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(54) **OPTIMISATION OF THE DESIGN OF A COMPONENT**

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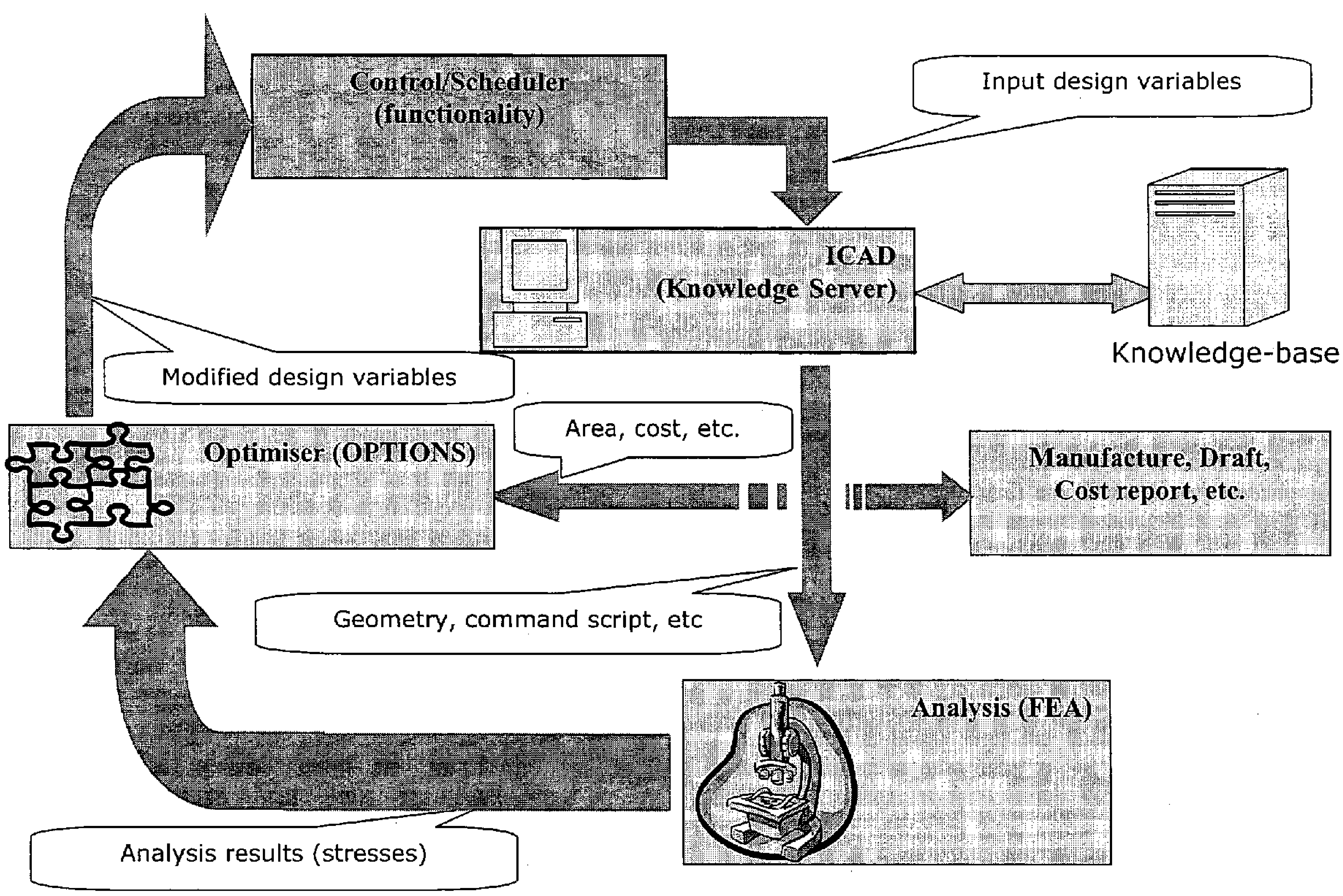
(57) **ABSTRACT**

A method of optimising a design of a component is described. The method comprises the steps of: representing a base design as a CAD model comprising a plurality of geometric entities, assigning a tag name to each geometric entity, transferring the design into an analysis code and determining an optimum design from the analysis code. The tag names associate boundary conditions to the geometric entities such as temperature, velocity or mesh density if the analysis code is a Finite Element Analysis code.



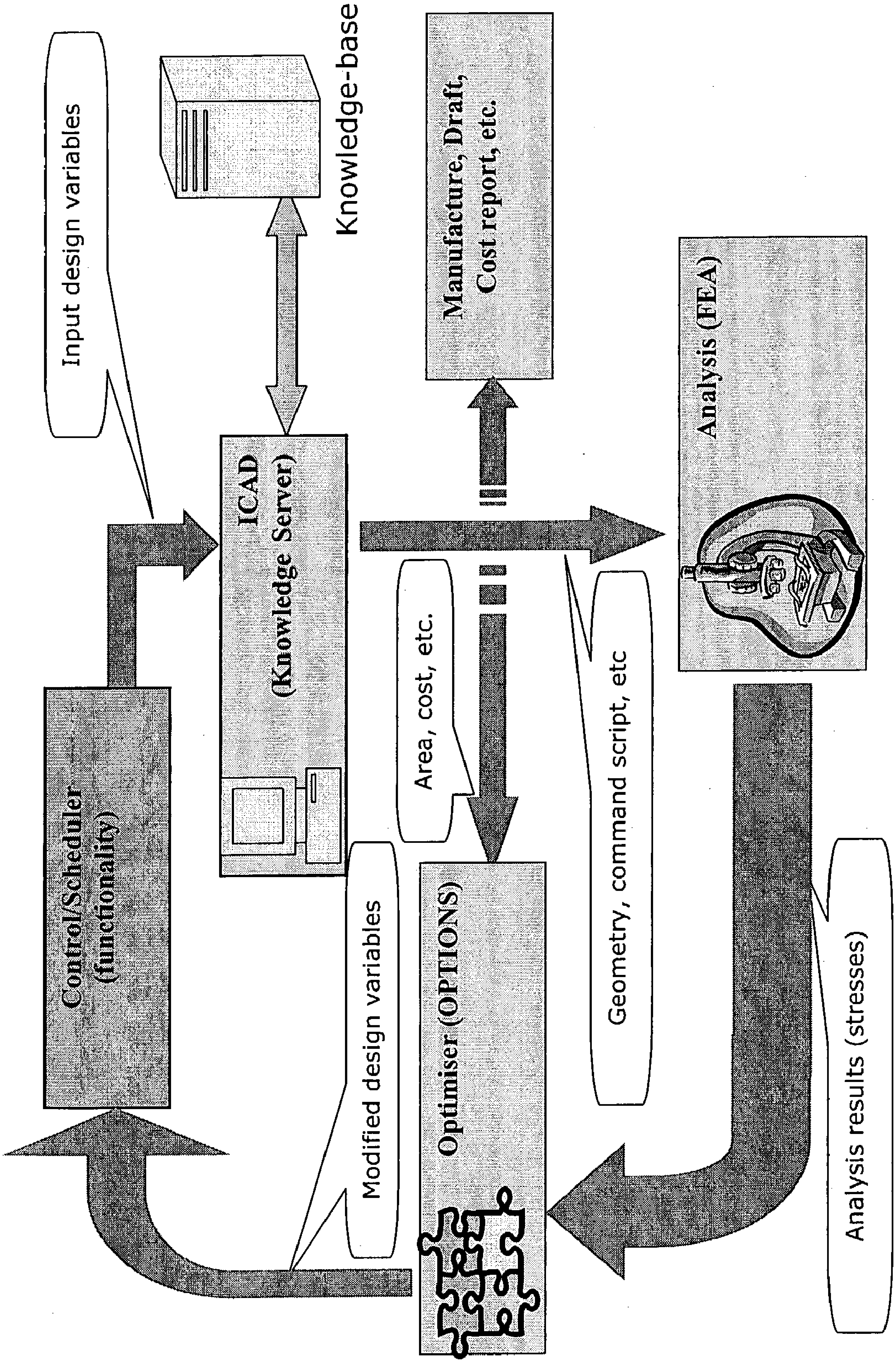

Input design variables
Knowledge-base
Manufacture, Draft, Cost report, etc.
ICAD (Knowledge Server)
Analysis (FEA)
Control/Scheduler (functionality)
Area, cost, etc.
Geometry, command script, etc
Modified design variables
Optimiser (OPTIONS)
Analysis results (stresses)

Figure 1.

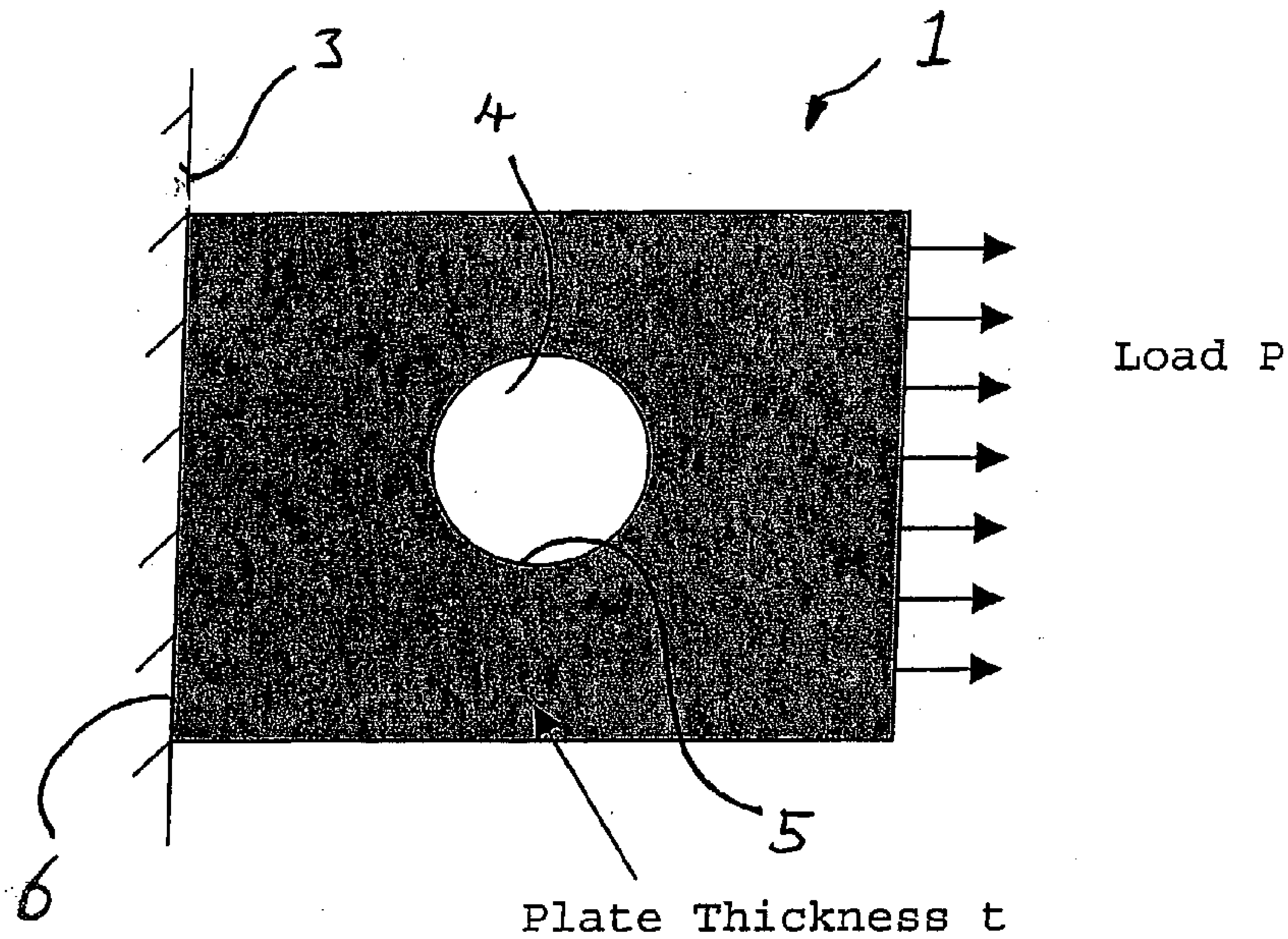



Figure 2

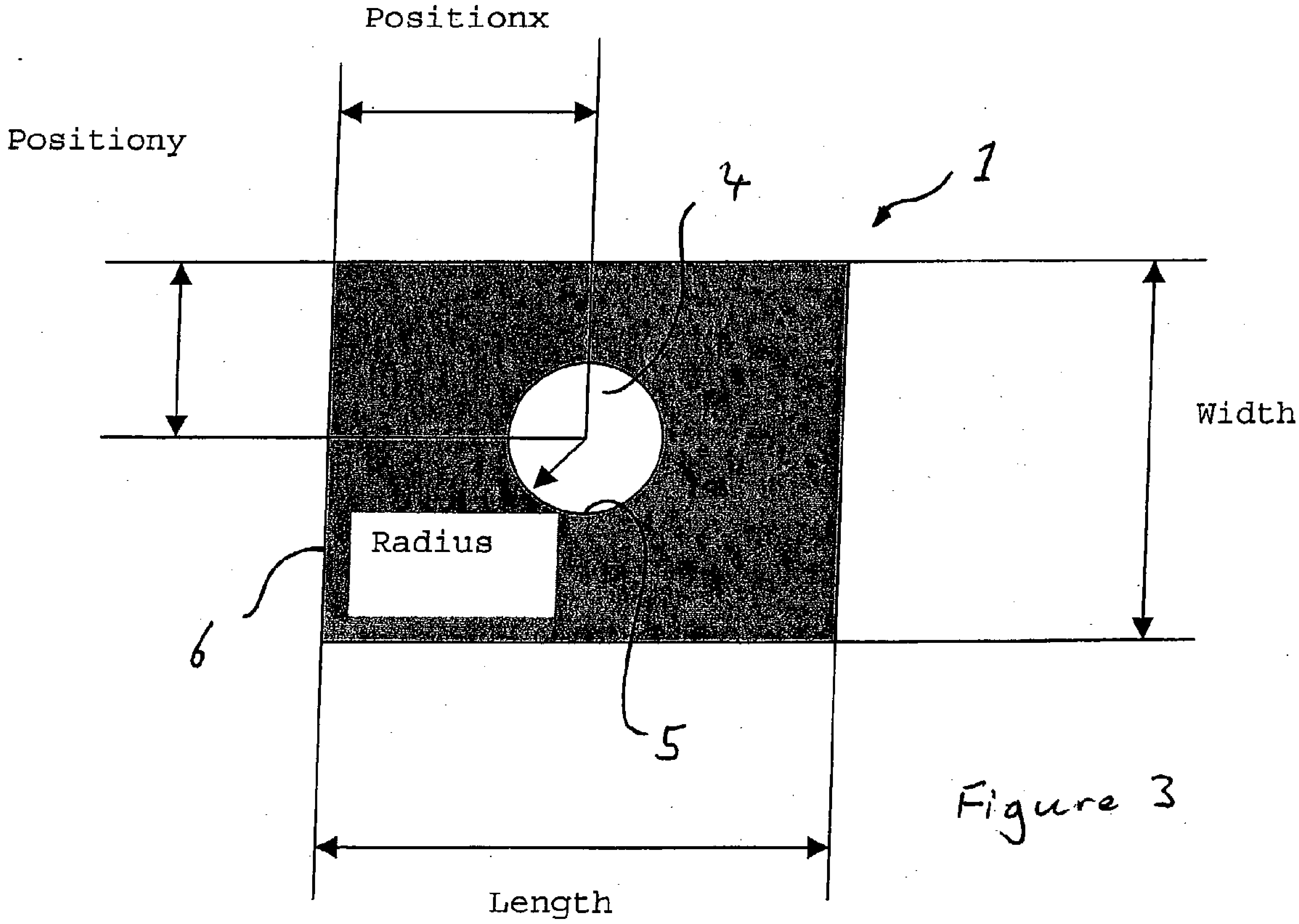



Figure 3

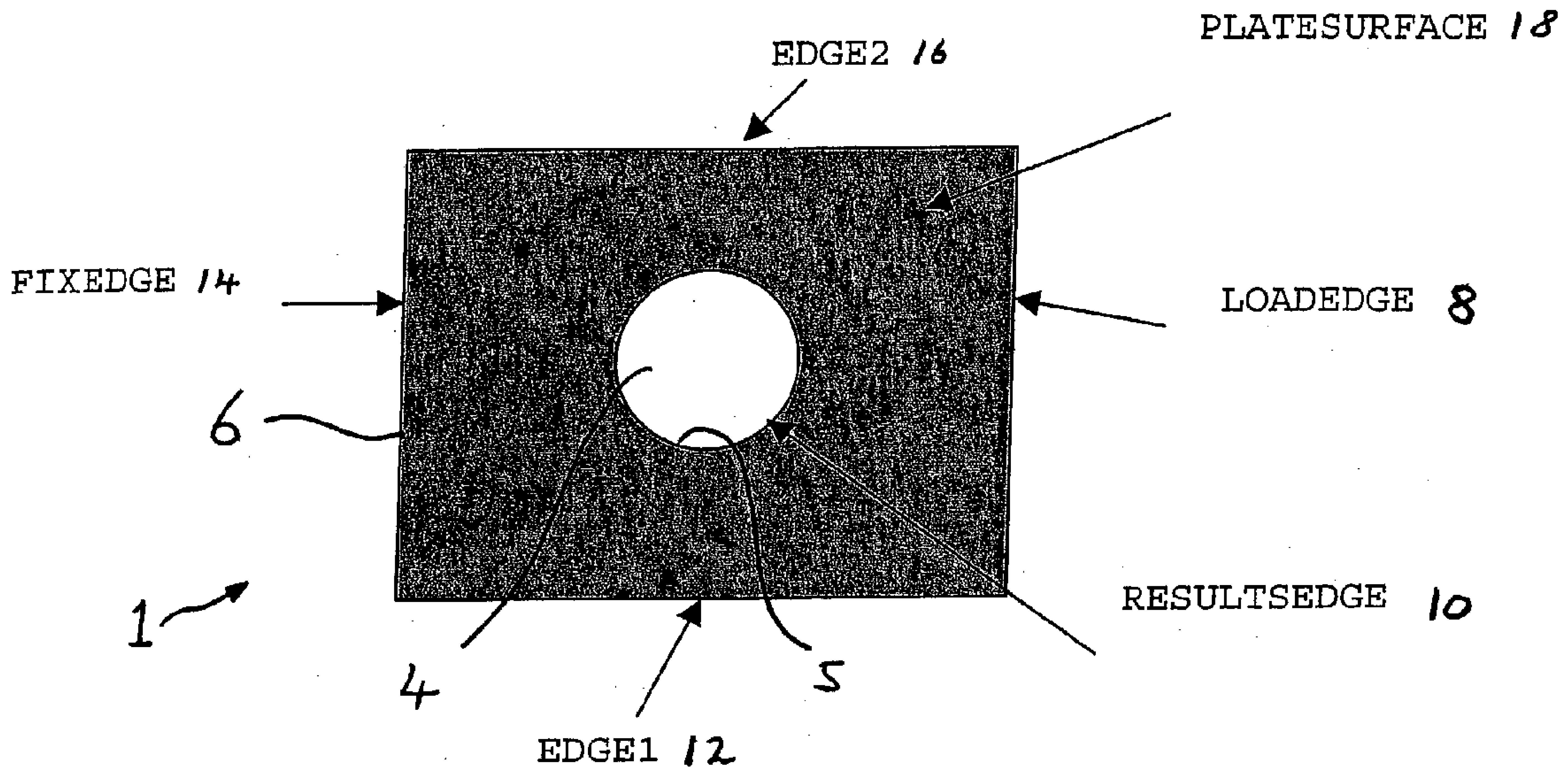



Figure 4

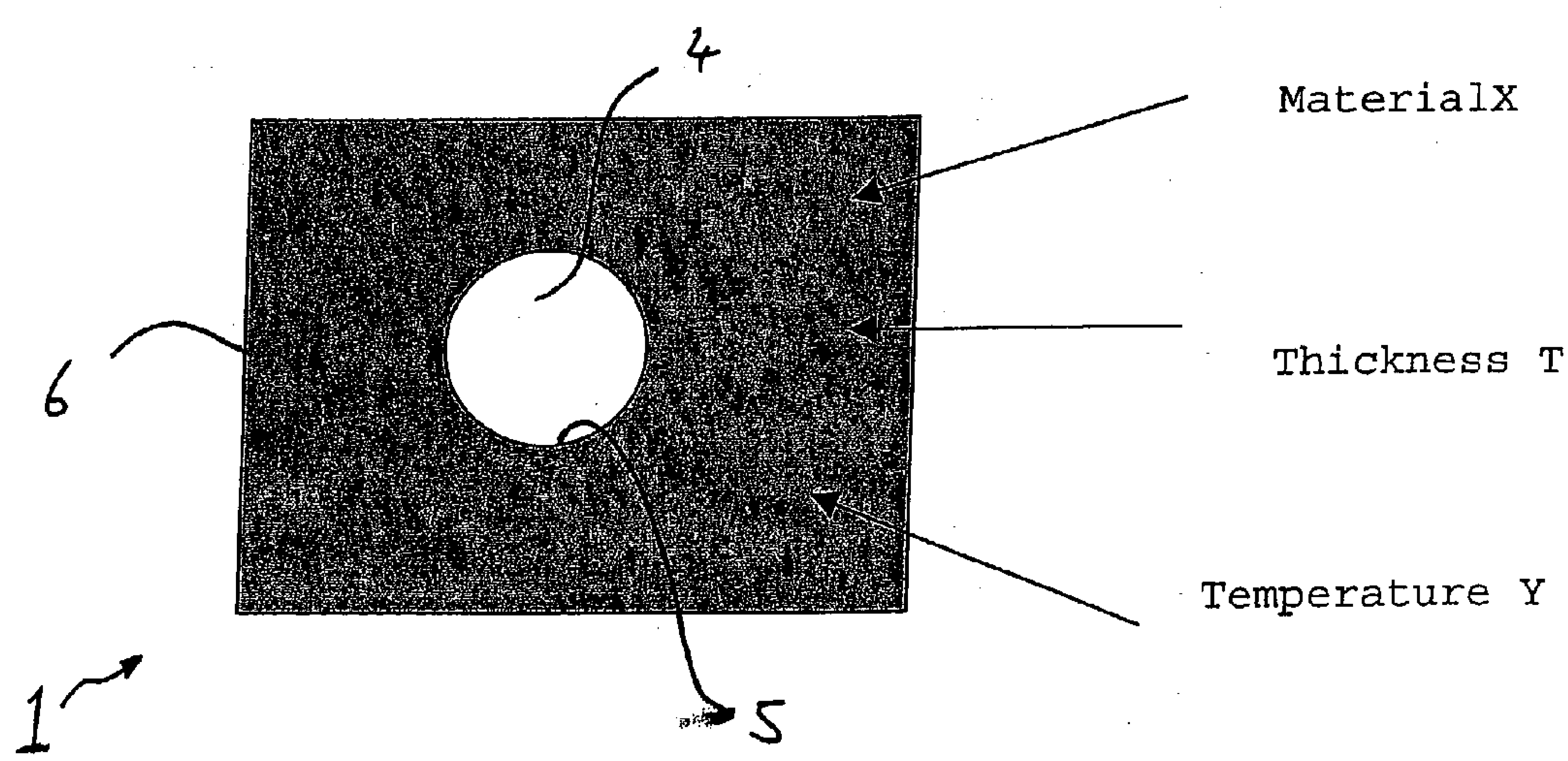



Figure 5

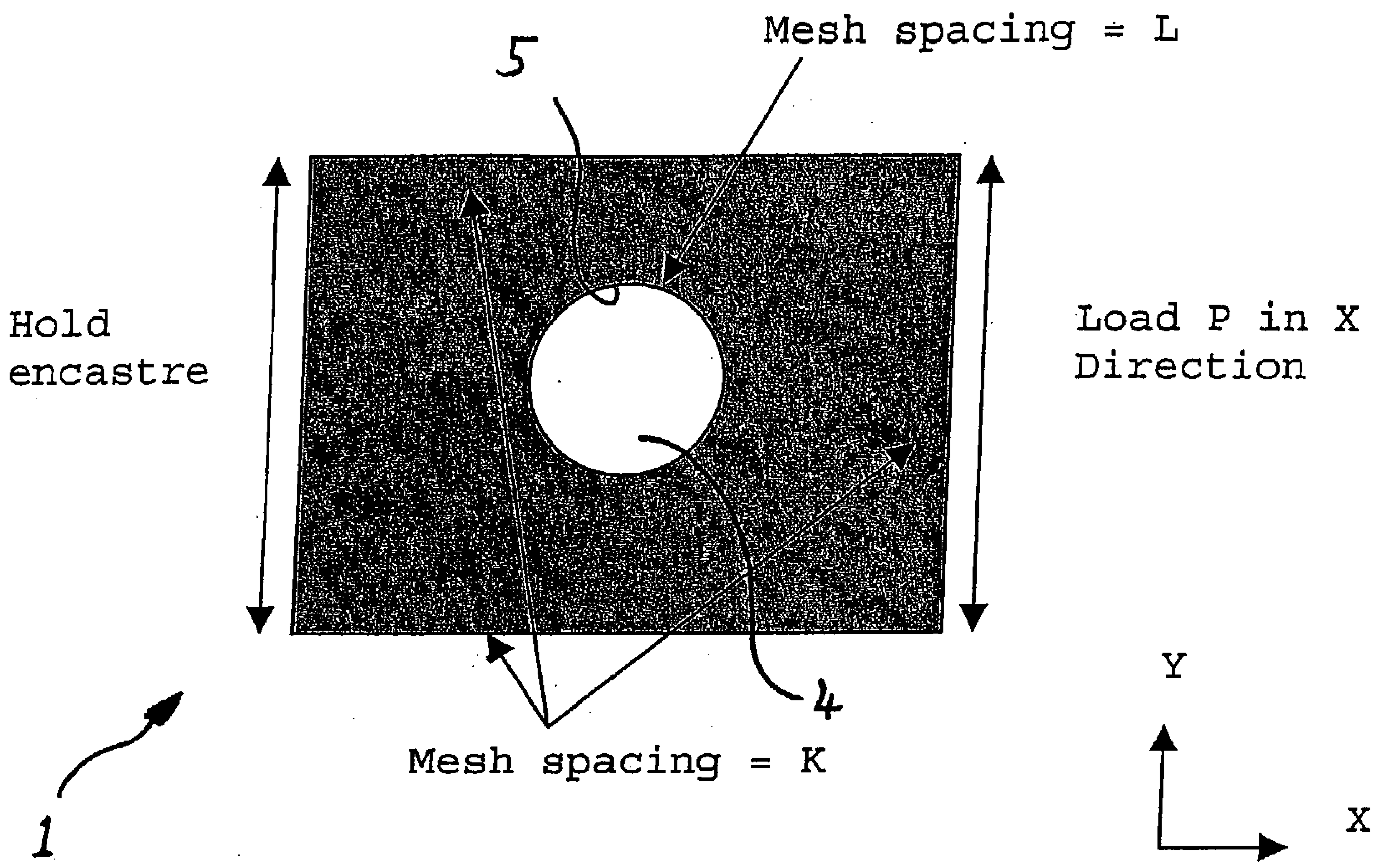

Mesh spacing = L
5
Hold
encastre
Load P in X
Direction
4
Y
Mesh spacing = K
X
1

Figure 6

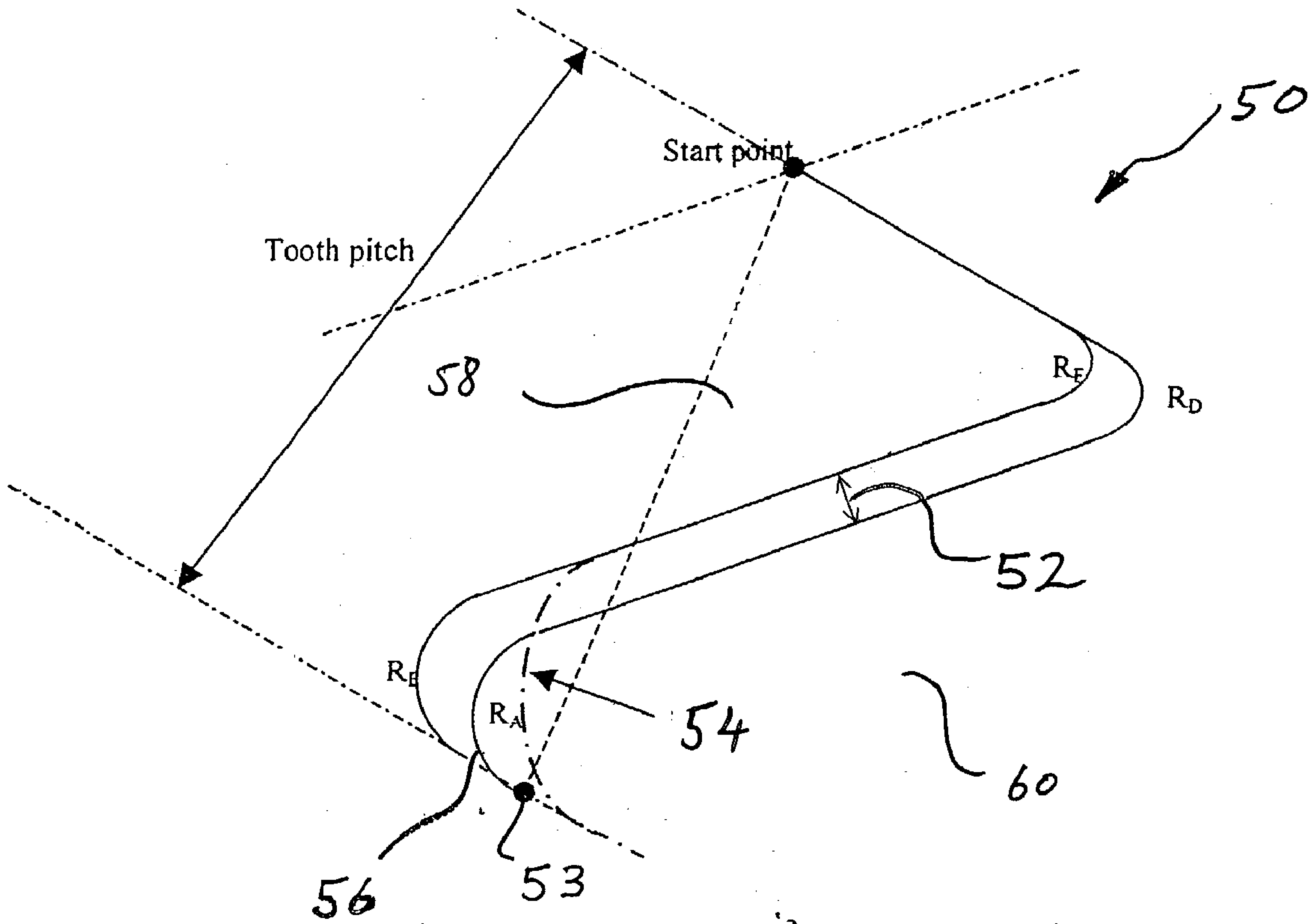



Figure 7

Blade root / Disk head geometry parameters

| Variable | Name | Units | Type |
|---|---|---|---|
| Skew (β) | Root-skew-angle | degree | Variable |
| Nteeth | Number-of-teeth | | Variable |
| Rwa | Root-wedge-angle | degree | Variable |
| Alor(L) | Axial-length-of-root | mm | Variable |
| Snw | Shank-neck-width | mm | Variable |
| Fsw | Fir-tree-shoulder-width | mm | Variable |
| Rcrest | Fir-tree-tooth-crest-radius | mm | Variable |
| Rtrough | Fir-tree-tooth-trough-radius | mm | Variable |
| Bp1, 2. | Blade-tooth-pitch | mm | Variable |
| Btcr | Bottom-tooth-crest-radius | mm | Variable |
| Cpw | Cooling-passage-width | mm | Variable |
| Bglr(R1) | Bucket-groove-lower-radius | mm | Variable |
| Bgur(R2) | Bucket-groove-upper-radius | mm | Variable |
| Dtp1,2. | Disk-tooth-pitch | mm | Variable |
| Fcrest | Disk-tooth-crest-radius | mm | Variable |
| Ftrough | Disk-tooth-trough-radius | mm | Variable |
| Nblades | Number-of-blades | | Parameter |
| Drad | Disk-radius | mm | Parameter |
| Ninc | Number-of-blades-inclusive | | Parameter |
| Rtsn | Radius-to-shank-neck | mm | Parameter |
| Snfr (R) | Shank-neck-fillet-radius | mm | Parameter |
| Tfa (ϕ) | Top-flank-angle | degree | Parameter |
| Ufa (γ) | Under-flank-angle | degree | Parameter |
| Ncfc | Non-contact-face-clearance | mm | Parameter |
| Bac (Ca) | Blade-axial-chord | mm | Parameter |
| Cpa | Cooling passage area | mm*2 | Parameter |
| Fdcr | First-disk-crest-radius | mm | Parameter |
| Inr | Inner-radius | mm | Parameter |
| Dhnw(D) | Disk-head-neck-width | mm | Derived |
| Bga | Bucket-groove-area | mm*2 | Derived |
| Bch (H) | Bottom-to-contact-height | mm | Derived |
| Bl | Bedding-length | mm | Derived |
| Fh | Fir-tree-height | mm | Derived |

Figure 8

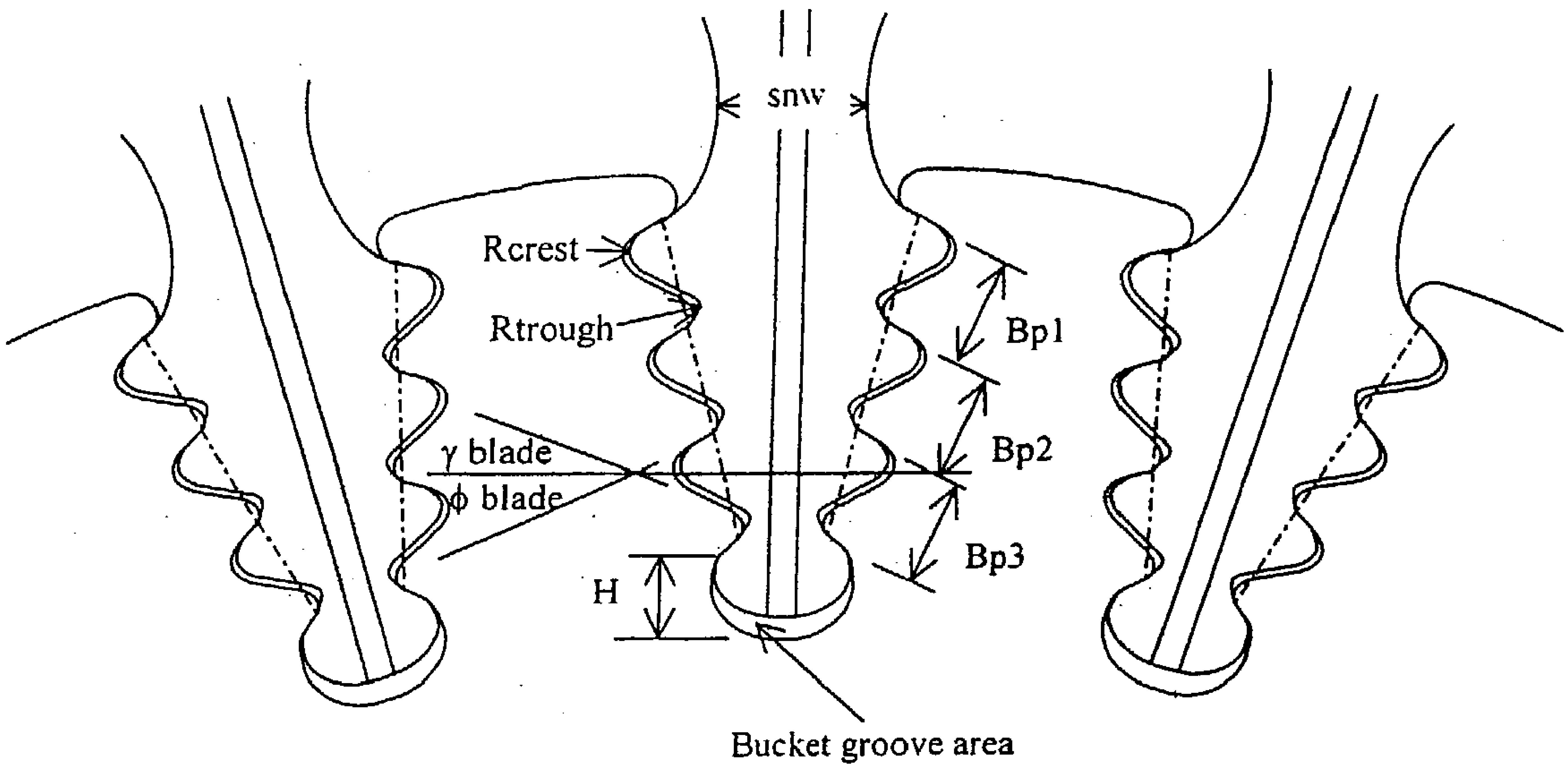



Figure 9a

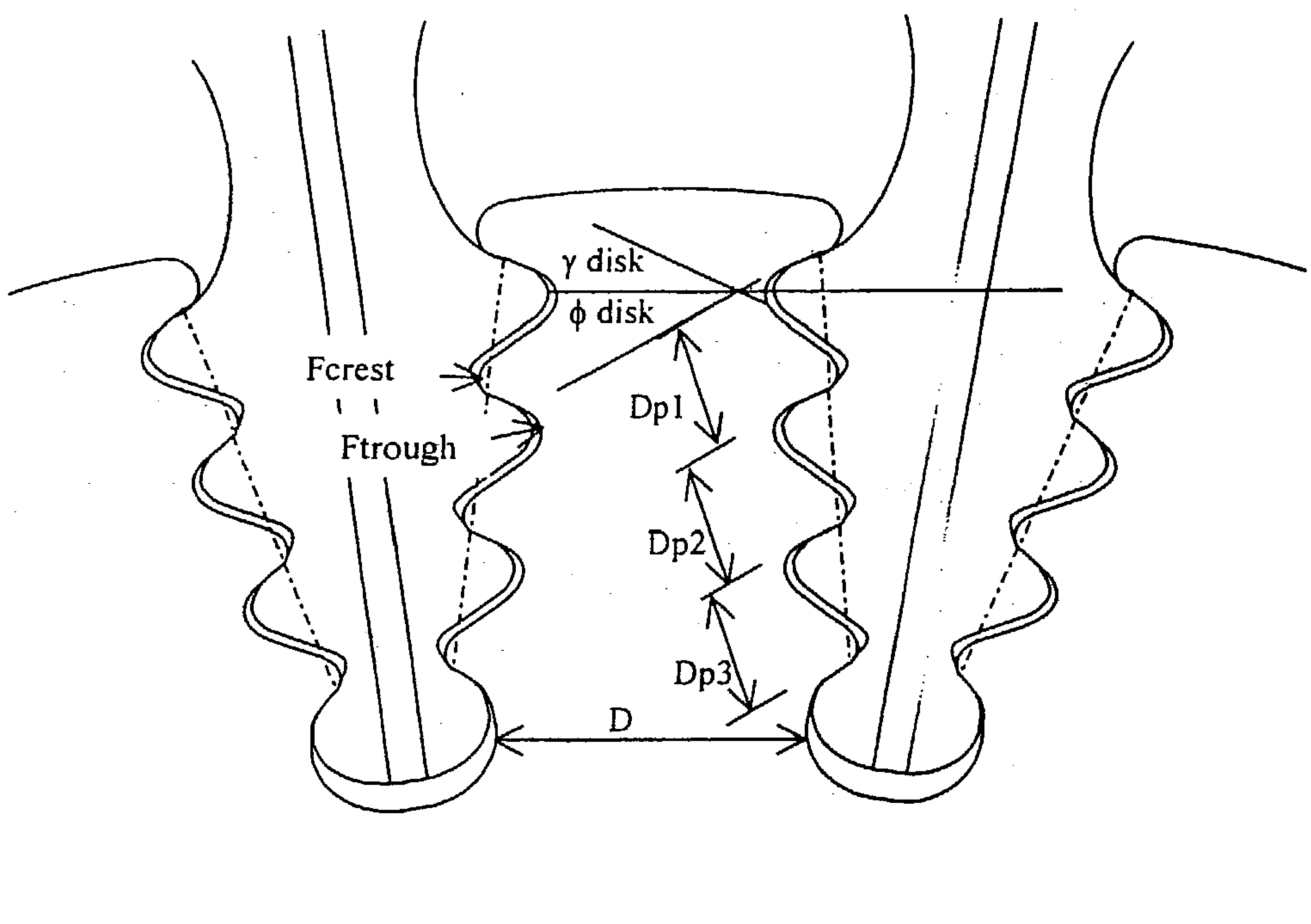



Figure 9b

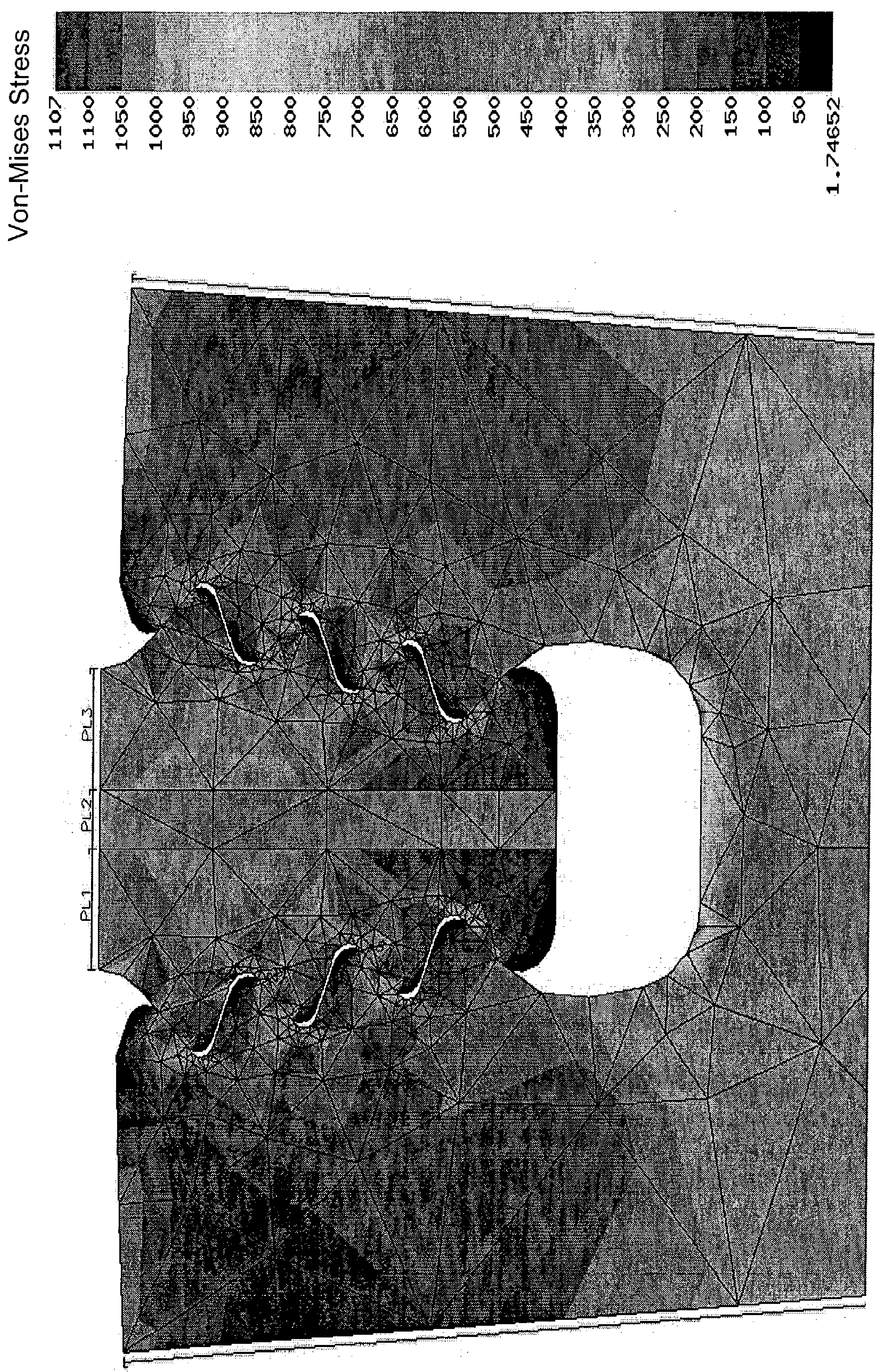

Von-Mises Stress
1107
1100
1050
1000
950
900
850
800
750
700
650
600
550
500
450
400
350
300
250
200
150
100
50
1.74652
PL1
PL2
PL3

Figure 10a

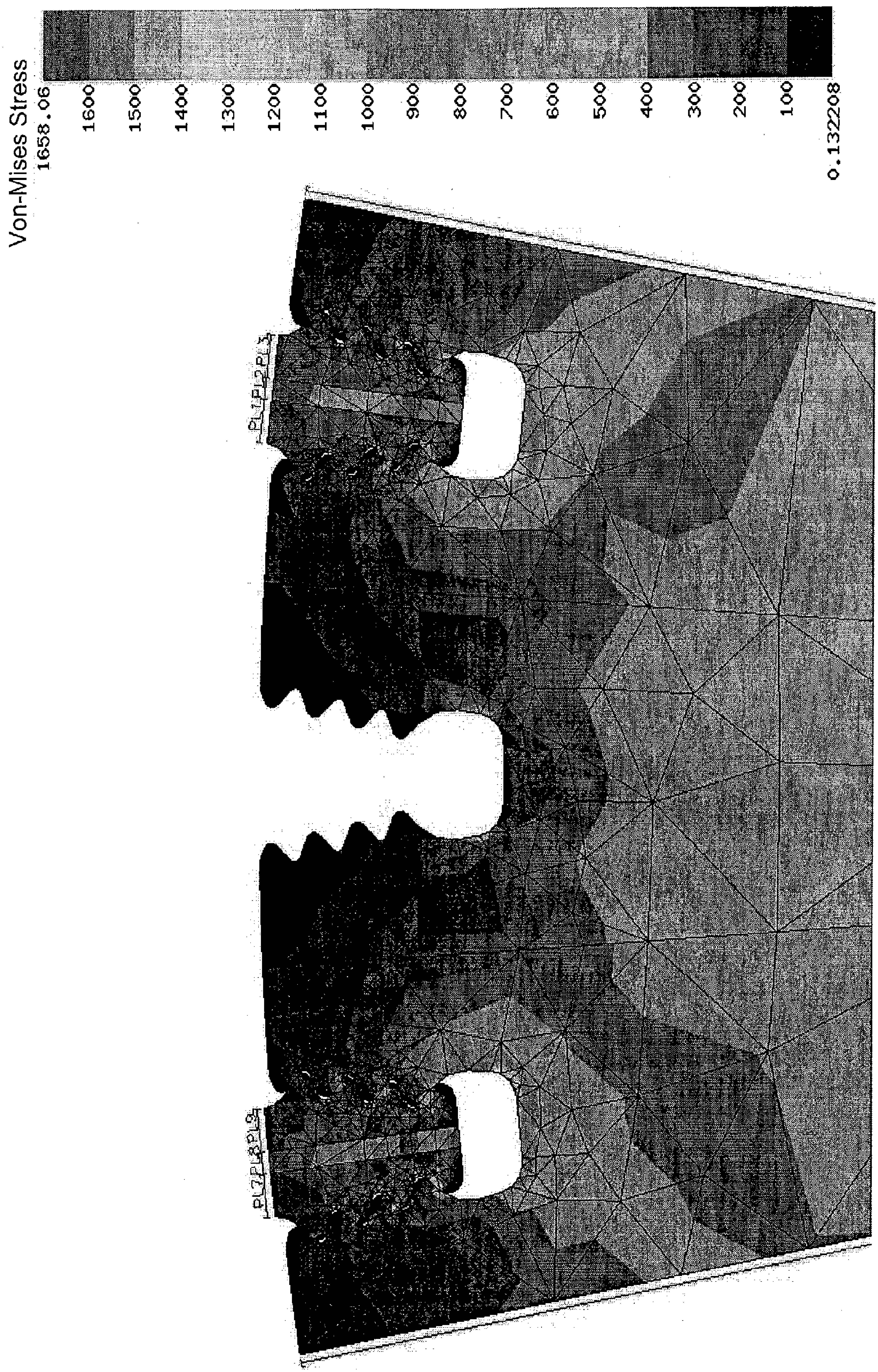

Von-Mises Stress
1658.06
1600
1500
1400
1300
1200
1100
1000
900
800
700
600
500
400
300
200
100
0.132208
PL1PL2PL3
PL7PL8PL9

Figure 10b

| Variables | number of variables | |
|---|---|---|
| | 6 | 14 |
| | *Tooth profile parameters* | |
| root wedge angle (degree) | 20-40 | 20-40 |
| Tooth pitch (mm) | 2.0-4.0 | 2.0-4.0 |
| Blade crest radius (mm) | 0.2-1.0 | 0.2-1.0 |
| Blade trough radius (mm) | 0.2-1.0 | 0.2-1.0 |
| disk crest radius (mm) | 0.2-1.0 | 0.2-1.0 |
| disk trough radius (mm) | 0.2-1.0 | 0.2-1.0 |
| | *Fir-tree root /disk head parameters* | |
| Skew angle (degree) | [15][1] | 10-20 |
| Axial length of root (mm) | [20] | 15-25 |
| shank neck width (mm) | [6.7615] | 6.5-7.5 |
| Firt-ree shoulder width (mm) | [9.8943] | 8-12 |
| bottom tooth crest radius (mm) | [1.0668] | 0.8-1.2 |
| cooling passage width (mm) | [1.3455] | 1.2-1.5 |
| bucket groove lower radius (mm) | [3.5] | 3.0-4.0 |
| bucket groove upper radius (mm) | [2.2] | 1.5-2.5 |

1. [] indicates value when not used in optimisation

Figure 11

| number | Constraints | number of variables | |
|---|---|---|---|
| | | 6 | 14 |
| 1 | R1/R2 | X | X |
| 2 | H/D | X | X |
| 3 | DP/F | X | X |
| 4-5 | min<DP<max | X | X |
| 6 | Lca | X | X |
| 7 | RSA | X | X |
| 8 | minimum serration pitch | X | X |
| 9 | bottom neck width > pitch | X | X |
| 10 | minimum wall thickness | X | X |
| 11 | Bucket groove area> cooling passage area | X | X |
| 12-23 | Notch stresses | | X |
| 24-35 | Section stresses | X | X |
| 36-43 | Crushing stresses | X | X |
| 44-45 | Bucket groove stresses | X | X |
| 46-53 | Unzipping stresses | X | X |
| Objective function | | Maximum notch stress | Fir-tree Frontal area |

Figure 12a

Normalized constraint vector for the base design

| No | Name of constraint | Numeric values | | |
|---|---|---|---|---|
| | | Lower Bound | Value | Upper Bound |
| 1 | Ratio of R1 to R2 [R1/R2] | -1.0 | -0.8642 | - |
| 2 | Ratio of H to D [H/D] | -1.0 | -0.8955 | - |
| 3 | Ratio of R1 to disk trough [R1/Ftrough] | -1.0 | -0.5268 | - |
| 4 | Maximum ratio of tooth pitch to disk trough [DP/Ftrough(max)] | - | 1.4779 | 1.0 |
| 5 | Minimum ratio of tooth pitch to disk trough [DP/Ftrough(min)] | -1.0 | -0.5467 | - |
| 6 | Ratio of axial length to blade axial chord[LCA] | -1.0 | -0.4961 | - |
| 7 | Root Stagger Angle[RSA] | - | 0.7499 | 1.0 |
| 8 | Ratio of Blade/Disk serration pitch[PMIN] | -1.0 | -0.4540 | - |
| 9 | Ration of blade bottom neck width to tooth pitch[BNP] | -1.0 | -1.1474 | - |
| 10 | Minimum wall thickness of bottom blade notch[BNMIN] | -1.0 | -1.3038 | - |
| 11 | Ratio of bucket groove region area to cooling passage area[AR] | -1.0 | -1.0900 | |
| 12-19 | Maximum blade notch stress[NBL(R)(2[1])][2] | -1.0 | 0.9270 | 1.0 |
| 20-23 | Maximum disk notch stress[NDL(R)(3)] | -1.0 | 0.9948 | 1.0 |
| 24-29 | Maximum blade section stress[SB(1)] | -1.0 | 0.6931 | 1.0 |
| 29-35 | Maximum disk section stress[SD(4)] | -1.0 | 0.5623 | 1.0 |
| 36-43 | Maximum crushing stress[CS(1)] | -1.0 | 0.6514 | 1.0 |
| 44-45 | Maximum bucket groove stress | -1.0 | 0.9023 | 1.0 |
| 46-53 | Maximum unzipping stress[UZP(1)] | -1.0 | 0.3688 | 1.0 |

1. The numbers in bracket indicate the no. of the tooth or the section where the maximum stress occurs.
2. For the purpose of compactness, only the maximum stresses are shown in the table.

Figure 12b

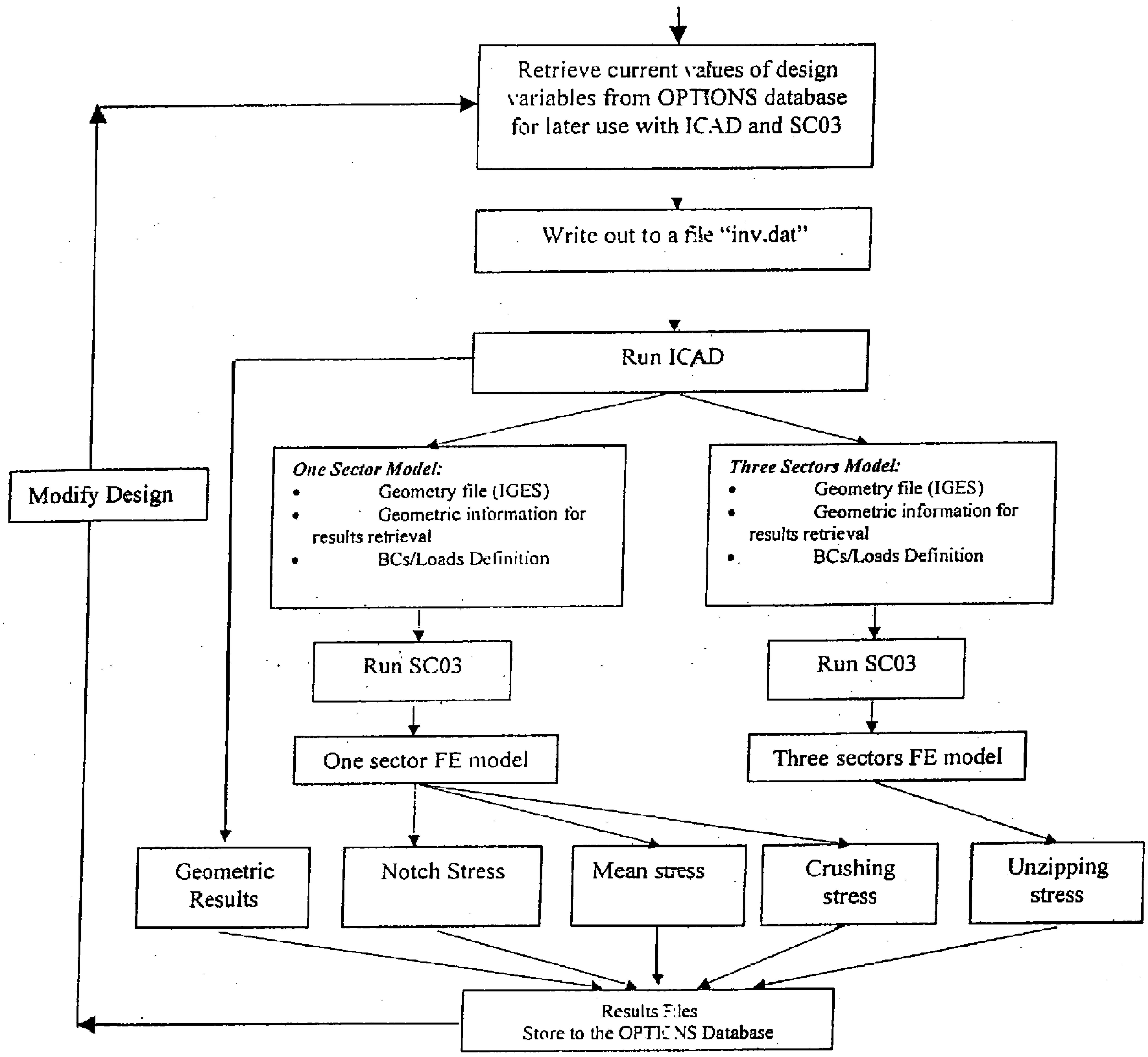
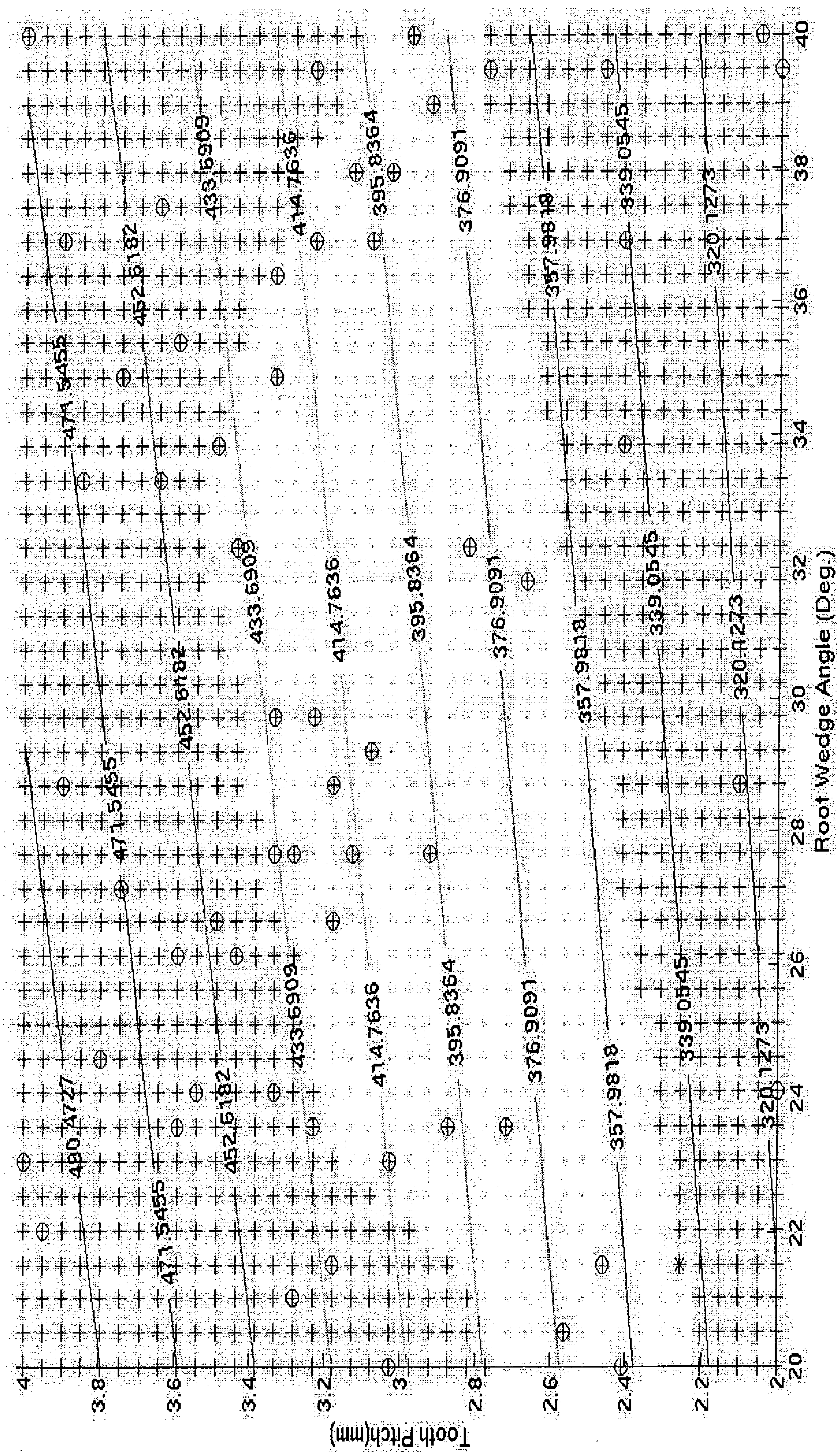

Retrieve current values of design variables from OPTIONS database for later use with ICAD and SC03
Write out to a file "inv.dat"
Run ICAD
Modify Design
One Sector Model:
• Geometry file (IGES)
• Geometric information for results retrieval
• BCs/Loads Definition
Three Sectors Model:
• Geometry file (IGES)
• Geometric information for results retrieval
• BCs/Loads Definition
Run SC03
Run SC03
One sector FE model
Three sectors FE model
Geometric Results
Notch Stress
Mean stress
Crushing stress
Unzipping stress
Results Files
Store to the OPTIONS Database

Figure 13




⊕ Geometry or analysis failure + infeasible points * minimum as shown

Figure 14

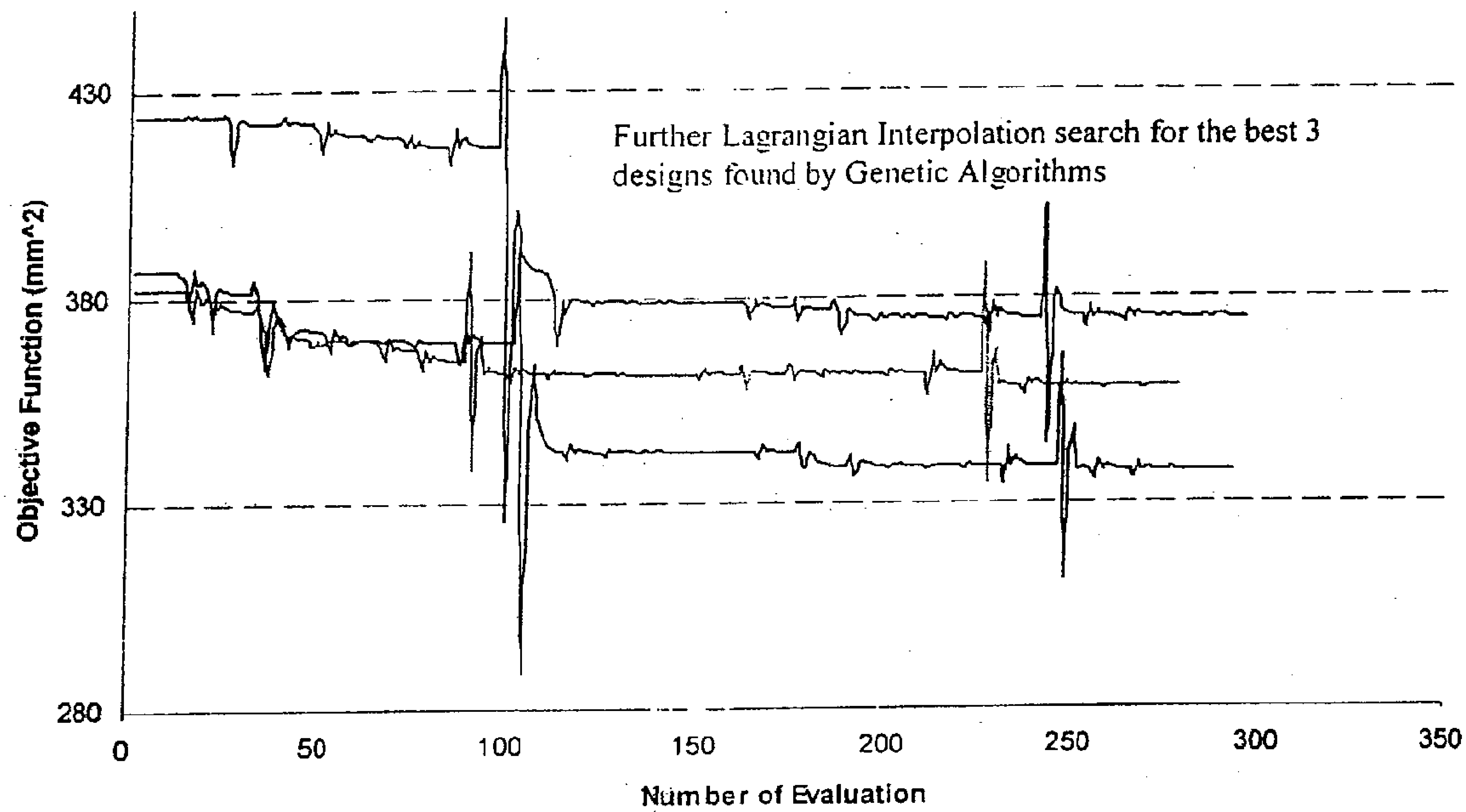

Further Lagrangian Interpolation search for the best 3 designs found by Genetic Algorithms
Objective Function (mm^2)
430
380
330
280
0
50
100
150
200
250
300
350
Number of Evaluation

Figure 15

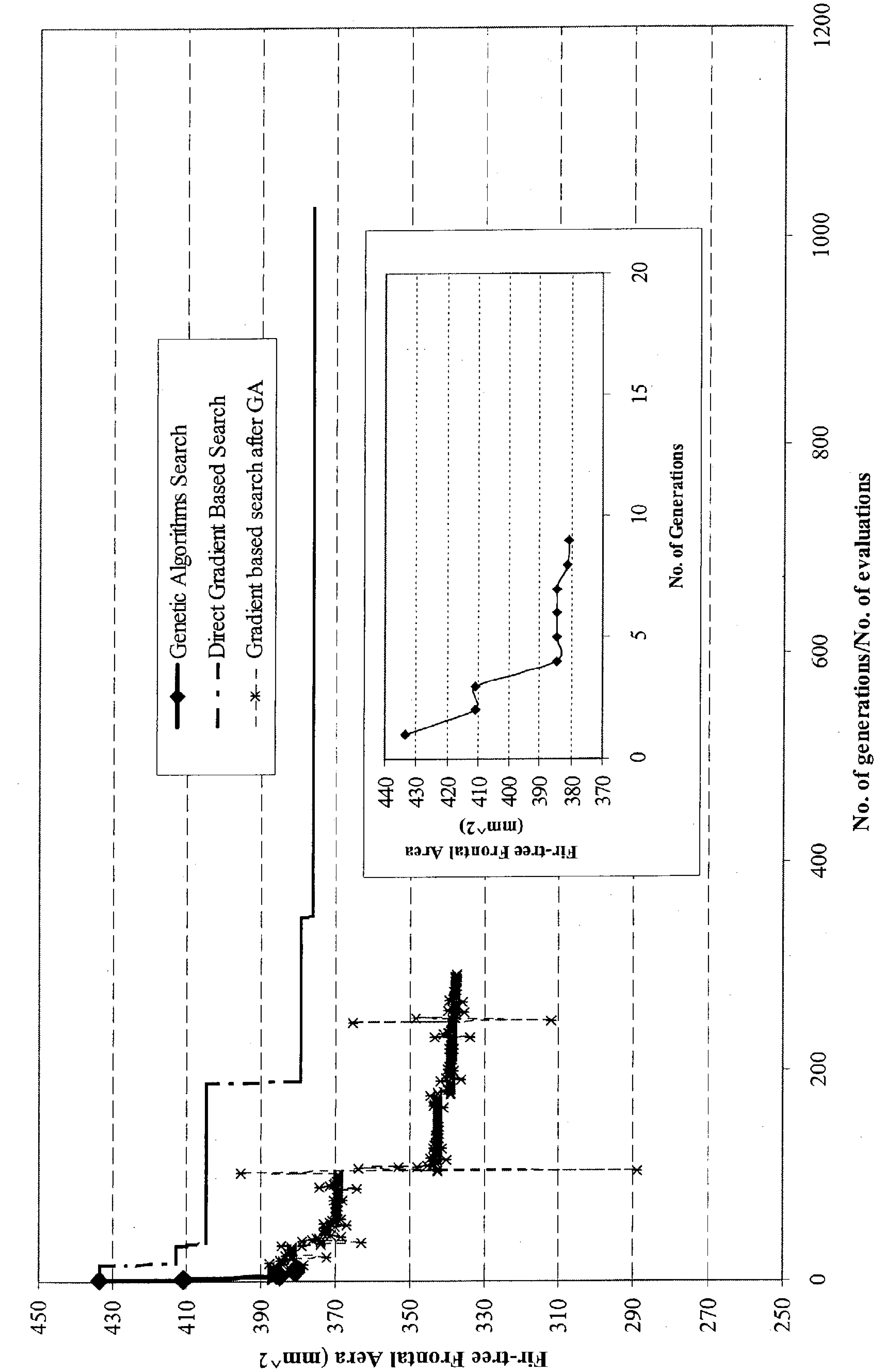

Genetic Algorithms Search
Direct Gradient Based Search
Gradient based search after GA
Fir-tree Frontal Aera (mm^2
450
430
410
390
370
350
330
310
290
270
250
0
200
400
600
800
1000
1200
No. of generations/No. of evaluations
Fir-tree Frontal Area (mm^2)
440
430
420
410
400
390
380
370
0
5
10
15
20
No. of Generations

Figure 16

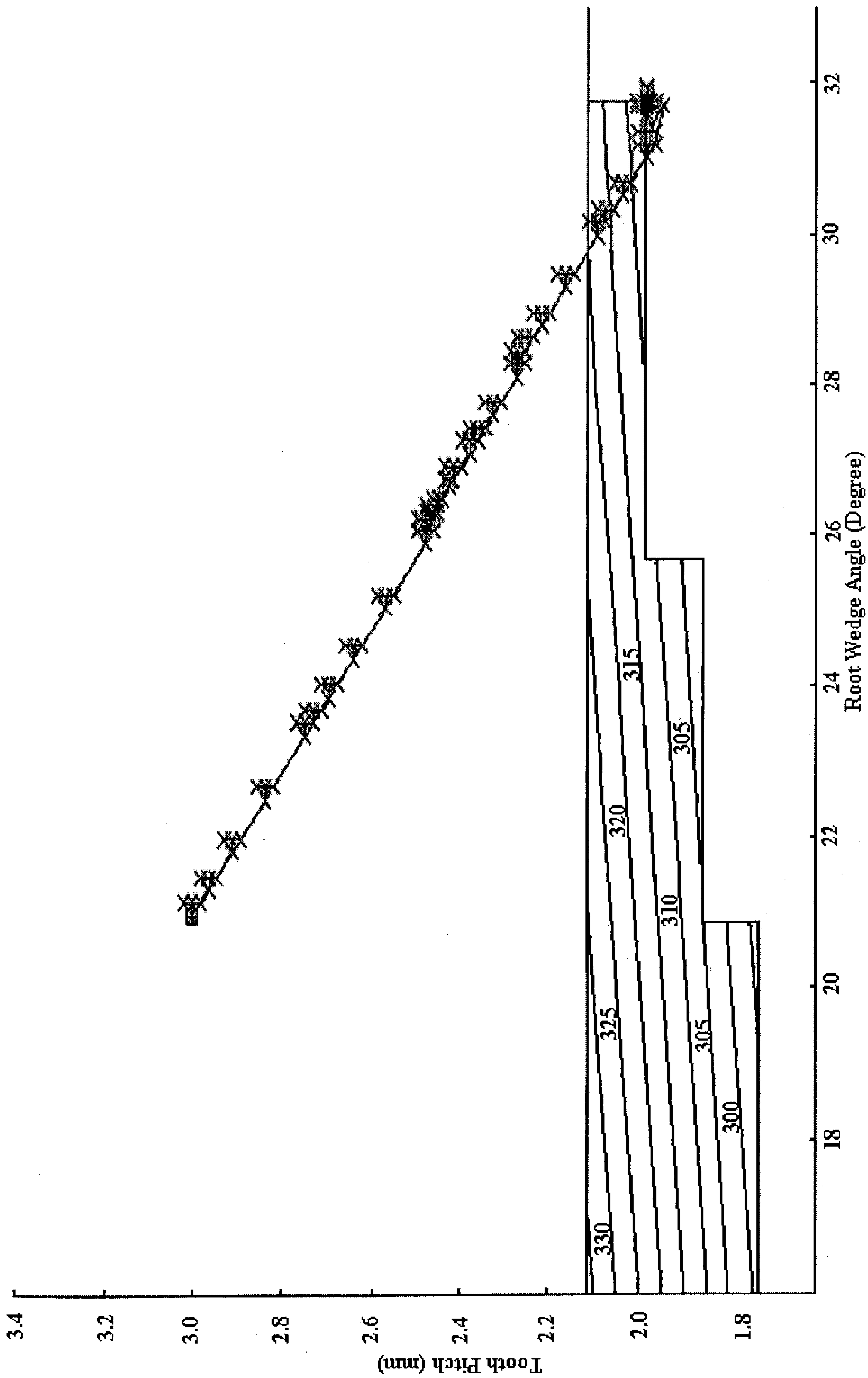

Tooth Pitch (mm)
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1.8
Root Wedge Angle (Degree)
18
20
22
24
26
28
30
32
330
325
320
315
310
305
305
300

Figure 17

Effect on stress distribution of design variables for the base design

| Geometry features | Unzip stress | Notch stress blade | Notch stress disk | Section stress blade | Section stress disk | Crushing stress |
|---|---|---|---|---|---|---|
| Skew angle | X | +[1] | + | + | X | 1+,restX |
| Shank neck width | - | X | 1+;restX | + | X | 1,4-;restX |
| Blade shoulder width | + | - | 2,3+;rest- | 1+;rest- | + | + |
| Bottom tooth crest radius | X | 2+, rest- | - | X | X | 4+;restX |
| Cooling passage width | X | 1,2+;rest- | 1-;2,3+ | + | X | 1,2+;3,4- |
| Bucket groove lower radius | X | X | X | X | X | X |
| Bucket groove upper radius | X | X | X | X | X | X |
| Tooth pitch | + | + | + | X | + | - |
| Blade crest radius | 1-;rest+ | - | + | X | X | + |
| Blade trough radius | - | + | - | X | X | 1,2-;3,4+ |
| Disk crest radius | 1-;rest+ | - | - | + | - | + |
| Disk trough radius | - | X | - | X | X | 4-;rest+ |

1. + indicates increase, - indicates decrease, X means no significant effect when variable increases, number indicates the tooth number in top-down order

Figure 18

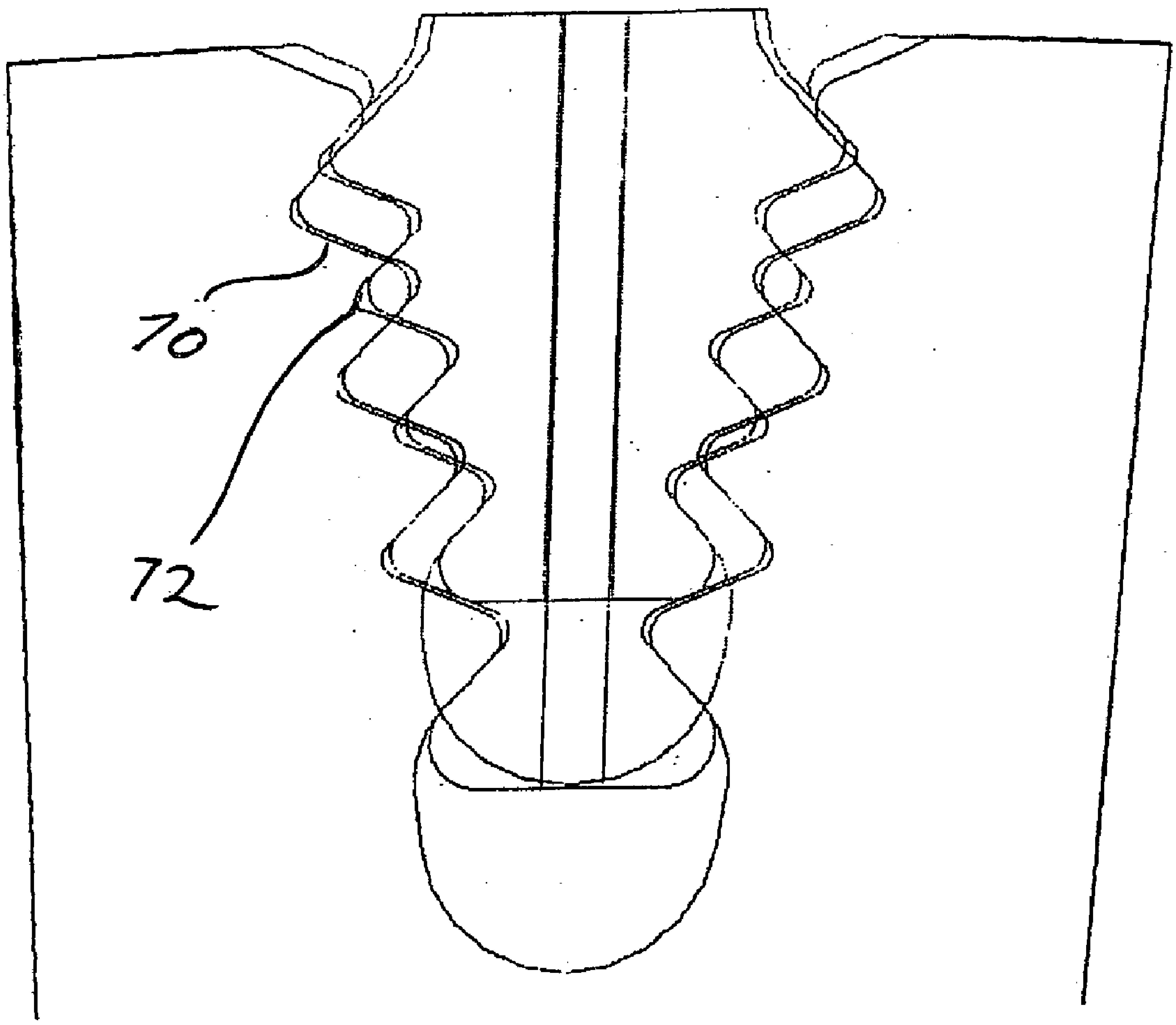



Figure 19

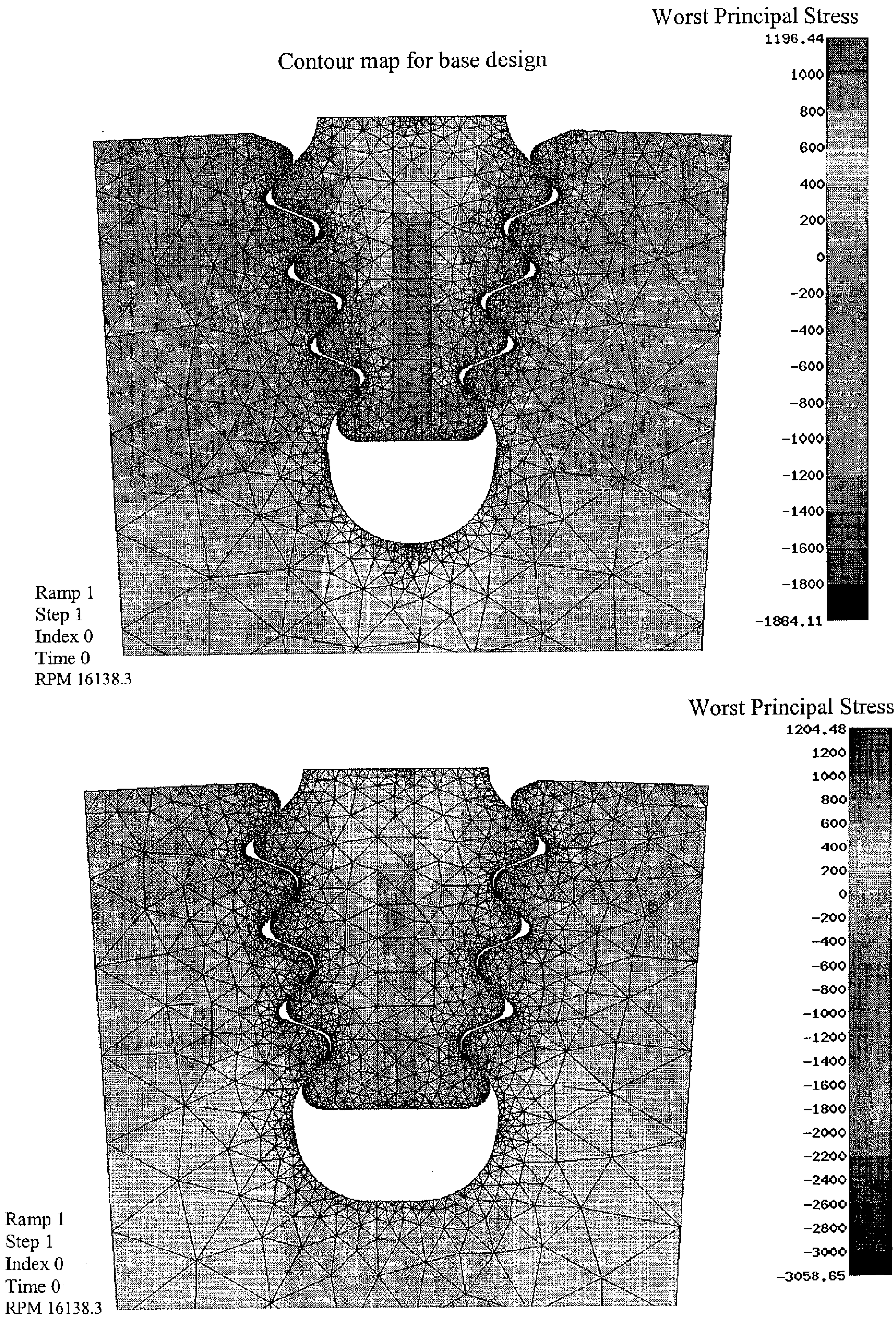

Contour map for base design
Worst Principal Stress
1196.44
1000
800
600
400
200
0
-200
-400
-600
-800
-1000
-1200
-1400
-1600
-1800
-1864.11
Ramp 1
Step 1
Index 0
Time 0
RPM 16138.3
Worst Principal Stress
1204.48
1200
1000
800
600
400
200
0
-200
-400
-600
-800
-1000
-1200
-1400
-1600
-1800
-2000
-2200
-2400
-2600
-2800
-3000
-3058.65
Ramp 1
Step 1
Index 0
Time 0
RPM 16138.3

Figure 20

## OPTIMISATION OF THE DESIGN OF A COMPONENT

[0001] This invention relates to a method of optimising the design of a component. More specifically, although not exclusively, the invention relates to the automation and optimisation of the design of a component using a computer aided design (CAD) system and a computer aided analysis system and a method of transferring data between the two.

[0002] It is well known to use CAD systems when designing a component. It is also common practice to use the CAD model as a basis for models to be analysed by computer to determine the suitability and limits of the design. Such analysis models may be, for example, a computational fluid dynamics model or a thermo-mechanical finite element analysis model. A single CAD representation of the geometry forms the basis of each analysis model. The use of parametric CAD has enabled this geometry to be automatically updated when the value of the dimension of a design entity, for example a fillet radius, is changed. In a parametric CAD tool design variables may be associated with these dimensions to enable the component geometry to be varied, creating a new design variant. For a given set of parameters a particular instance of the geometry may be generated for export to an analysis code. Geometry is typically transferred to the analysis code using a neutral data standard file format, eg IGES, Step, or a custom written interface which provides a link between a specific CAD and analysis package.

[0003] In order to automate this process successfully the geometry model and the analysis model need to be linked such that any change to the geometry is automatically reflected in the analysis model. This process must maintain the associativity between the geometry and the analysis model definition, eg boundary conditions and domain properties. The ability to extract results based on the new geometry needs to be provided to enable the design criteria to be automatically evaluated.

[0004] Known techniques are based on the assumption that the CAD package will export the entities in a consistent order during the translation process. Each geometric entity is assigned an entity number during the translation process, which is used within the analysis code to identify the geometry. This identification number is used to assign boundary conditions and extract results.

[0005] This process breaks down where topology changes occur, for example the addition and deletion of entities, or where the ordering changes. In such cases, the entity number assigned to a geometric entity during translation may not be the same as in a previous iteration. The process, therefore, is not robust where significant shape changes are required.

[0006] For example, one iteration of a design during an optimisation process may have a central hole which is absent in a subsequent iteration. The loads, mesh densities and other domain properties and boundary conditions are stored in sequence alongside the geometric entities of the component. If one of these geometric entities, such as the hole, is removed, the listing of the domain properties and boundary conditions may lose their correct associations with the geometric entities, and the system would fail.

[0007] According to a first aspect of the present invention there is provided a method of optimising a design of a component by conducting analyses on a set of design variants, each analysis comprising the steps of:

[0008] (a) representing the design variant as a CAD model comprising a plurality of geometric entities,

[0009] (b) assigning a tag name to each geometric entity,

[0010] (c) creating a computerised analysis model from the CAD model wherein the tag names remain associated with the respective geometric entities,

[0011] (d) assigning boundary conditions to at least one of the geometric entities in the analysis model by reference to the tag name, and

[0012] (e) determining an output condition of the analysis model in response to the boundary conditions,

[0013] the method further comprising the step of selecting an optimum variant on the basis of the results of the analyses.

[0014] The method may further comprise the step of determining an output condition of at least one of the geometric entities in the analysis model by reference to the tag name. The set of design variants may be generated using a computer algorithm, and this step may be achieved by modifying a dimension of at least one of the plurality of geometric entities, or by adding and/or removing at least one geometric entity.

[0015] The tag name may associate a mesh density with the geometric entity to which it is assigned.

[0016] The computerised analysis model may be a finite element analysis model or a computational fluid dynamics model.

[0017] A model property of at least one of the geometric entities is preferably associated with the tag name of that geometric entity. This model property may be a material property, a temperature or a speed of the geometric entity.

[0018] According to a second aspect of the present invention there is provided a component having a design optimised by conducting analyses on a set of design variants, each analysis comprising the steps of:

[0019] (a) representing the design variant as a CAD model comprising a plurality of geometric entities,

[0020] (b) assigning a tag name to each geometric entity,

[0021] (c) creating a computerised analysis model from the CAD model wherein the tag names remain associated with the respective geometric entities,

[0022] (d) assigning boundary conditions to at least one of the geometric entities in the analysis model by reference to the tag name, and

[0023] (e) determining an output condition of the analysis model in response to the boundary conditions; and

[0024] selecting an optimum variant on the basis of the results of the analyses.

[0025] According to a third aspect of the present invention, there is provided a component manufactured by optimising the design of the component, by conducting analyses on a set of design variants, each analysis comprising the steps of:

[0026] (a) representing the design variant as a CAD model comprising a plurality of geometric entities,

[0027] (b) assigning a tag name to each geometric entity,

[0028] (c) creating a computerised analysis model from the CAD model wherein the tag names remain associated with the respective geometric entities,

[0029] (d) assigning boundary conditions to at least one of the geometric entities in the analysis model by reference to the tag name, and

[0030] (e) determining an output condition of the analysis model in response to the boundary conditions;

[0031] selecting an optimum variant on the basis of the results of the analyses, and manufacturing the component in accordance with the optimised design.

[0032] The component may be a component of a gas turbine engine, and may be a turbine blade having a fir tree root.

[0033] According to a forth aspect of the present invention there is provided a computer program product comprising code for carrying out a method of optimising a design of a component by conducting analyses on a set of design variants, each analysis comprising the steps of:

[0034] (a) representing the design variant as a CAD model comprising a plurality of geometric entities,

[0035] (b) assigning a tag name to each geometric entity,

[0036] (c) creating a computerised analysis model from the CAD model wherein the tag names remain associated with the respective geometric entities,

[0037] (d) assigning boundary conditions to at least one of the geometric entities in the analysis model by reference to the tag name, and

[0038] (e) determining an output condition of the analysis model in response to the boundary conditions,

[0039] the method further comprising the step of selecting an optimum variant on the basis of the results of the analyses.

[0040] According to a fifth aspect of the present invention, there is provided a computer system adapted to carry out a method of optimising a design of a component by conducting analyses on a set of design variants, each analysis comprising the steps of:

[0041] (a) representing the design variant as a CAD model comprising a plurality of geometric entities,

[0042] (b) assigning a tag name to each geometric entity,

[0043] (c) creating a computerised analysis model from the CAD model wherein the tag names remain associated with the respective geometric entities,

[0044] (d) assigning boundary conditions to at least one of the geometric entities in the analysis model by reference to the tag name, and

[0045] (e) determining an output condition of the analysis model in response to the boundary conditions,

[0046] the method further comprising the step of selecting an optimum variant on the basis of the results of the analyses.

[0047] This invention thus provides a novel step within the design automation loop, in which a unique text string in the form of a tag name is assigned to each geometric entity within the CAD tool. This tag name is then used within the analysis code to define the associativity between the analysis model properties (boundary conditions, result locations) and the geometry.

[0048] A component geometry is described in a CAD system by a number of geometric features, eg lines, arcs, NURBS etc whose relationship and dimension are prescribed by the designer. In a parametric CAD tool design variables may be associated with these dimensions to enable the component geometry to be varied. For a given set of these parameters a particular instance of the geometry may be generated for export to an analysis code. This export may utilise a neutral data standard, eg IGES or STEP or a custom written translator, written to link a particular CAD and Analysis package. A number of CAD tools have the ability to assign a unique text string, or tag name, to each geometric entity, eg line, surface, volume. In this invention the analysis code utilises this information to generate the associativity between the geometry and the analysis model properties.

[0049] The analysis code provides the ability to define all the properties of the model by tag name. These include boundary conditions, mesh densities, and domain boundaries and properties (a domain is a user-defined region of a model; the region is represented by a set of surfaces in 2D or a set of volumes in 3D.) Examples of domain properties include speeds, temperatures, material properties and thicknesses (2D models only).

[0050] The model can then be automatically regenerated based on this definition and the new geometry (plus tag names) output from the parametric CAD model. The analysis code also provides the capability to extract the results required, using these tag names to identify the region of interest, for example, peak stress over an entity (line, surface or volume) or average stress along a section constituting the minimum section length between two entities in the form of edges. Introducing this facility enables a robust link to be generated between the parametric CAD model and the analysis model, without the restriction imposed by a constant topology. This enables the design/analysis loop to be run in batch, which is a requirement for geometric shape optimisation.

[0051] For a better understanding of the present invention and to show how it may be carried into effect, reference will now be made by way of example to the accompanying drawings, in which:—

[0052] **FIG. 1** is a general block diagram of an optimisation process;

[0053] **FIG. 2** is a diagram of a component with boundary conditions applied;

[0054] **FIG. 3** is an illustration of the component of **FIG. 2** with design variables shown;

[0055] **FIG. 4** is the component of **FIGS. 2 and 3**, with assigned tag names shown;

[0056] **FIG. 5** is an illustration of the component of FIGS. **2** to **4**, with boundary conditions and main properties applied;

[0057] **FIG. 6** is an illustration of the component with boundary conditions applied;

[0058] **FIG. 7** is an example of a fir tree joint illustrating the associated blade and disc geometry (both partially shown).

[0059] **FIG. 8** is a table of quantities used to describe the geometry of a fir-tree root component;

[0060] **FIG. 9***a* is a simplified cross section view of a fir-tree root component with blade root geometry;

[0061] **FIG. 9***b* is a simplified cross section view of a fir-tree root component with disk head geometry;

[0062] **FIG. 10***a* is a FE stress diagram for a single blade installed in the disk;

[0063] **FIG. 10***b* is a FE stress diagram for three blade sections of a disk with the middle blade removed;

[0064] **FIG. 11** is a table of design parameters of a fir tree root and tooth used in the optimisation process of a fir tree joint component;

[0065] **FIG. 12***a* is a table of design constraints used in the optimisation of a fir tree joint component;

[0066] **FIG. 12***b* is a table of geometric and mechanical constraints and normalised values;

[0067] **FIG. 13** is a diagram of the optimisation program structure;

[0068] **FIG. 14** is a contour map of Fir tree frontal area for root-wedge-angle and tooth-pitch based on Genetic Algorithm results;

[0069] **FIG. 15** is a graph of results obtained from a gradient based search for use in the optimisation process;

[0070] **FIG. 16** is a graph of results obtained from a direct gradient based search for use in the optimisation process;

[0071] **FIG. 17** is a graph of results obtained from the Hooke and Jeeves gradient based search for use in the optimisation process;

[0072] **FIG. 18** is a table illustrating which stress distributions are affected by various design variables;

[0073] **FIG. 19** is a comparison between the original geometry and the optimal geometry resulting from the genetic algorithm search results; and

[0074] **FIG. 20** is a FE stress diagram for two profiles after being optimised towards different goals.

[0075] The overall architecture of the design optimisation process is illustrated in **FIG. 1**. In this structure, ICAD (Intelligent Computer Aided Design) knowledge server **100** is used to generate the model definition based on rules that may be stored in a knowledge database **102**. The model is defined in a descriptive form using the ICAD design language, which is a derivation of, and extension to Common LISP, designed for geometric modelling. This model is used to produce geometry and related information. The geometry is then passed to the analysis code **104** along with any geometry dependent properties to evaluate the design performance. Tag names are assigned to each geometric entity as described below:

[0076] With reference to **FIG. 2, a** two dimensional component is shown generally at **1**, the component being attached to a surface **3** along its left edge **5** as viewed in **FIG. 2**. The component also has a load applied to its right edge and a circular hole **4** provided in its centre defined by interior circular edge **5**.

[0077] As a first step in a design process with the objective of optimising the design of the component **1**, an initial design variant of the component is created in a CAD system by entering values for each design variable (Position y, Position x, Width, Radius, Length) as shown in **FIG. 3**. Geometric features of the component, such as the inner circular surface **5** and loaded edge **8** are assigned tag names, as indicated in **FIG. 4**.

[0078] Data relating to the geometry and the associated tag names is then exported to the preferred analysis code. This is done either by transferring the data in a neutral format, or via a translator used to convert the CAD data into a format recognised by the analysis software.

[0079] The analysis model defined by the analysis code is then used to simulate the behaviour of the component. This is done by applying boundary conditions and domain data to the model. As shown in **FIG. 5**, domain data such as the type of material, the thickness of the material and the temperature of the material is entered at this stage for use in the analysis. In **FIG. 6**, boundary conditions are also applied to the analysis model. Each domain property and boundary condition is associated with the relevant part or parts of the component via the tag names. For example, the mesh spacing of K would be applied to EDGE1 and EDGE2 and LOADEDGE whereas different mesh spacing L is applied to the edge **5** surrounding the hole **4**. Information such as "load P in x direction applied to LOADEDGE", and "material X applied to domain PLATESURFACE" is also added. In accordance with the present invention, the mesh densities, for example, are assigned to each tag name representing each geometric entity. Therefore, the removal of one entity and hence tag name does not confuse the system.

[0080] If the CAD system used does not provide the capability to export tagged data, a new facility in the analysis code which automatically tags the geometry on import may be used. These tags would then be used throughout the analysis for each corresponding entity. The tags would be used as a reference when, for example, the mesh density or a model property such as temperature is applied to the geometric entities.

[0081] The use of tag names creates the possibility of automating the results extraction during analysis. A new facility in the analysis code enables the user to define the output results locations using the tag names. For example, the system could be asked to extract the peak worst principal stress which occurs on the entity "RESULTSEDGE".

[0082] As an example, if the user wished to modify the hole radius to Z and output the new peak stress on the edge **5** of the hole **4**, the following steps would be run:

[**0083**] (1) open the CAD tool **100**, open the CAD model, set the design variable radius to Z and export the geometry and tag data;

[**0084**] (2) open the analysis tool **104**, import the geometry and tags (or run automatic tagging if necessary) and import the model definition;

[**0085**] (3) run the analysis and extract the peak stress on the tagged entity "RESULTSEDGE".

[**0086**] This process may be controlled via a batch script without user intervention. This enables the whole process to be linked to an optimiser to identify the best set of design variable values to meet a set of design limits, for example to achieve minimum component mass while meeting the requirement that the peak stress at the edge **5** of the hole **4** is less than a predetermined value established for the material of the component.

[**0087**] As a detailed example, an optimisation process incorporating the present invention will now be described with reference to the optimisation of a fir tree root component as used in the turbine engines to attach a blade to a turbine disk.

[**0088**] Here, the design of the fir-tree geometry is carried out using ICAD. The basic procedure of the geometry design falls into two steps: first identification of the features and rules used to define the geometry and secondly the breaking down of the whole model into several modules, each of which becomes a building block in a hierarchical structure. In ICAD, each of these basic blocks is described using the ICAD design language (IDL) as a generic definition which can be implemented in the ICAD browser using a specific set of parameter values. Thus the model is defined parametrically: different sets of parameter values will result in different designs from the same template. In addition, multi-modality and backward compatibility can be achieved by incorporating different behaviours into one model with a single interface while only the internal implementation is modified.

[**0089**] A single basic tooth geometry **50** is illustrated in **FIG. 7**, which is defined in such a way as to allow the designer to explicitly control the non-contact clearance **52** and to avoid duplicate entities in the model. This latter feature eases the application of boundary conditions and loads during analysis.

[**0090**] The acceptability of any fir-tree geometry needs to be checked since some particular combination of parameters may result in unacceptable features such as intersections between entities or the collapse of very short entities. The handling of unacceptable features is important to the optimisation process as well as to the analysis code. Using ICAD, geometry features can be checked within the modelling process, as part of the whole model and appropriate actions can then be taken using preset default values, while signalling which parameter is causing the problem. Taking the modelling of the base tooth **50** as an example, in every step the modelling process is checked to make sure an acceptable geometry can be produced, otherwise, a geometry failure is signalled to the optimiser **106** to cancel the analysis. An example of unacceptable base tooth geometry, in which two circular arcs **54**, **56** are intersected, is also illustrated in **FIG. 7**. In this condition, the radius RE has been increased to a value which results in the profile failing to meet the end point **53** of the section shown.

[**0091**] Here, the blade root **58** and disk head **60** geometry are defined in the same way as the basic tooth, with further parameters and rules being needed. Details are covered in the following sections. Some of the quantities used in the definition are not expected to change and are thus held constant during an optimisation run and are referred to as design parameters, while others which are identified as being more influential to the design will be varied by the optimiser in the optimisation loop and are referred to as design variables.

[**0092**] The first fir-tree geometry **50** shown is a simplified version of an existing fir-tree model, which is composed of straight lines and circular arcs only. The complete geometry is described by approximately 30 quantities, as shown in **FIG. 8**. The resultant blade/disk geometry is illustrated in **FIGS. 9***a* and **9***b*. Some of these quantities are design variables and are identified as playing an important role in the stress distribution and thus will be optimised against known constraints. Others are design parameters which will be kept at constant values based on previous experience.

[**0093**] Every entity within the ICAD model can have additional non-geometric properties which will ease the use of the geometry in other applications such as analysis and manufacture. This object-oriented feature enables various information related to a product design to be integrated into a single model. For example, to apply boundary conditions and loads to entities during the analysis stage, it is desirable to name the entities with unique tag names which can then be referenced later. Using tag names on each entity in the geometry enables the boundary conditions, load properties and mesh parameters to be specified in batch mode.

[**0094**] Geometric quantities such as the minimum thickness of the blade root, the distance between the centre of the contact face of the tooth on each side, etc, are calculated in the ICAD model based on a mathematical representation of the geometry. Some of these are treated as constraints in the optimisation problem and some are used to retrieve analysis results. For example, point coordinates are normally required to get the stress values at those points. Alternatively, if a tagged geometric entity is specified the worst principal stress at that entity may be found.

[**0095**] The fir-tree joint used to hold a blade in place in a turbine structure is usually identified as a critical component which is subject to high mechanical loads. Most often the attachment is a multi-lobe construction used to transfer loads from blade to disk. It is generally assumed that there are two forms of loading which act on the blade, the primary radial centrifugal tensile load resulting from the rotation of the disk, and bending of the blade as a cantilever which is produced by the action of the gas pressure on the airfoil and forces due to tilting of the airfoil. The resulting stress distribution in the root attachment area is a function of geometry, material and loading conditions (which are of course related to the speed of rotation). It is known that some critical geometry features exist for the stress distribution in the blade disk interface.

[**0096**] Many studies into the stress state of the blade root attachment have been reported, originally using photo-elastic methods, now mainly using finite element analysis.

Modern finite element codes already have the capability of dealing with thermal-mechanical coupling and contact analysis between blade root and disk head. It is now relatively easy to obtain the stress distribution in the attachment area using commercial FE codes. Also many in-house FE codes exist to handle corporate-specific problems (these have some advantages over commercial tools among which the most notable is complete control over the source code). Although there are many kinds of code available, the general procedure of finite element analysis is almost always as follows:

[0097] (1) Create the geometry, or import the geometry from another CAD system;

[0098] (2) Apply the boundary conditions and loads;

[0099] (3) Mesh the geometry;

[0100] (4) Solve the problem and retrieve the results.

[0101] Most FE codes support batch running of the analysis and this allows the analysis to be embedded into the overall optimisation loop. Smooth coupling of the modelling process and analysis, however, is not an easy task. It involves the transfer of the geometry itself and related geometry dependent properties to the analysis code, in this case, the finite element software **104**. Using unique tag names for each entity allows the correct geometry dependent properties to be associated with the respective geometric entity, even if the number of entities is changed or a value of an entity changes.

[0102] The loading on the root is mainly due to centrifugal load which is dependent on the mass of the whole blade. The design of the fir-tree root involves an iterative process of controlling the blade mass, which incorporates the root mass. Also some key features, such as the fillet radius, play very important roles in the stress distribution in notch regions. Thus a set of competitive constraints ranging from geometrical, mechanical, cooling requirements, etc, is established for use in exploration of various design candidates for the fir-tree root. Finite element analysis is then utilized to obtain the resulting stress distributions. This further complicates the situation. A traditional manual method is now too slow for this process and thus automation is required. Four types of constraints are used to check the design:

[0103] Crushing stress describes the direct tensile stress on the teeth: bedding width is the main factor affecting the stress.

[0104] Unzipping can occur after a blade release: the disk post on either side of the released blade are then subject to high tensile and bending stresses. The disk post must be able to withstand these stresses in order to avoid a progressive 'unzipping and release' of all the blades

[0105] Disk neck creep: the disk posts are subject to direct tensile stress which causes material creep. Too much creep, combined with low cycle fatigue, can dramatically reduce the component life.

[0106] Peak stresses: peak stresses occur at the inner fillet radii of both the blade and the disk. If the fillet radii are too small and produce unacceptable peak stresses, some bedding width has to be sacrificed to make them bigger.

[0107] Apart from the above constraints, which are used to check the candidate designs, some others are used to check the optimised result. These include vibration limits, neck stress, etc From a preliminary blade number optimisation, these criteria are not deemed a significant constraint here.

[0108] As the fir-tree geometry is constant along the root centre line, it is possible to think of the stresses as two dimensional. However, the loading applied along the root centre line is not uniform, so strictly speaking, the distribution of stresses will be three dimensional. Nonetheless, it is still possible to assume that each section behaves essentially as a two dimensional problem with different loadings applied to it. The difference of loading on each section is affected by the existence of skew angle which will increase the peak stresses in the obtuse corners of the blade root and the acute corners of the disk head. From previous root analysis research, it is feasible and convenient to use a factor to estimate the peak stresses at each notch of the blade and disk, and this factor takes different values for different teeth.

[0109] Also, it is known from previous work using photoelastic and finite element methods, that the distribution of centrifugal load between the teeth is very non-uniform and the top tooth may take a significant portion of the load. This feature allows the possibility of using different tooth sizes. The system implemented here also allows designers to explore the effect of varying the number of teeth, but this may cause difficulties when gradient-based methods are used for optimisation.

[0110] Both a one sector model and a three sector model are considered when estimating the mechanical constraints, the one sector model for the estimation of maximum notch point stresses, crushing stresses and blade/disk neck mean stresses and the three sector model for the estimation of unzipping stresses. Typical FE results are illustrated in **FIG. 10***a* and **FIG. 10***b*, for the one sector model and three sector model, respectively. In general, finite element analysis is computationally expensive, thus a compromise between accuracy and computation cost should always be made to obtain acceptable results as quickly as possible when this is embedded in an optimisation run. This compromise is made by an appropriate choice of mesh density.

[0111] It will be appreciated that the condition shown in **FIG. 10***b* represents a secondary design variant, in which one blade is entirely missing from the analysis model. This enables an evaluation to be made, during the optimisation process, of an altered geometry created by a rule-based process.

[0112] The whole process from the importing of geometry, application of boundary conditions and loading, to results retrieval is implemented here as a SC03 Piugin, which is a facility provided by the Rolls-Royce in-house FEA code SC03 to extend the capability of its core functionality. A command file is used by SC03 to carry out jobs ranging from importing geometry from IGES files, applying boundary conditions and loads, to retrieving stress results.

[0113] Two different optimisation problems were tackled using population based genetic algorithms (GAs) and gradient-based methods. One was to minimize the area outside of the last continuous radius of the turbine disk, which is proportional to the rim load by virtue of the constant axial length. This quantity is referred to as the fir-tree frontal area in the following sections. The number of teeth is treated as a design variable in this problem and the number of constraints is dependent on the number of teeth. The other was to find the optimum tooth profile to minimize the maximum notch stress. The design variables (see **FIG. 11**) and constraints (see **FIG. 12***a*) used in the second problem are a subset of those defined in the first problem (which has **14** design variables and up to **53** constraints for a three-tooth design) although the goals are different.

[**0114**] The constraints are divided into two categories, geometric and mechanical, which are summarized in **FIG. 12***b* along with the normalized values for the initial design. For the meaning of symbols used in this section, please see **FIGS. 9***a* and **9***b*. The normalization adopted here is described as follows for upper and lower limits u and l, respectively:

[**0115**] a. constraints with upper bounds only:

$$y_{norm} = \begin{cases} y/u, (u \neq 0) \\ y(u = 0) \end{cases};$$

[**0116**] b. constraints with lower bounds only:

$$y_{norm} = \begin{cases} -l/y, (y \neq 0, l > 0) \\ -y/l, (l/0 \text{ or } y = 0); \\ y(l = 0) \end{cases}$$

[**0117**] c. constraints with both upper and lower bounds:

$$y_{norm} = 2\left(\frac{y-l}{u-l}\right) - 1.$$

[**0118**] These different formula make it possible for all normalized constraints to have consistent behaviour when the design is moving from an infeasible region towards feasibility, and to have the values of −1 or +1 at the boundary of the constraints.

[**0119**] It is necessary to establish the appropriate values for mesh control parameters. The purpose is to find a compromise between the high computational costs that are incurred for very fine meshes and the accuracy required to capture the maximum stresses in the notch area. Therefore, the local and global effects of mesh density must be studied.

[**0120**] Following the set up of the system, a series of systematic evaluations is carried out to establish appropriate mesh density parameter values and to gain experience on the effects of design variables changes. From varying the mesh density control parameters while holding all others constant, for example the global and local edge node spacing, it is found that reducing the notch edge node spacing increases the mesh density and therefore reduces the perturbations in maximum notch stress. In this example the use of 0.001 mm for both global and local edge node spacing has been chosen as a suitable value. The effect of different geometric features on the stress distribution within the structure are summarized in **FIG. 18**.

[**0121**] From parameter study results it can be seen that the notch stress on the second tooth takes the largest value, as already implied from previous work: this aspect makes it desirable to design each tooth using different values of tooth profile parameters.

[**0122**] With reference to **FIG. 13**, the optimisation is performed in this preferred embodiment using the OPTIONS software package which provides designers with a flexible structure for incorporating problem specific code as well as more than forty optimisation algorithms. The critical parameters to be optimised, or design variables, are stored in a design database, which also includes the objective, constraints and limits. The design variables are transferred to ICAD by means of a property list file which contains a series of pairs with alternating names and values. This file is updated during the process of optimisation and reflects the current configuration, constituting a principal design variant. The geometry file produced by ICAD, containing the geometry definition and tagnames, is then passed to the FE code SC03. The model creation, running and results extraction is executed by a command file. The analysis results are written out to another file, which is read in by the optimisation code. The design variables are then modified according to the optimisation strategy in use until convergence or a specified number of loops has been executed. In this way the optimisation is an iterative process. A design variable is changed, the effect analysed and the process repeated. The program structure is illustrated in **FIG. 13**.

[**0123**] In the process illustrated in **FIG. 13**, the "One Sector Model" and "Three Sectors Model" constitute principal design variants. Using a rule-based engine, secondary design variants can be modelled for analysis by applying mathematical operations on the principal design variant at each iteration, in order to simulate for example, a damaged blade, a missing blade (as in **FIG. 10***b*), or a geometry at the design tolerance limits.

[**0124**] Owing to the presence of a discrete design variable (the number of teeth), most gradient based optimisation techniques will not work directly here. Therefore a two-stage strategy of combining a Genetic Algorithm (GA) with gradient search may be used in this problem. A typical GA is first employed in an attempt to give a fairly even coverage on the search space, and then gradient based search methods are applied on promising individuals with the number of teeth fixed. One of the considerations here is that generic algorithms are capable of dealing with discrete design variables. Another consideration is that as the GA proceeds, the population tends to saturate with designs close to all the likely optima including sub-optimal and globally optimum designs, while gradient based methods are more suited to locating the exact position of individual optimum given suitable starting points. Here the GA is used to give good starting points for the gradient search methods.

[**0125**] In this example an initial analysis on the base design reveals that several geometric and mechanical constraints are violated. These include geometric constraints **4**, **9**, **10** and **11** (see **FIG. 12***a*), and the disk notch stress constraints. **FIG. 12***b* shows the resulting normalized constraint values for the base design. Note that this means that the GA must first locate feasible designs before it can begin optimisation.

[**0126**] Genetic algorithms often require large number of evaluations of the objective function and constraints. The computational cost involved can soon become prohibitively high when computationally intensive finite element analysis is used to calculate the stresses in the structure. In this problem each evaluation takes about 5-6 minutes to finish, and most of this time is engaged in finite element analysis. This means that it takes about 80 hours to finish a 10 generation GA search with a population size of 100 using serial processing. (Note that for some specific sets of parameters, there is no viable geometry that can be constructed, and when this occurs SC03 is simply signalled to cancel the analysis).

[**0127**] Because of the large number of design variables, the optimisation trace may only be plotted on contour maps of two variables if these maps are produced while holding all the other variables constant. Furthermore, if only a small number of quantities are chosen as design variables, there may be no feasible designs at all. For an infeasible starting design, it is easier for the optimiser to find a feasible region if a large number of quantities are left as design variables and broad exploratory searches are used.

[**0128**] A contour map for two design variables has been generated using results from the GA search (**FIG. 14**). Infeasible geometries and possible analysis failures are also illustrated in the Figure. It is noted that identification of this type of failure is useful for identifying any problems with the implementation of the system (sometimes calculations fail simply because of delays due to overloading of the network), but this is not a concern for optimisation as long as appropriate measures are employed to avoid misleading the optimiser. This is important especially when approximations such as response surfaces are introduced to improve run speeds.

[**0129**] A gradient based search is illustrated in **FIG. 15**. It can be seen that better starting points do not always converge to better results, depending on the location in the design space. This justifies the use of the whole final population of the GA results instead of just the best one as starting points for gradient search. A comparison between this simple two-stage strategy and a direct gradient-based search is provided in **FIG. 16**, which shows that this two-stage strategy, although simple, works better than a direct gradient search as the initial GA search offers a better chance of steering the optimiser towards global optima, while a direct gradient-based search will more likely get stuck in a local optimum.

[**0130**] Several steepest descent search methods have been applied to the problem after the initial GA search: these include the Hooke and Jeeves direct search method plus various other methods discussed in Schwefel's book. The first method is very fast when the number of design variables is small, as shown in **FIG. 17**, in which only 6 variables are chosen as design variables (the full scale problem contains 14 design variables). Although the complexity of this problem is only modest, the computational cost in terms of the thousands of evaluations required for some search techniques is still an obstacle for a detailed search. It can be seen from the contour maps that the objective function is rather smooth, and this may justify the use of approximation techniques alongside the accurate finite element model.

[**0131**] A 20% reduction in the objective function is achieved in this example using the above methods while satisfying all the geometric and mechanic constraints. By looking at the trace data of the search process, it can be seen that the three geometric constraints and the disk notch stresses (identified earlier) remain the major factors affecting the optimisation results. Note that the primary changes to geometry occurred during the GA search and are as illustrated in **FIG. 19**, where the base geometry shown at 70 and the optimised geometry at 72. These are a decrease in tooth pitch, an increase in shank neck width and a decrease in three of the four fillet radii. In addition, the root wedge angle is slightly increased.

[**0132**] Although minimising the frontal area, and thus the rim-load will reduce overall weight, the life of blade/disk is highly dependent on the notch stresses, and so the notch stress may be minimised to achieve required life targets. Therefore, following the search on the full scale problem, a second optimisation problem to minimize the maximum notch stress has been carried out, starting from the best design found in the previous search. It is expected that this search will drive the geometry in a different direction given the changed goal. The result is shown in **FIG. 20**. In this case only the six tooth profile parameters were chosen as design variables. It can be seen that although a 25% reduction in the maximum notch stress can be achieved, the fir-tree area is now increased by approximately 11%. Also note that the root wedge angle has dropped significantly while the pitch and all the radii have risen. Clearly, the choice of objective function has significant impact on the final design.

[**0133**] The generative modelling facility provided by the ICAD system enables the rapid evaluation of different design alternatives in an engineering environment. Incorporating such capabilities into a FEA-based structural optimisation process has been shown to be an effective way to reduce design time scales and at the same time improve the quality of the end product. Other information such as cost evaluation or manufacturing requirements could be further included without sacrificing the compatibility of the existing model. A complete and consistent product model could then be achieved to be set up for evaluation in the design optimisation process.

**1** A method of optimising a design of a component by conducting analyses on a set of design variants, each analysis comprising the steps of:

(a) representing the design variant as a CAD model comprising a plurality of geometric entities,

(b) assigning a tag name to each geometric entity,

(c) creating a computerised analysis model from the CAD model wherein the tag names remain associated with the respective geometric entities,

(d) assigning boundary conditions to at least one of the geometric entities in the analysis model by reference to the tag name, and

(e) determining an output condition of the analysis model in response to the boundary conditions,

the method further comprising the step of selecting an optimum variant on the basis of the results of the analyses.

**2** A method of optimising a design of a component as claimed in claim 1, wherein the determination of an output condition comprises determining an output condition of at least one of the geometric entities in the analysis model by reference to the tag name of that entity.

**3** A method of optimising a design of a component as claimed in claim 1, wherein the set of design variants is generated by use of a computer algorithm.

**4** A method of optimising a design of a component as claimed in claim 3, the geometric entities comprising at least one dimension, wherein the set of design variants is generated by modifying a dimension of at least one of the geometric entities.

**5** A method of optimising a design of a component as claimed in claim 3, wherein the set of design variants is generated by the addition or removal of at least one geometric entity.

**6** A method of optimising a design of a component as claimed in claim 1, wherein the generation of the set of design variants includes generating a design variant by modifying a previous design variant in response to the output condition of the previous design variant.

**7** A method of optimising a design of a component as claimed in claim 5, wherein the association between each tag name and corresponding geometric entity is unaffected by the removal or addition of a geometric entity.

**8** A method of optimising a design of a component as claimed in claim 1, wherein a mesh density is associated with at least one of the geometric entities by reference to the tag name of that entity.

**9** A method of optimising a design of a component as claimed in claim 1, wherein the analysis model is a thermomechanical finite element analysis model.

**10** A method of optimising a design of a component as claimed in claim 1, wherein the analysis model is a computational fluid dynamics model.

**11** A method of optimising a design of a component as claimed in claim 1, the geometric entities having at least one model property applied to it, wherein at least one of said model properties of at least one of the geometric entities is associated with the tag name of that geometric entity.

**12** A method of optimising a design of a component as claimed in claim 11, wherein the model property is a material property of said geometric entity.

**13** A method of optimising a design of a component as claimed in claim 11, wherein the model property is a temperature of said geometric entity.

**14** A method of optimising a design of a component as claimed in claim 1, which is performed as a batch process, the design variants being generated automatically as the process proceeds until an optimum design variant is achieved.

**15** A method of manufacturing a component, the method comprising:

(a) optimising the design of the component by a method in accordance with claim 1;

(b) manufacturing the component in accordance with the optimised design.

**16** A method of manufacturing a component as claimed in claim 15, in which the component is a component of a gas turbine engine.

**17** A method as claimed in claim 16, in which the component is a turbine blade having a fir tree root, the design of at least the fir tree root being optimised by a method in accordance with claim 1.

**18** A component having a design optimised by conducting analyses on a set of design variants, each analysis comprising the steps of:

(a) representing the design variant as a CAD model comprising a plurality of geometric entities,

(b) assigning a tag name to each geometric entity,

(c) creating a computerised analysis model from the CAD model wherein the tag names remain associated with the respective geometric entities,

(d) assigning boundary conditions to at least one of the geometric entities in the analysis model by reference to the tag name, and

(e) determining an output condition of the analysis model in response to the boundary conditions; and

selecting an optimum variant on the basis of the results of the analyses.

**19** A component manufactured by optimising the design of the component by conducting analyses on a set of design variants, each analysis comprising the steps of:

(a) representing the design variant as a CAD model comprising a plurality of geometric entities,

(b) assigning a tag name to each geometric entity,

(c) creating a computerised analysis model from the CAD model wherein the tag names remain associated with the respective geometric entities,

(d) assigning boundary conditions to at least one of the geometric entities in the analysis model by reference to the tag name, and

(e) determining an output condition of the analysis model in response to the boundary conditions;

selecting an optimum variant on the basis of the results of the analyses, and manufacturing the component in accordance with the optimised design.

**20** A computer program product comprising code for carrying out a method of optimising a design of a component by conducting analyses on a set of design variants, each analysis comprising the steps of:

(a) representing the design variant as a CAD model comprising a plurality of geometric entities,

(b) assigning a tag name to each geometric entity,

(c) creating a computerised analysis model from the CAD model wherein the tag names remain associated with the respective geometric entities,

(d) assigning boundary conditions to at least one of the geometric entities in the analysis model by reference to the tag name, and

(e) determining an output condition of the analysis model in response to the boundary conditions,

the method further comprising the step of selecting an optimum variant on the basis of the results of the analyses.

**21** A computer system adapted to carry out a method of optimising a design of a component by conducting analyses on a set of design variants, each analysis comprising the steps of:

(a) representing the design variant as a CAD model comprising a plurality of geometric entities,

(b) assigning a tag name to each geometric entity,

(c) creating a computerised analysis model from the CAD model wherein the tag names remain associated with the respective geometric entities,

(d) assigning boundary conditions to at least one of the geometric entities in the analysis model by reference to the tag name, and

(e) determining an output condition of the analysis model in response to the boundary conditions,

the method further comprising the step of selecting an optimum variant on the basis of the results of the analyses.

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