assume that the original program source code is referred to as P, the AST of the program loop view **850** is AST^P, the code generation optimizations used to generate the AST^P are denoted P_{CG}, and the stable program generated from the AST^P using regeneration is P'. Using this notation, the transition graph for the iterative polyhedral loop transformation optimizations of the illustrative embodiments is shown in FIG. **15** where the goal, which is achieved by the illustrative embodiments, is to ensure $AST^P \equiv AST^{P'}$ in order to build a stable, reentrant and iterative framework that does not rely on any other part of the global compilation chain to iterate.

[0147] Consider a simple code fragment such as shown in FIGS. **16**A-**16**C, with statements S**1**, S**2**, and S**3** covering the iteration points depicted in the graphs below the code fragments. Considering the code in FIG. **16**A, a typical represen-

tation of this program generated by the loop optimizer module **830** is shown in FIG. **16**B. Polyhedral code generator algorithms of the code generation optimization/parallelism detection module **860** isolate the i==N data point from the others. After one or more iterations of the loop optimizer module **830** and the code generation optimization/parallelism detection module **860** using the optimizations described above, and without imposing the stability requirements discussed herein, the code shown in FIG. **16**C is obtained where additional data points have been isolated.

[0148] FIGS. **16**A-**16**C illustrate the large code growth that may occur as a byproduct of successive polyhedral rescan operations by the polyhedral rescan module **870** and the successive operation of the code generation optimization/parallelism detection module **860**. This code growth will obviously impact the quality of the final code. In addition, by looking carefully at FIG. **16**C, one notices that all the conditionals have migrated from inside the loops to the top level (outside of any loops). This is a hard to reverse process that may prevent other optimizations from being performed.

[0149] This code bloat and conditional migration occurs because of the following reasons. Each time that the code goes through a polyhedral regeneration operation, and the statements are split, the subsequent polyhedral code generation operation by the polyhedral code generator **840** has less flexibility to combine the same original statements into convex areas with uniform sets of statements within itself. For example, in FIG. **16**B, all the statements are under a unique "for i" loop because there are instances of S**1**/S**2**/S**3** in the full i=1 . . . n range. Consider what happens to S**1** the next time around. The original S**1** has been split into 3 distinct S**1**.**1**, S**1**.**2**, and S**1**.**3** sub-statements with, respectively, ranges for i=1 . . . n−2, i=1 . . . n−1, and i=n. These distinct ranges will result in a top-level conditional with the resulting replication seen in FIG. **16**C. Thus, what is needed is a way to control this replication in order to prevent highly optimized code with poor single thread performance due to instability conditions that leads to highly duplicated code with a high degree of redundant replication and control overheads.

[0150] This goal is achieved by the mechanisms of the illustrative embodiments by recombining instances of statements prior to the polyhedral rescan operation being performed so as to prevent the code growth shown in FIG. **16**C. These mechanisms are heuristically driven in that none of the operations discussed hereafter are necessary but, if followed, the code growth for the particular statement operated will be prevented.

[0151] In order to perform this recombining of instances of a statement, an operation is performed, such as by the polyhedral rescan module **870**, on the new AST generated by the code generation optimization/parallelism detection module **860**. As discussed above, the AST is an encoded hierarchical ordered graph where each inner node corresponds to an iteration domain at a given depth in the loop nest structure. Each leaf node has also a list of statements that are enclosed by the loop nest. For a given node N at depth d in the AST, the node is associated with a domain D^N, which is a polyhedral representation of the domain associated with the enclosed statements, and projected to reflect the depth d of the node in the AST.

[0152] The requirements imposed by stability under the separation algorithm are less straightforward. Consider a node N of depth d in AST^P. If its scattering domain D^N projected on depths **1** . . . d−1 and simplified under the parent domain is not the universe domain, it means node N holds constraints that can be hoisted. A hoistable condition is a constraint at depth j appearing in a polyhedron of depth k in the AST such that j<k (i.e. a constraint in which the time dimension k does not appear in the constraint). Recall that the time t is represented by a vector of time elements, with a lexicographical interpretation of the times. In the above statement, it is stated that a constraint in the time dimension k does not appear if the k's position in the vector (starting from the left) is null. If such constraints were to appear in the regenerated program, they would trigger the same separation behavior from Quillere's algorithm as shown in FIG. **16**A-**16**C. In turn, this would result in the code bloat as shown in FIG. **16**C. Therefore, in order to perform a stable polyhedral rescan operation to generate the program statement view **820** from the newly optimized program loop view **850**, the polyhedral rescan operation must not specialize a statement's representation past any node containing such a hoistable condition.

[0153] Thus, the mechanisms of the illustrative embodiments use the following three phase operation to ensure stability of the program code being returned to the program statement view **820**. A first pass on the AST^P is used to detect the nodes containing hoistable conditionals by traversing the tree of the AST^P in a depth-first search traversal order (i.e. visiting/processing each parent node before each of its own children. A parent, or father, node F is then marked as a boundary node for re-entrance if one or more of its direct children are detected as having one or more hoistable conditionals. Then, for each such boundary node of a given depth d, all the instances of a given statement S' are identified and a single compound statement S in P' is generated to represent them. The set of instances of S under parent node F is denoted $\{S_{inst}{}^F\}$ where inst implicity denotes an enumeration of the different instances of the given statement S under F. The new domain for the compound statement is set as D^S=union over all instances inst(⊥dom iters $(D_{inst}{}^{SinstF})$.

[0154] In other words, the new domain associated with a given statement S is formed as follows. From node F, each of the leaf nodes (nodes without children) for which F is a (direct or indirect) parent is searched. The union of the domain associated with statement S at each of these leaf nodes is generated. This union of domain defines the final domain associated with statement S. The tradeoff with the above solution is that statements in P' do not correspond to the leaves of AST^P. This means that when the rescan process of the polyhedral rescan module **870** is complete, and the program loop view **850** has been successfully translated back into the program statement view **820**, each part of the statements associated with each of the leaves under node F we will not be able to optimized separately as they will have been grouped together as a single statement in order to avoid the problem associated with the hoistable condition. However, the new $AST^{P'}$ resulting from application of the code generation optimizations is guaranteed to be the same syntax tree as the original AST^P. This means that by doing such grouping of statements under node F, the hoistable condition problem has been avoided and thus, the overall quality of the generated code has not been degraded by a cycle through elements **840** and **870**.

[0155] Returning again to FIG. **8**, as discussed above, after iterating the optimizations performed by the loop optimizer **830** and the code generation optimization/parallelism detection module **860** via the re-entrance path comprising the

polyhedral rescan module **870** to achieve a desired level of optimization, the resulting optimized code must be emitted, by the code emitter **880**, to the compiler **805**.

[0156] Determination on whether to go along the re-entrance path or not depends on various factors. First, it may be desirable to transform the code to the program loop view **850** representation before completing all optimizations in the program statement view **820** representation in order to gather some knowledge about the code. For example it may be desirable to use the program loop view **850** to determine which loops are parallel, to see if kernels need to be extracted, to evaluate the complexity of the current code, and/or any other qualitative information that may be gathered from the program loop view **850** representation. Once this information is gathered, it is desirable to go back to the program statement view **820** representation to exploit this additional knowledge for further optimization of the code upon determination, based on this additional knowledge, that particular optimizations are advantageous.

[0157] Second, it may be desirable to apply all optimizations in the program statement view **820** representation at once. In this framework, some loop optimizations from the loop optimization module **830** may be applied and then the program statement view **820** representation may be converted to the program loop view or AST **850** representation. Specific code optimizations may be applied by the code generation optimization/parallel detection module **860**, such as kernel extraction and/or other code generation optimizations, and then the re-entrance path may be traversed to go back to the program statement view **820**. Further optimizations of specific aspects of the program loop view or AST **850** representations (after being modified by the code generation optimizations) may then be performed. Both approaches above are not exclusive and may be jointly applied or applied repetitively in some alternating fashion.

[0158] It is important that the code that is emitted back to the compiler **805** be of good quality even in the presence of highly optimized loop transformations, such as those of the illustrative embodiments, used for data locality and parallelism where statements are executed at "different speed" from the original program.

[0159] FIGS. **17**A-**17**C illustrate an example of code optimization where two statements have had their speed accelerated by a factor of 3. FIG. **17**A is an example of the original code having a complex schedule. FIG. **17**B is an example of the original code after optimization and regeneration for emitting back to the compiler. As can be seen from FIG. **17**B, obviously something very wrong occurred as the code size has been significantly increased. Thus, significant code bloat is again found with typical optimizations.

[0160] With the illustrative embodiments, to avoid such code bloat, a domain stretching operation is performed to augment the code generation optimization/parallelism detection operations that transform the program statement view **820** into the program loop view **850**. This operation essentially normalizes the domains associated with each statement by stretching them to their largest possible values without adding any execution points, which would otherwise change the semantics of the program. As a result of this optimization, which may be typically implemented in the code generation optimization/parallelism detection module **860** for example, high quality output code may be generated while enabling a path in which a statement can be still meaningfully split into distinct sub-statements as previously described above. As discussed above, these sub-statements may then be optimized as if they were original statements in the original program, namely the full range of optimizations such as loop fusion, loop splitting, loop skewing, loop tiling, (non) unimodular loop transformations, and the like, may be applied to these sub-statements as well.

[0161] As discussed above, the schedule of loops in a program may be represented as a structured matrix having three sub-matrices: (1) the Alpha matrix, which represents the speed at which statements are fired along a given time dimension; (2) the Beta matrix, which represents the sequential interleaving of statements along the different loop depths; and (3) the Gamma matrix, which represents the constant parametric shifting along each time dimension. The values of the Beta matrix will differ for each instance of an original statement S. The values of this Beta matrix may be read from the inner data representation of the AST in either the program statement view **820** or the program loop view **850**.

[0162] When the loop optimizer **830** accelerates a statement with respect to another, this yields a matrix Alpha with strides greater than 1 along with constant shiftings, and additional stability interplays occur with the Quillere separation algorithm. For example, as shown in FIGS. **17**A-**17**B, both S**1** and S**2** are slowed by a factor 3 on a first time dimension and statement S**2** is shifted by 1.

[0163] When considering the Alpha matrix, or A for short, transformations with stride greater than 1 along with shifting, the domains in the transformed space become very unfriendly for re-entrance. For example, consider the simplified schedules in FIGS. **18**A-**18**F. Initially, scattering domains are constructed in the time (i.e. transformed) space by applying the schedule function to the iteration domain for each statement in the program. This step actually expands the size of the domain by a factor 3 producing the time bounds observed in FIG. **18**B. The separation phase proceeds and yields the code obtained from the code generation optimization/parallelism detection module **860** as shown in FIG. **18**C. Now, when the program P' is regenerated in FIG. **18**D, the new iteration domains are obtained by shrinking back the separated time domains into the original space by a factor 3. The new expansion phase performed when reconstructing the new scatter domains in FIG. **18**E returns different scattering domains that will be split further and make the code size grow as in FIG. **18**F. It is also important to notice how the loop bounds on the first and last loops change between FIGS. **18**C and **18**F when using re-entrance and going back and forth from time space to original space. Actually, if no special care is taken, this process is recurrent at each regeneration attempt and stability will never be reached in the context of such Alpha matrix schedules.

[0164] Thus, the mechanisms of the illustrative embodiments define a new transformation, the scatter domain stretching transformation, to apply on domain constraints at scattering construction time. For each statement S the following operations are performed. First, the loop depth Ds associated with statement S is determined. Then the Hermite Normal Form (Hnf) matrix is calculated from the Alpha scheduling matrix. The Hermite Normal Form matrix is constructed using a standard matrix transformation (or linear algebra) that separates a given matrix X into a product of two matrices Y*Z, where Y is a matrix in Hermite Normal Form and Z is a unimodular matrix. The Hermite Normal Form Y matrix is a non-negative, non-singular, lower triangle matrix such that for each row i, the maximal element is $Y_{i,i}$ (i.e. the

diagonal element is larger than any others on that row). A unimodular matrix is a rectangle matrix whose determinant is either plus or minus one.

[0165] The scattering matrix Theta is computed using Alpha, Beta, Gamma matrices, and the domain of the statement S. For each time dimension Td (from 1 to Ds) the following operations are performed. The stride factor is computed as Sf=Hnf[Td, Td]. Namely the stride factor is the diagonal element at row/column number Td in the Hermite Normal Form matrix. Upon a determination that the stride factor Sf>1 then a determination is made as to whether this stride factor Sf divides every component (i.e. time domain, and parametric dimensions) in the scattering matrix for every row that contains a non-null Td entry.

[0166] If this check succeeds, then proceed as follows for each domain constraints Cd that include Td. If Cd is determined to be a lower bound constraint of the form "f(time, parameters)>=const", then const is replaced by floor((const−1)/Sf)*Sf+1 in the original domain matrix. Alternatively, if Cd is determined to be an upper bound constraint of the form "f(time, parameters)<=const", then const is replaced by floor ((const+1) Sf)*Sf−1 in the original domain matrix.

[0167] Once the above algorithm runs its course, the following post-processing is performed. The scattering matrix Theta is recalculated using Alpha, Beta, and Gamma matrices, and the modified domain. The resulting new scattering matrix is then void of the stretching constraint issue.

[0168] Note that the above example is only one exemplary way to process the time constraints, as they are alternative ways to derive some of the coefficients and/or other values that the constraints can be normalized to. Those of ordinary skill in the art will readily understand, in view of the present description, the manner by which the mechanisms of the illustrative embodiment may be modified for other implementations in which the coefficients and other values are represented differently. The present invention is not limited to the particular illustrative embodiments set forth above.

[0169] This transformation has the effect of stretching each constraint, encompassing into the domain every integer point of the time space that is strictly non integrate in the original space. It provides normalization for the scatter domains while guaranteeing that no new point is added to the original space. It further keeps the exact same number of executed instances for each statement while maximizing the overlapping of time domains.

[0170] The scatter domain with stretching transformation of the illustrative embodiments receives, as input, the Alpha, Beta, and Gamma matrices for a given statement as well as the domain for the statement. The scatter domain with stretching transformation outputs a modified scattering matrix Theta'. An example of pseudocode for implementing a scatter domain with stretching transformation in accordance with one illustrative embodiment is provided as follows:

```
Determine the loop depth Ds associated with statement S;
Compute the Hnf (Hermite Normal Form) matrix from the alpha matrix;
Compute the scattering matrix Theta using Alpha, Beta, Gamma, and
Domain for each time dimension Td (from 1 to Ds)
  stride factor Sf = Hnf[Td, Td] (the diagonal element at row/column
number Td in the Hermite Normal Form matrix);
  if stride factor Sf > 1 then
    check if this stride factor Sf divides every component (i.e. time
domain, and parametric dimensions) in the scattering matrix for every
row that contain a non-null Td entry;
    if this check succeeds, then
      for each domain constraint Cd that includes Td:
        if it is determined that Cd is a lower bound constraint of the
form "f(time, parameters) >= const",
          replace const by floor( (const−1) / Sf) * Sf +1 in the origina
domain matrix;
        if it is determined that Cd is an upper bound constraint of the form
"f(time, parameters) <= const",
          replace const by floor((const+1) / Sf) * Sf −1 in the original
domain matrix;
Recompute the scattering matrix Theta using Alpha, Beta, Gamma, and
the modified domain.
```

[0171] In the above pseudocode, the Hermite Normal Form matrix is a matrix obtained from using the known Hermite Normal Form decomposition method but which is restricted to the case of a single transformation, or at best to harshly constrained multiple transformations. The Hermite Normal Form matrix may be defined as follows: Given an integer matrix H of size m×n and full rank, H is in Hermite Normal Form if and only if H=[B 0] where B is a non-negative, non-singular lower triangular matrix such that for each row I, the unique maximal element is $b_{i,i}$(i.e. Vj<i, $b_{i,j}<b_{i,i}$). Moreover, in the above pseudocode, the generation of the scattering matrix Theta from the Alpha, Beta, and Gamma matrices, and the domain, is generally known in the art.

[0172] FIGS. **19**A-**19**B illustrate the scattering domains for S**1** and S**2** and resulting stable $AST^{P'}$ obtained using the scatter domain with stretching transformation of the illustrative embodiments. The overlapping portion of the scattering domains is much friendlier to re-entrance stability but will still generate a cut for statement S**2** and $3N+3 \leqq t1 \leqq 3N+4$. However, this cut is actually non-integral in the original dimension and it will indeed be removed from the resulting program loop view **850**, i.e. $AST^{P'}$. While domain stretching is described as a transformation for re-entrance, it also provides very efficient reduction in the number of separations performed by the separation phase because it helps in avoiding spurious border cuts.

[0173] FIGS. **20**A-**20**C illustrate an example of domain stretching under re-entrance in accordance with one illustrative embodiment. This example considers the schedule equations:

$$i \in [1,M] \wedge t_1=3i+2M \; t_2=3i+3M$$

The respective scatter domains are thus: $2M+3 \leqq t_1 \leqq 5M+2 \wedge 3M+3 \leqq t_2 \leqq 6M$. This in turn yields, after stretching: $2M+3 \leqq t_1 \leqq 5M+2 \wedge 3M+3 \leqq t_2 \leqq 6M+2$. While the constraints on t**2** have been successfully stretched and will provide less opportunities for separation with other statements, the ones on t**1** could not be stretched because of the statically unknown value of 2M %3. Under such schedules, the interleaving of the statements changes with the values of 2M %3 and cannot be expressed without outermost modulo case distinction. However, no disruption on the re-entrance stability is experienced as can be seen from FIGS. **20**A-**20**C. The reason behind the stability is that the stretched loop bounds are parametric and cover all the different modulo cases with a single expression. The lack of knowledge of the exact modulo remainder forces the domains to overlap and does not generate spurious cuts.

[0174] FIGS. **21-24** are flowcharts that illustrate various operations according to the illustrative embodiments. It will be understood that each block of the flowchart illustrations,

and combinations of blocks in the flowchart illustrations, can be implemented by computer program instructions. These computer program instructions may be provided to a processor or other programmable data processing apparatus to produce a machine, such that the instructions which execute on the processor or other programmable data processing apparatus create means for implementing the functions specified in the flowchart block or blocks. These computer program instructions may also be stored in a computer-readable memory or storage medium that can direct a processor or other programmable data processing apparatus to function in a particular manner, such that the instructions stored in the computer-readable memory or storage medium produce an article of manufacture including instruction means which implement the functions specified in the flowchart block or blocks.

[0175] Accordingly, blocks of the flowchart illustrations support combinations of means for performing the specified functions, combinations of steps for performing the specified functions and program instruction means for performing the specified functions. It will also be understood that each block of the flowchart illustrations, and combinations of blocks in the flowchart illustrations, can be implemented by special purpose hardware-based computer systems which perform the specified functions or steps, or by combinations of special purpose hardware and computer instructions.

[0176] Furthermore, the flowcharts are provided to demonstrate the operations performed within the illustrative embodiments. The flowcharts are not meant to state or imply limitations with regard to the specific operations or, more particularly, the order of the operations. The operations of the flowcharts may be modified to suit a particular implementation without departing from the spirit and scope of the present invention.

[0177] FIG. **21** is a flowchart outlining an exemplary operation for utilizing a re-entrance path to obtain further optimization of code in accordance with one illustrative embodiment. The operation outlined in FIG. **21** may be performed, for example, by a polyhedral loop optimization mechanism, such as element **800** in FIG. **8** described above. The polyhedral loop optimization mechanism may work in conjunction with a compiler, such as compiler **805** in FIG. **8**, to optimize source code in an iterative manner using a re-entrance path of the illustrative embodiments. The optimized code may then be provided back to the compiler **805** for use in generating executable code for execution on a computing device, such as server **304** or client **310** in FIG. **3**, or the like.

[0178] As shown in FIG. **21**, the operation starts with source code being received from the compiler (step **2110**). A program statement view of the source code is generated using a known methodology (step **2112**). For example, as discussed above, a known polyhedral scan operation may be performed on an intermediate representation of the source code to generate a program statement view of the source code. One or more loop optimizations are then optionally performed on the program statement view of the source code (step **2113**).

[0179] For each statement in the program statement view of the source code, program statement information, such as the Alpha, Beta, and Gamma matrices, the Domain, Access Function(s), and the statement expression, are obtained (step **2114**). A scattering matrix is built for each statement based on the program statement information (step **2116**) and a portion of the program statement information, such as the Alpha, Beta, and Gamma matrices, for example, is stored for later use (step **2118**). It should be noted that the present invention is in no way limited to a specific representation of the scheduling function associated with a given statement. The Alpha, Beta, and Gamma matrix structure is used herein as one example embodiment for illustrative purposes only. Many other types of representations may be utilized without departing from the spirit and scope of the present invention. For example, other possible representations may include a unified matrix representing the scheduling information that maps a specific iteration to a specific (possibly multi-dimensional) date or the like.

[0180] A program loop view, or AST, of the source code is generated based on the program statement view and the scattering matrix (step **2120**). For each node in the program loop view, a list of statements included in the node is stored and a reference to each statement's original program statement information is also stored in association with the node (step **2122**).

[0181] One or more code generation optimizations may be performed on the program loop view (step **2124**) and a determination is made as to whether the re-entrance path is to be taken (step **2126**). As mentioned above, the decision to take the re-entrance path is dependent upon the particular circumstances and whether or not re-entrance will be beneficial to the overall optimization of the code. This decision may be made based on user input or an automated mechanism, as discussed previously above.

[0182] If the re-entrance path is not to be taken, then the operation terminates. If the re-entrance path is to be taken, then the statements in the nodes of the program loop view are split, if possible, into sub-statements upon which loop optimizations may be performed individually (step **2128**). The nodes, which may include the split sub-statements, are then rescanned to construct new program statement information (step **2130**). The rescanning of the nodes in the program loop view may involve, for example, selecting a set of boundary nodes. The set of boundary nodes may be a set of interior nodes (node with children) provided none of the interior nodes have parent nodes that are already a boundary node or a set of leaf nodes (node without children) provided that nod of the leaf nodes have parent nodes that are already boundary nodes. Then for a given boundary node B at depth d in the program loop view, for each statement S associated with B, new program statement information is constructed as follows. The Alpha and Gamma matrices are maintained the same as they were for the original statement S (as stored by step **2118**). The Beta matrix is reactualized to reflect the ordering in the program loop view. For example, if a beta value in the Beta matrix is 1, the depth d is set to the node number at each level in the program loop view. If a beta value in the Beta matrix has a value of the depth d+1, the last beta value is set to the corresponding value in the original beta values associated with S and stored in the program loop view. The domain may then be constructed as the union of all the domains associated with leaf nodes that contain S and have node B as a parent node.

[0183] The new program statement information is then used to generate a new program statement view of the source code (step **2132**). This new program statement view of the source code may then be subjected to additional loop optimizations, converted into a new program loop view of the code to which additional code generation optimizations may be applied, and the like, in an iterative manner, if desired. The

operation then either terminates if no further optimization is required or returns to step **2113** if further optimization is desired.

[0184] It should be noted that the above embodiment is only one possible application of a scheme in which a code generation step (such as the AST generation in step **2120**) is used in order to split original statements for further optimizations in a Program Statement Representation. Alternative embodiments could simply build an AST and analyze it using some generic inspector in order to determine suitable cuts directly in the original Program Statement Representation. While it is believed that the process in FIG. **21** is an efficient way to proceed, such alternative process generating an AST or similar code representation followed by an inspection phase to split statements in the original Program Statement View are equally applicable and suitable in some implementations of the present invention.

[0185] FIG. **22** is a flowchart outlining an exemplary operation for applying a code generation transformation algorithm in accordance with one illustrative embodiment. The operation outlined in FIG. **22** may be performed, for example, as part of the step **2124** in FIG. **21**. It should be appreciated that the operation outlined in FIG. **22** may be performed for each of a plurality of code generation optimizations.

[0186] As shown in FIG. **22**, the operation starts with receiving the program loop view of the source code (step **2210**). This may be obtained, for example, from step **2122** in FIG. **21**. A definition of the types of nodes in the program loop view to which a particular code generation optimization is to be applied may be generated (step **2220**). It should be appreciated that such a definition may have been previously defined prior to the operation outlined in FIG. **22** being executed, for example. The nodes of the program loop view are then traversed to mark the nodes meeting the definition for the code generation optimization (step **2230**). The code generation transformation algorithm is then applied such that the code generation optimization core function is applied to the statements in the marked nodes (step **2240**). The operation then terminates.

[0187] The code generation transformation algorithm applied in step **2240** may be of the type shown in FIG. **11**, described previously. An alternative code generation transformation algorithm may be as shown in the following pseudocode where AST refers to the program loop view of the source code currently undergoing the code generation optimizations:

```
ApplyCodegenTransformation: applies a transformation on AST
  Input :
    node: AST, where the nodes that initiate a transformation are marked.
    outer_visitor_type: method that order the nodes of the AST
according to its definition (e.g. depth first search, ...)
    apply_core_function: method to be applied on the marked node
    propagate_changes: method to be applied on the children below a
    marked node
Output : transformed AST
  visitor = new visitor(outer_visitor_type, root of AST)
  for each currentNode in visitor, according to the order defined by the
outer_visitor_type
    if currentNode is marked
      boolean changed = false
1     core_node_list = apply_core_function(currentNode, changed)
      if changed
        sort core_node_list under parent context
        foreach newNode in core_node_list, accorind to the sorted
        order
2         newNode.children = a replicate copy of currentNode.children
3         newNode = propagate_changes(newNode.children)
          if newNode.children size is 0
4           delete newNode
return transformed AST
```

It should be appreciated that with the application of the code generation transformation algorithm of the illustrative embodiments, rather than having to apply code generation optimizations to the program loop view as a whole, i.e. at only the root of the program loop view, or at all of the nodes at the same depth as a whole, the mechanisms of the illustrative embodiments allow the code generation optimizations to be applied to individual arbitrary sets of one or more nodes in the program loop view.

[0188] FIG. **23** is a flowchart outlining an exemplary operation for preserving stability of code in the presence of conditionals for re-entrance in accordance with one illustrative embodiment. The operation outlined in FIG. **23** may be performed, for example, as part of step **2130** in FIG. **21** to ensure stability of the code, i.e. minimizing growth of the code or code bloat.

[0189] As shown in FIG. **23**, the operation starts with receiving the program lop view of the source code (step **2310**). A next node in the program loop view is identified in a depth first search order with processing of parent nodes before child nodes (step **2320**). The depth d of the node is retrieved and the immediate parent of the node is identified (step **2330**). The scattering domain for the node is projected on a depth of 1 to d−1 (step **2340**) and the projected scattering is simplified under the domain of the parent node (step **2350**). A determination is made as to whether the domain is the universe (step **2360**). Stating that a domain is the universe is equivalent to stating that the domain corresponds to the entire space with no constraints. For example, in the one dimensional space of integer or entire numbers, a domain formed by the two constrains "x>−5 and x<10" is not the universe as it has constrains. However, if a domain with no constrains is the universe, it includes any possible integer numbers, from minus infinity to plus infinity. If the domain is not the universe, then the parent node is marked as a boundary node (step **2370**).

[0190] Thereafter, or if the node is the universe, a determination is made as to whether more nodes are present that need to be processed (step **2380**). If so, the operation returns to step **2320** and repeats with the next node. If no more nodes are to be processed, the operation performs a rescan operation (step **2390**) such as in step **2130** of FIG. **21**. The operation then terminates.

[0191] FIG. **24** is a flowchart outlining an exemplary operation for performing scatter domain stretching in accordance with one illustrative embodiment. As shown in FIG. **24**, the operation starts with receiving the program statement view of the source code with the program statement information for the various statements (step **2410**). A next statement in the program statement view to process is identified (step **2412**) and a loop depth Ds of the statement is determined (step **2414**). The Hermite Normal Form (HNF) matrix for the statement is computed from the Alpha matrix of the statement (step **2416**). The scattering matrix Theta for the statement is

computed using the Alpha, Beta, and Gamma matrices and the Domain of the statement (step **2418**).

[0192] A next time dimension to be processed is identified (step **2420**). A stride factor for that time dimension is determined based on the HNF matrix (step **2422**). A determination is made as to whether the stride factor is greater than one (step **2424**). If the stride factor is greater than one, then a determination is made as to whether the stride factor divides every component in the scattering matrix for every row that contains a non-null time dimension entry Td (step **2426**). If so, then for each domain constraint Cd that includes the time dimension entry Td, if Cd is a lower bound constraint of the form f(time, parameters)>=const, then const is replaced by floor(const−1/Sf)*Sf+1 in the original domain matrix, where Sf is the scatter factor. If Cd is an upper bound constraint of the form f(time, parameters)<=const, then const is replaced by floor((const+1)/Sf)*Sf−1 in the original domain matrix (step **2428**).

[0193] Thereafter, or if the stride factor does not divide every component in the scattering matrix (step **2426**), or if the stride factor is less than or equal to 1, then a determination is made as to whether there are additional time dimensions to process (step **2430**). If there are additional time dimensions to process, the operation returns to step **2420** and proceeds with the next time dimension. If there are no additional time dimensions to process, the operation determines if there are more statements to process (step **2432**). If there are more statements to process, the operation returns to step **2412** and proceeds with the next statement. If there are no more statements to process, then the scattering matrix Theta is recomputed using the Alpha, Beta, and Gamma matrices and the modified domain (step **2434**). The operation then terminates.

[0194] Again, this invention is not constrained to a particular representation of the schedule (the Alpha/Beta/Gamma matrices here). While they are used in the above embodiments, other alternative representations can be used in the illustrative embodiments without departing from the spirit and scope of the present invention, as discussed above.

[0195] Thus, the illustrative embodiments provide a mechanism for optimizing source code that permits individual statement instances within a program loop view of the source code to be operated upon by code generation optimizations and loop optimizations. A re-entrance path is provided through which the code may undergo optimizations in an iterative manner. The re-entrance path allows a program loop view of the code to be transformed back into a program statement view so that program loop optimizations may be applied to the program statement view after code generation optimizations have been applied to the previous program loop view. Moreover, mechanisms are provided for ensuring the stability of the code when traversing the re-entrance path by projecting and simplifying scattering domains, performing polyhedral rescans of the code based on such scattering domains, and minimizing code bloat.

[0196] It should be appreciated that the illustrative embodiments may take the form of an entirely hardware embodiment, an entirely software embodiment or an embodiment containing both hardware and software elements. In one exemplary embodiment, the mechanisms of the illustrative embodiments are implemented in software, which includes but is not limited to firmware, resident software, microcode, etc.

[0197] Furthermore, the illustrative embodiments may take the form of a computer program product accessible from a computer-usable or computer-readable medium providing program code for use by or in connection with a computer or any instruction execution system. For the purposes of this description, a computer-usable or computer-readable medium can be any apparatus that can contain, store, communicate, propagate, or transport the program for use by or in connection with the instruction execution system, apparatus, or device.

[0198] The medium may be an electronic, magnetic, optical, electromagnetic, infrared, or semiconductor system (or apparatus or device) or a propagation medium. Examples of a computer-readable medium include a semiconductor or solid state memory, magnetic tape, a removable computer diskette, a random access memory (RAM), a read-only memory (ROM), a rigid magnetic disk and an optical disk. Current examples of optical disks include compact disk—read-only memory (CD-ROM), compact disk—read/write (CD-R/W) and DVD.

[0199] A data processing system suitable for storing and/or executing program code will include at least one processor coupled directly or indirectly to memory elements through a system bus. The memory elements can include local memory employed during actual execution of the program code, bulk storage, and cache memories which provide temporary storage of at least some program code in order to reduce the number of times code must be retrieved from bulk storage during execution.

[0200] Input/output or I/O devices (including but not limited to keyboards, displays, pointing devices, etc.) can be coupled to the system either directly or through intervening I/O controllers. Network adapters may also be coupled to the system to enable the data processing system to become coupled to other data processing systems or remote printers or storage devices through intervening private or public networks. Modems, cable modems and Ethernet cards are just a few of the currently available types of network adapters.

[0201] The description of the present invention has been presented for purposes of illustration and description, and is not intended to be exhaustive or limited to the invention in the form disclosed. Many modifications and variations will be apparent to those of ordinary skill in the art. The embodiment was chosen and described in order to best explain the principles of the invention, the practical application, and to enable others of ordinary skill in the art to understand the invention for various embodiments with various modifications as are suited to the particular use contemplated.

What is claimed is:

1. A method, in a data processing system, for optimizing program code, comprising:

receiving source code for a program in a compiler;

transforming the source code into a program statement view of the source code;

transforming the program statement view of the source code into a program loop view of the source code;

applying one or more code generation optimizations to the program loop view of the source code to generate optimized code; and

outputting the optimized code to a compiler for use in generating executable code for execution on a computing device, wherein transforming the program statement view of the source code into a program loop view of the source code comprises applying a domain stretching operation to domains of statements in the program statement view of the source code to normalize the domains

by stretching each domain to its largest possible value without adding execution points.

2. The method of claim **1**, wherein applying a domain stretching operation to domains of statements in the program statement view comprises, for each statement and each time dimension:

statically determining a factor f that is greater than one and will divide the time dimension t from a remapping matrix;

checking the factor f to ensure that the factor f divides every factor on all time, domain, and parametric dimensions; and

stretching a constraint associated with the domain of the statement based on the factor.

3. The method of claim **2**, wherein stretching the constraint comprises stretching the constraint to a closest multiple of the factor f minus 1 if the constraint is a lower bound on the time dimension t.

4. The method of claim **2**, wherein stretching the constraint comprises stretching the constraint to a closest multiple of the factor f plus 1 if the constraint is not a lower bound on the time dimension t.

5. The method of claim **1**, wherein applying a domain stretching operation to domains of statements in the program statement view comprises, for each statement and each time dimension:

receiving an Alpha matrix, Beta matrix, and Gamma matrix associated with the statement, wherein the Alpha matrix represents a speed at which an associated statement is performed along a given time dimension, the Beta matrix represents a sequential interleaving of the associated statement along different loop depths, and the Gamma matrix represents a constant parametric shifting of the associated statement along each time dimension;

receiving a domain for the statement;

applying a scatter domain with stretching transformation to the statement based on the Alpha, Beta, and Gamma matrices and the domain; and

receiving as output of the scatter domain with stretching transformation, a first scattering matrix.

6. The method of claim **5**, wherein applying the scatter domain with stretching transformation comprises:

determining a depth of the statement within the program statement view;

computing a second scattering matrix based on the Alpha, Beta, and Gamma matrices and the domain of the statement;

determining a modified domain for the statement based on the Alpha matrix; and

generating the first scattering matrix based on the Alpha, Beta, and Gamma matrices and the modified domain.

7. The method of claim **6**, wherein determining a modified domain for the statement comprises:

computing a Hermite Normal Form matrix based on the Alpha matrix;

determining a stride factor based on the Hermite Normal Form matrix; and

modifying constraints associated with the domain of the statement based on the stride factor.

8. A computer program product comprising a computer useable medium having a computer readable program, wherein the computer readable program, when executed on a computing device, causes the computing device to:

receive source code for a program in a compiler;

transform the source code into a program statement view of the source code;

transform the program statement view of the source code into a program loop view of the source code;

apply one or more code generation optimizations to the program loop view of the source code to generate optimized code; and

output the optimized code to a compiler for use in generating executable code for execution on a computing device, wherein transforming the program statement view of the source code into a program loop view of the source code comprises applying a domain stretching operation to domains of statements in the program statement view of the source code to normalize the domains by stretching each domain to its largest possible value without adding execution points.

9. The computer program product of claim **8**, wherein the computer readable program causes the computing device to apply a domain stretching operation to domains of statements in the program statement view by, for each statement and each time dimension:

statically determining a factor f that is greater than one and will divide the time dimension t from a remapping matrix;

checking the factor f to ensure that the factor f divides every factor on all time, domain, and parametric dimensions; and

stretching a constraint associated with the domain of the statement based on the factor.

10. The computer program product of claim **9**, wherein the computer readable program causes the computing device to stretch the constraint by stretching the constraint to a closest multiple of the factor f minus 1 if the constraint is a lower bound on the time dimension t.

11. The computer program product of claim **9**, wherein the computer readable program causes the computing device to stretch the constraint by stretching the constraint to a closest multiple of the factor f plus 1 if the constraint is not a lower bound on the time dimension t.

12. The computer program product of claim **8**, wherein the computer readable program causes the computing device to apply a domain stretching operation to domains of statements in the program statement view by, for each statement and each time dimension:

receiving an Alpha matrix, Beta matrix, and Gamma matrix associated with the statement, wherein the Alpha matrix represents a speed at which an associated statement is performed along a given time dimension, the Beta matrix represents a sequential interleaving of the associated statement along different loop depths, and the Gamma matrix represents a constant parametric shifting of the associated statement along each time dimension;

receiving a domain for the statement;

applying a scatter domain with stretching transformation to the statement based on the Alpha, Beta, and Gamma matrices and the domain; and

receiving as output of the scatter domain with stretching transformation, a first scattering matrix.

13. The computer program product of claim **12**, wherein the computer readable program causes the computing device to apply the scatter domain with stretching transformation by:

determining a depth of the statement within the program statement view;

computing a second scattering matrix based on the Alpha, Beta, and Gamma matrices and the domain of the statement;

determining a modified domain for the statement based on the Alpha matrix; and

generating the first scattering matrix based on the Alpha, Beta, and Gamma matrices and the modified domain.

14. The computer program product of claim **13**, wherein the computer readable program causes the computing device to determine a modified domain for the statement by:

computing a Hermite Normal Form matrix based on the Alpha matrix;

determining a stride factor based on the Hermite Normal Form matrix; and

modifying constraints associated with the domain of the statement based on the stride factor.

15. A system, comprising:

a processor; and

a memory coupled to the processor, wherein the memory comprises instructions which, when executed by the processor, cause the processor to:

receive source code for a program in a compiler;

transform the source code into a program statement view of the source code;

transform the program statement view of the source code into a program loop view of the source code;

apply one or more code generation optimizations to the program loop view of the source code to generate optimized code; and

output the optimized code to a compiler for use in generating executable code for execution on a computing device, wherein transforming the program statement view of the source code into a program loop view of the source code comprises applying a domain stretching operation to domains of statements in the program statement view of the source code to normalize the domains by stretching each domain to its largest possible value without adding execution points.

16. The system of claim **15**, wherein the instructions cause the processor to apply a domain stretching operation to domains of statements in the program statement view by, for each statement and each time dimension:

statically determining a factor f that is greater than one and will divide the time dimension t from a remapping matrix;

checking the factor f to ensure that the factor f divides every factor on all time, domain, and parametric dimensions; and

stretching a constraint associated with the domain of the statement based on the factor.

17. The system of claim **16**, wherein the instructions cause the processor to stretch the constraint by stretching the constraint to a closest multiple of the factor f minus 1 if the constraint is a lower bound on the time dimension t.

18. The system of claim **16**, wherein the instructions cause the processor to stretch the constraint by stretching the constraint to a closest multiple of the factor f plus 1 if the constraint is not a lower bound on the time dimension t.

19. The system of claim **15**, wherein the instructions cause the processor to apply a domain stretching operation to domains of statements in the program statement view by, for each statement and each time dimension:

receiving an Alpha matrix, Beta matrix, and Gamma matrix associated with the statement, wherein the Alpha matrix represents a speed at which an associated statement is performed along a given time dimension, the Beta matrix represents a sequential interleaving of the associated statement along different loop depths, and the Gamma matrix represents a constant parametric shifting of the associated statement along each time dimension;

receiving a domain for the statement;

applying a scatter domain with stretching transformation to the statement based on the Alpha, Beta, and Gamma matrices and the domain; and

receiving as output of the scatter domain with stretching transformation, a first scattering matrix.

20. The system of claim **19**, wherein the instructions cause the processor to apply the scatter domain with stretching transformation by:

determining a depth of the statement within the program statement view;

computing a second scattering matrix based on the Alpha, Beta, and Gamma matrices and the domain of the statement;

determining a modified domain for the statement based on the Alpha matrix; and

generating the first scattering matrix based on the Alpha, Beta, and Gamma matrices and the modified domain.

21. The system of claim **20**, wherein the instructions cause the processor to determine a modified domain for the statement by:

computing a Hermite Normal Form matrix based on the Alpha matrix;

determining a stride factor based on the Hermite Normal Form matrix; and

modifying constraints associated with the domain of the statement based on the stride factor.

* * * * *