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Yeo et al.

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(54) **ENCRYPTED SEARCH WITH NO ZERO-DAY LEAKAGE**

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H04L 29/06 (2006.01)
G06F 16/22 (2019.01)
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(58) **Field of Classification Search**
None
See application file for complete search history.

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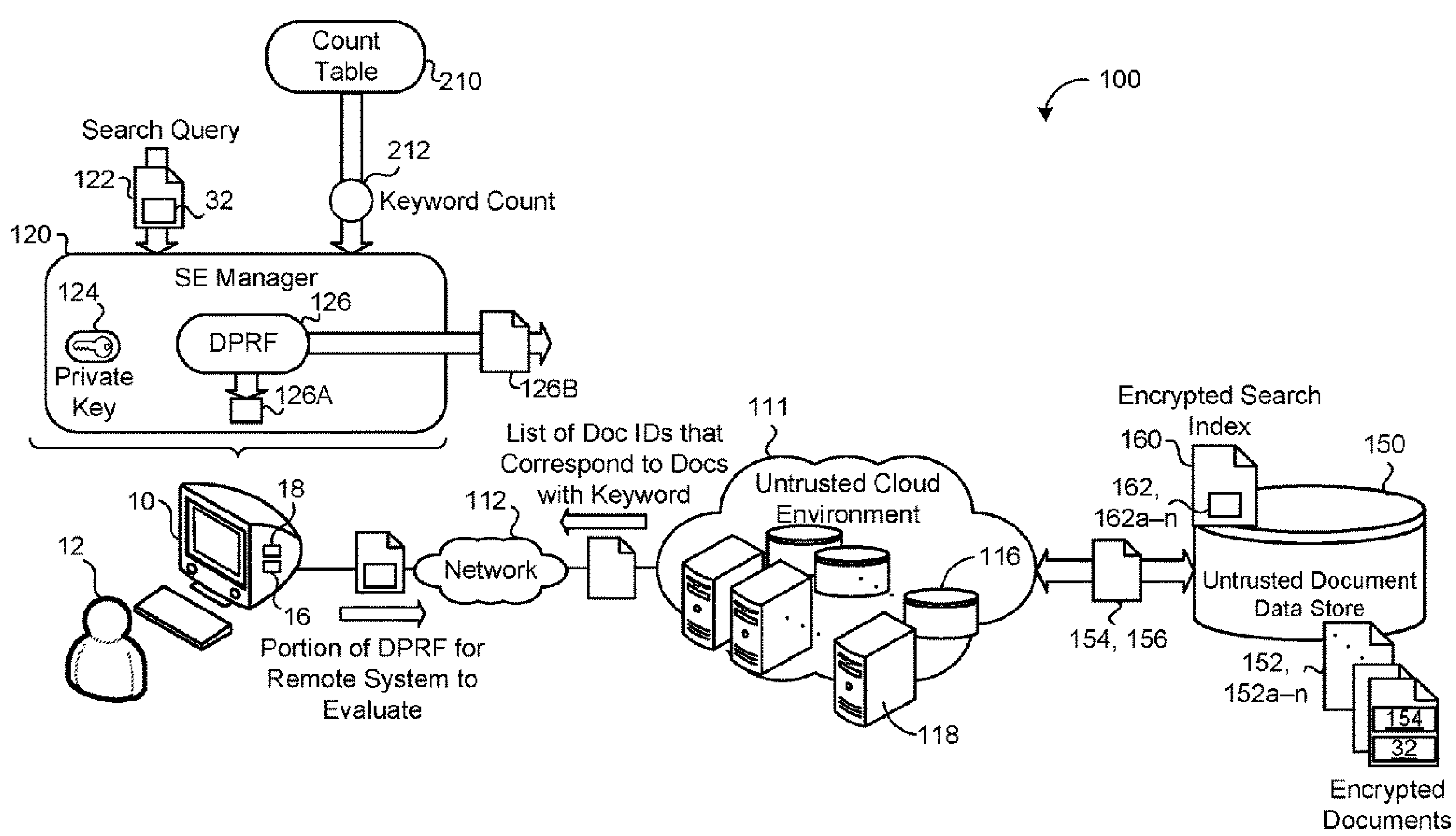
(57) **ABSTRACT**

A method for providing encrypted search includes receiving, at a user device associated with a user, a search query for a keyword that appears in one or more encrypted documents stored on an untrusted storage device and accessing a count table to obtain a count of documents that include the keyword. The method also includes generating a delegatable pseudorandom function (DPRF) based on the keyword, a private cryptographic key, and the count of documents. The method also includes evaluating a first portion of the DPRF and delegating a remaining second portion of the DPRF to the untrusted storage device which causes the untrusted storage device to evaluate the DPRF and access an encrypted search index associated with the documents. The untrusted storage device determines one or more encrypted documents associated with DPRF and returns, to the user device, an identifier for each encrypted document associated with the DPRF.

20 Claims, 10 Drawing Sheets



(51) **Int. Cl.**

G06F 16/242	(2019.01)
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H04L 9/06	(2006.01)
H04L 9/08	(2006.01)
H04L 9/00	(2022.01)
G06F 21/60	(2013.01)

(52) **U.S. Cl.**

CPC ***G06F 16/2455*** (2019.01); ***G06F 21/602*** (2013.01); ***H04L 9/006*** (2013.01); ***H04L 9/0656*** (2013.01); ***H04L 9/0861*** (2013.01)

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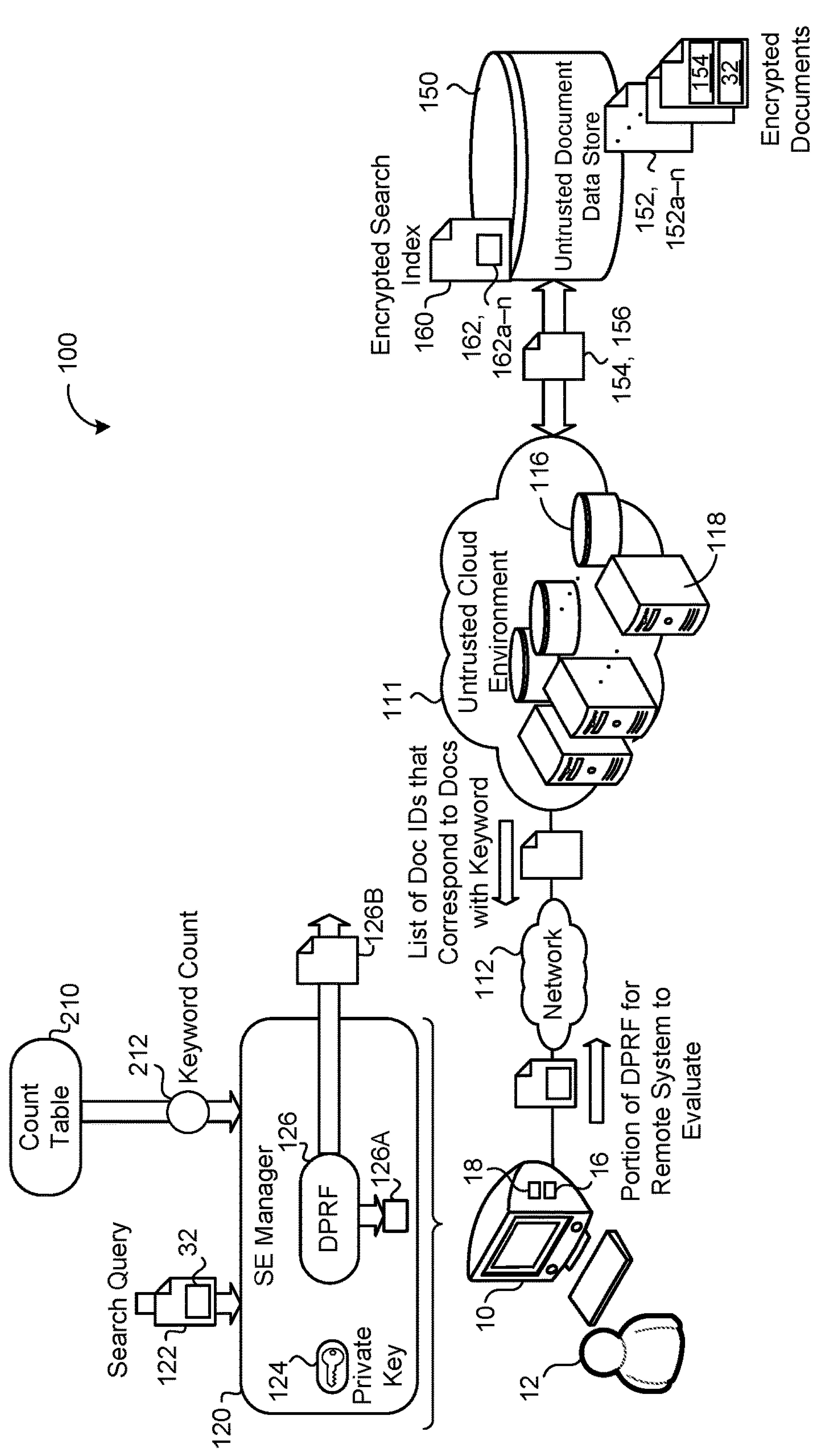

100
Search Query
32
122
Count Table
210
212
Keyword Count
120
SE Manager
124
Private Key
DPRF
126
126A
126B
12
10
18
16
Portion of DPRF for Remote System to Evaluate
112
Network
List of Doc IDs that Correspond to Docs with Keyword
111
Untrusted Cloud Environment
116
118
154, 156
Encrypted Search Index
160
162, 162a–n
150
Untrusted Document Data Store
152, 152a–n
154
32
Encrypted Documents

FIG. 1

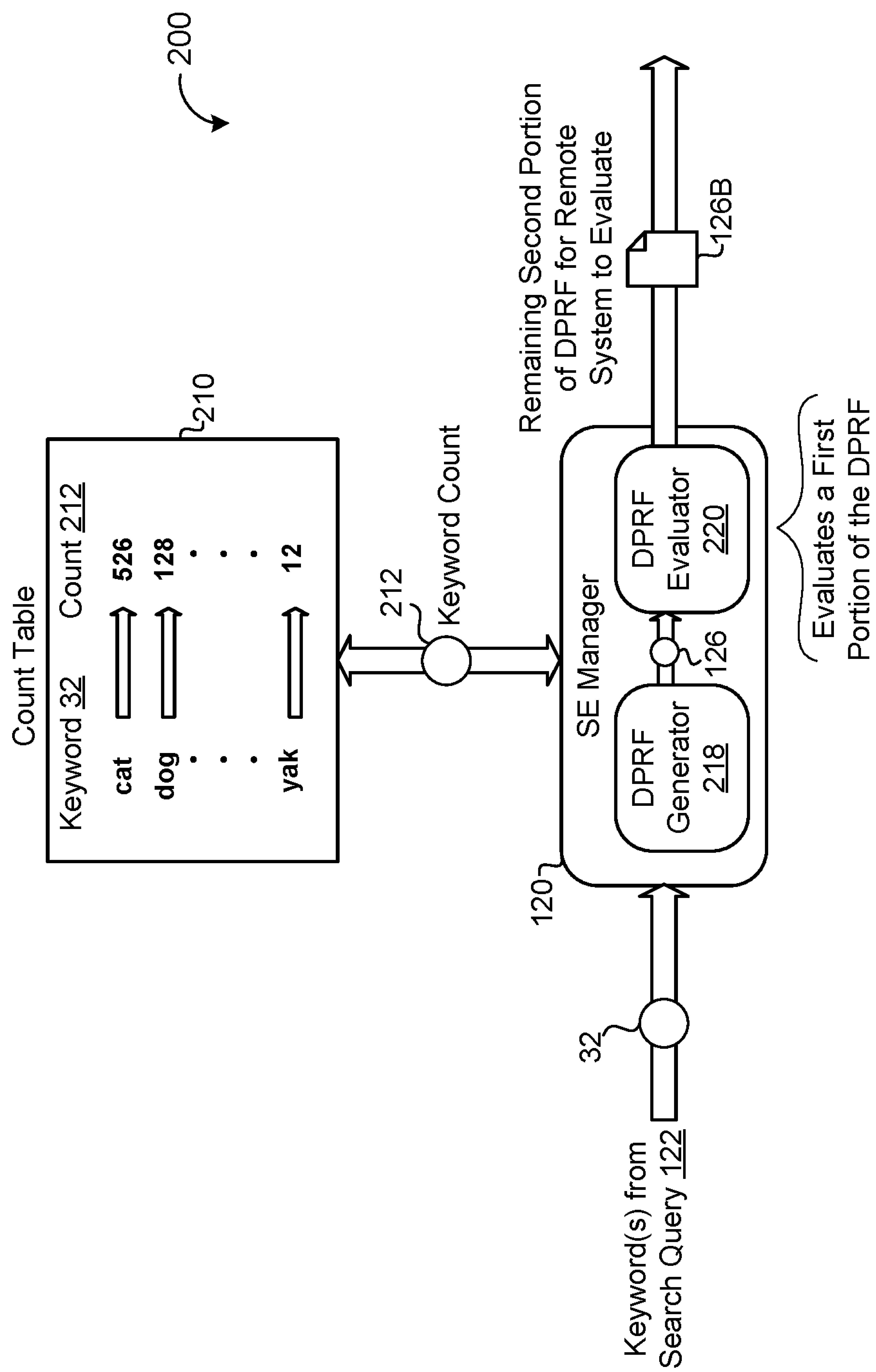

200
Count Table
210
Keyword 32
Count 212
cat
526
dog
128
yak
12
212
Keyword Count
Remaining Second Portion of DPRF for Remote System to Evaluate
126B
SE Manager
120
DPRF Generator 218
126
DPRF Evaluator 220
Evaluates a First Portion of the DPRF
32
Keyword(s) from Search Query 122

FIG. 2

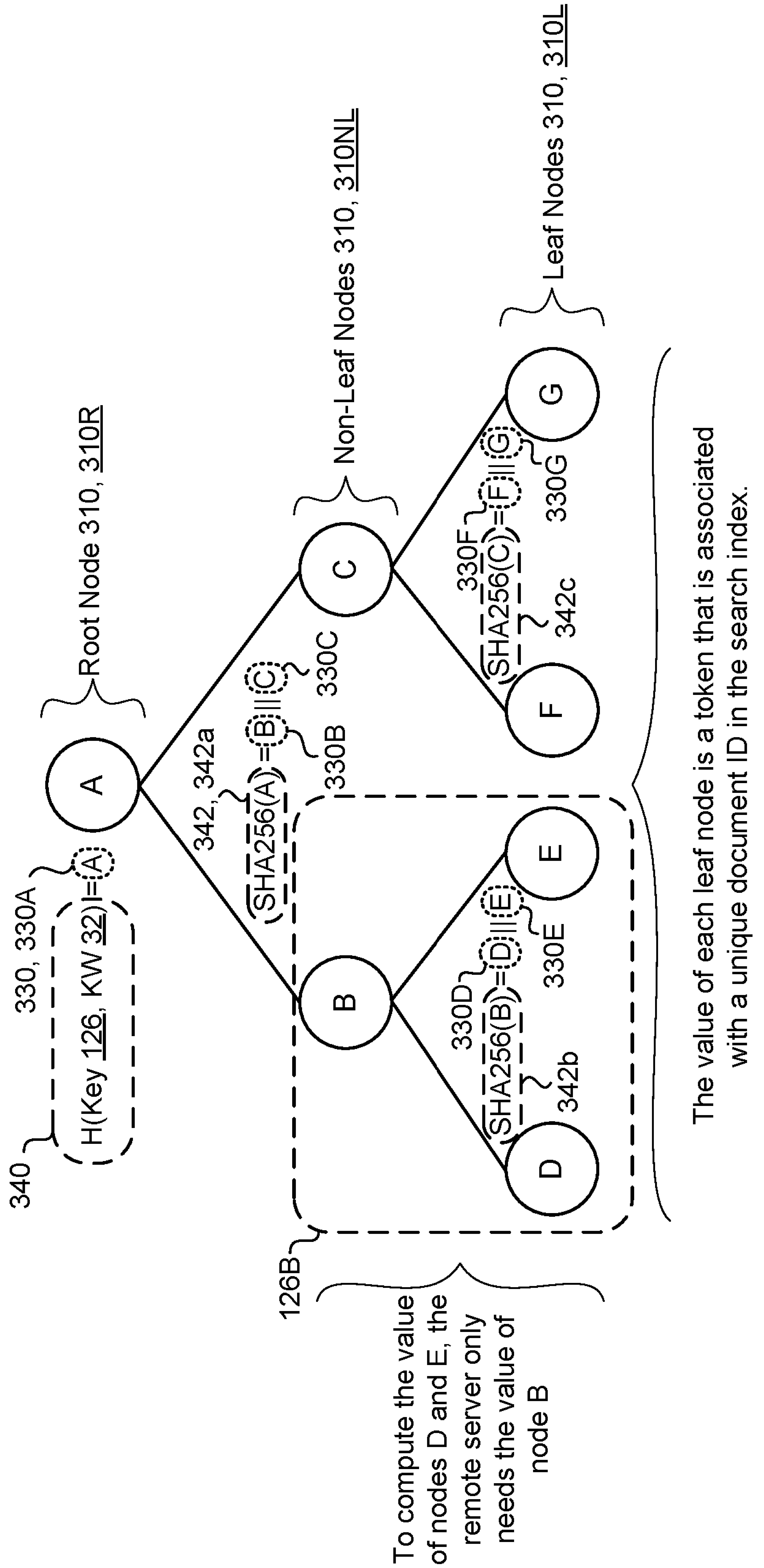

300
Root Node 310, 310R
Non-Leaf Nodes 310, 310NL
Leaf Nodes 310, 310L
340
330, 330A
H(Key 126, KW 32)I=A
A
342, 342a
SHA256(A)=B||C
330B
330C
B
C
330D
SHA256(B)=D||E
342b
330E
330F
SHA256(C)=F||G
342c
330G
D
E
F
G
126B
To compute the value of nodes D and E, the remote server only needs the value of node B
The value of each leaf node is a token that is associated with a unique document ID in the search index.

FIG. 3

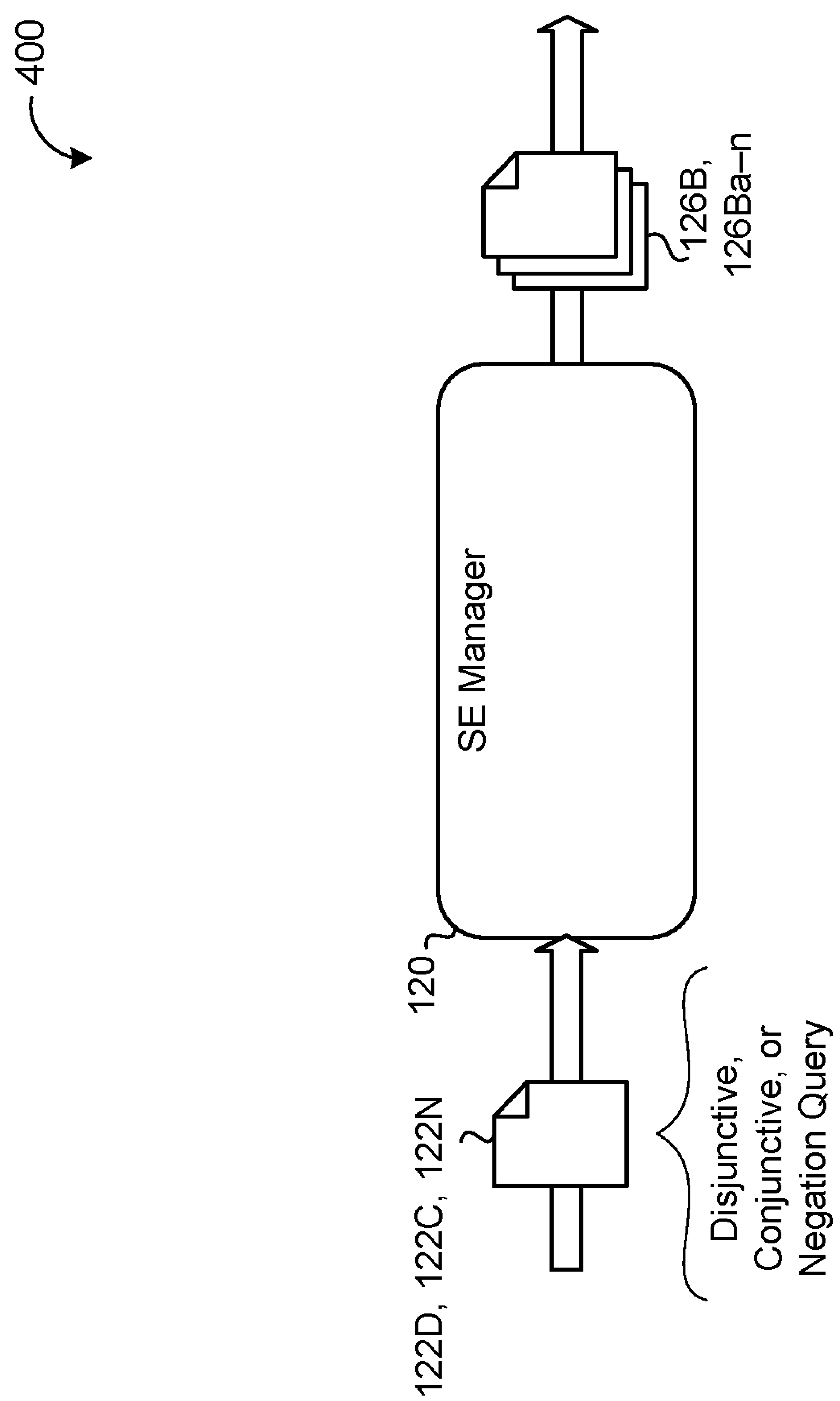

400
SE Manager
120
126B,
126Ba–n
122D, 122C, 122N
Disjunctive,
Conjunctive, or
Negation Query

FIG. 4

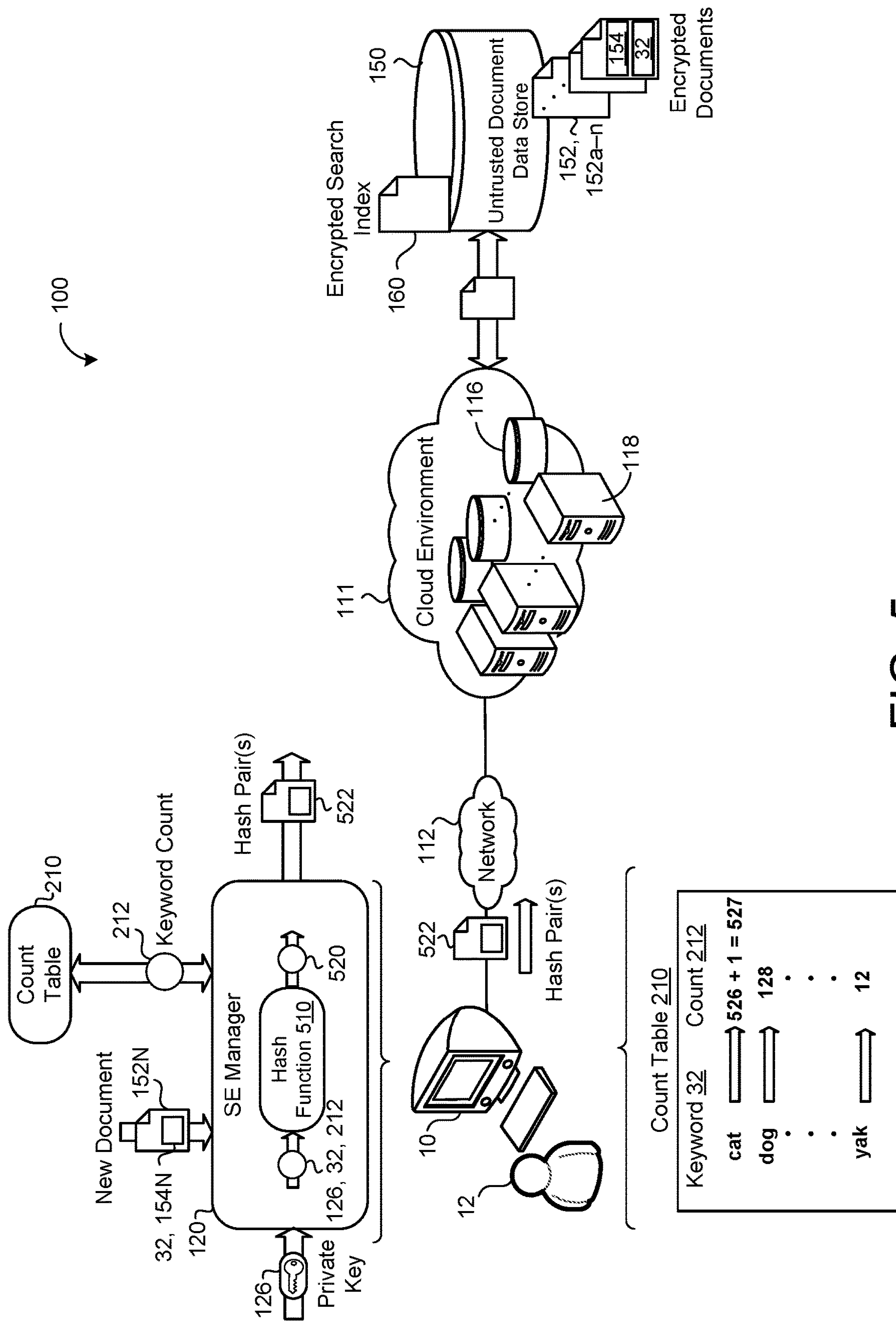
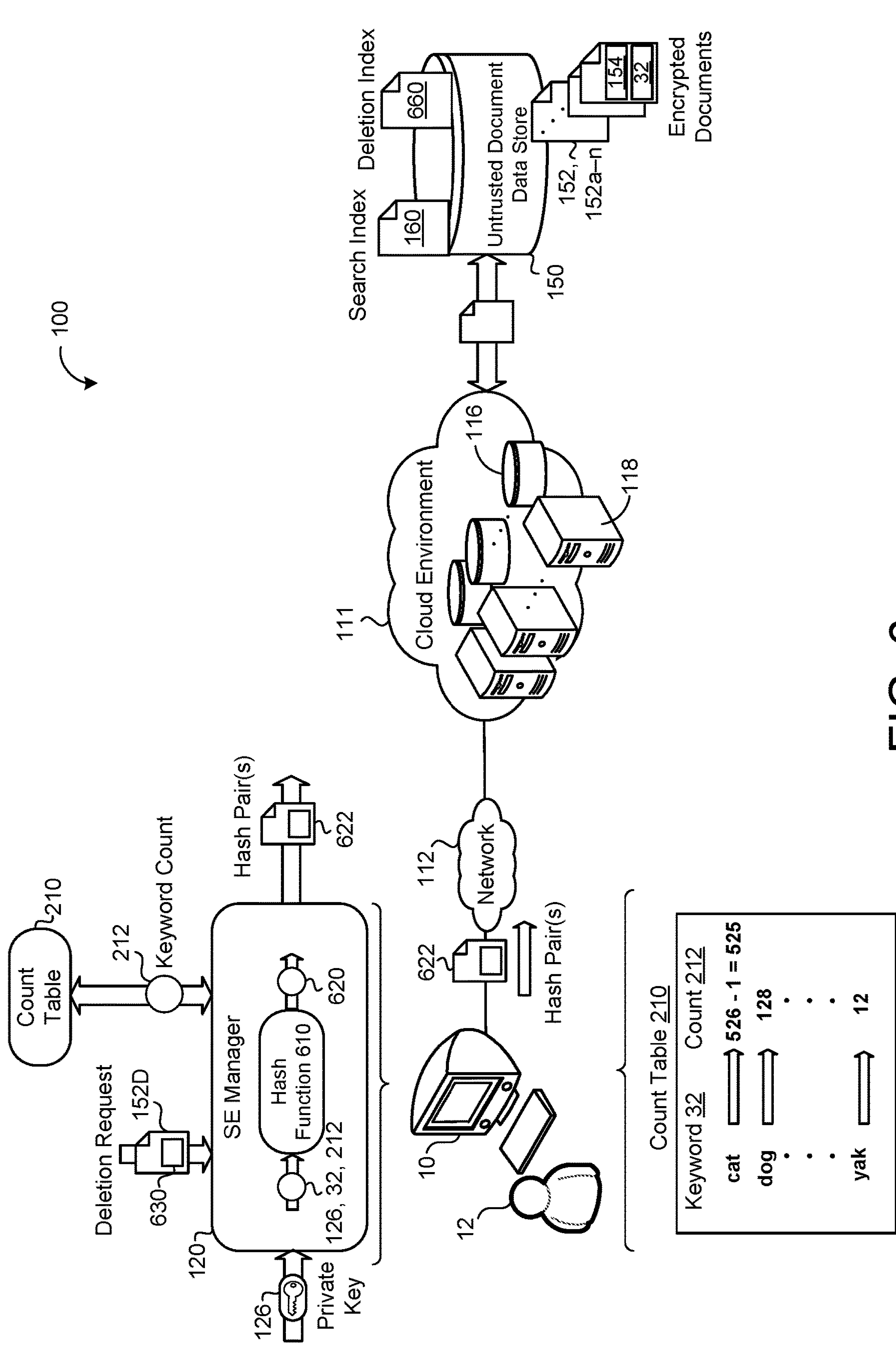

100
Count Table
210
212
Keyword Count
New Document
152N
32, 154N
120
SE Manager
Hash Function 510
126, 32, 212
520
Hash Pair(s)
522
126
Private Key
10
12
522
Hash Pair(s)
112
Network
Count Table 210
Keyword 32
Count 212
cat
526 + 1 = 527
dog
128
yak
12
111
Cloud Environment
116
118
160
Encrypted Search Index
150
Untrusted Document Data Store
152, 152a–n
154
32
Encrypted Documents

FIG. 5



100
Deletion Request
152D
630
Count Table
210
212
Keyword Count
SE Manager
Hash Function 610
126, 32, 212
620
120
126
Private Key
Hash Pair(s)
622
112
Network
622
Hash Pair(s)
10
12
Count Table 210
Keyword 32
Count 212
cat
526 - 1 = 525
dog
128
yak
12
Cloud Environment
111
116
118
Search Index
160
Deletion Index
660
Untrusted Document Data Store
150
152, 152a–n
154
32
Encrypted Documents

FIG. 6

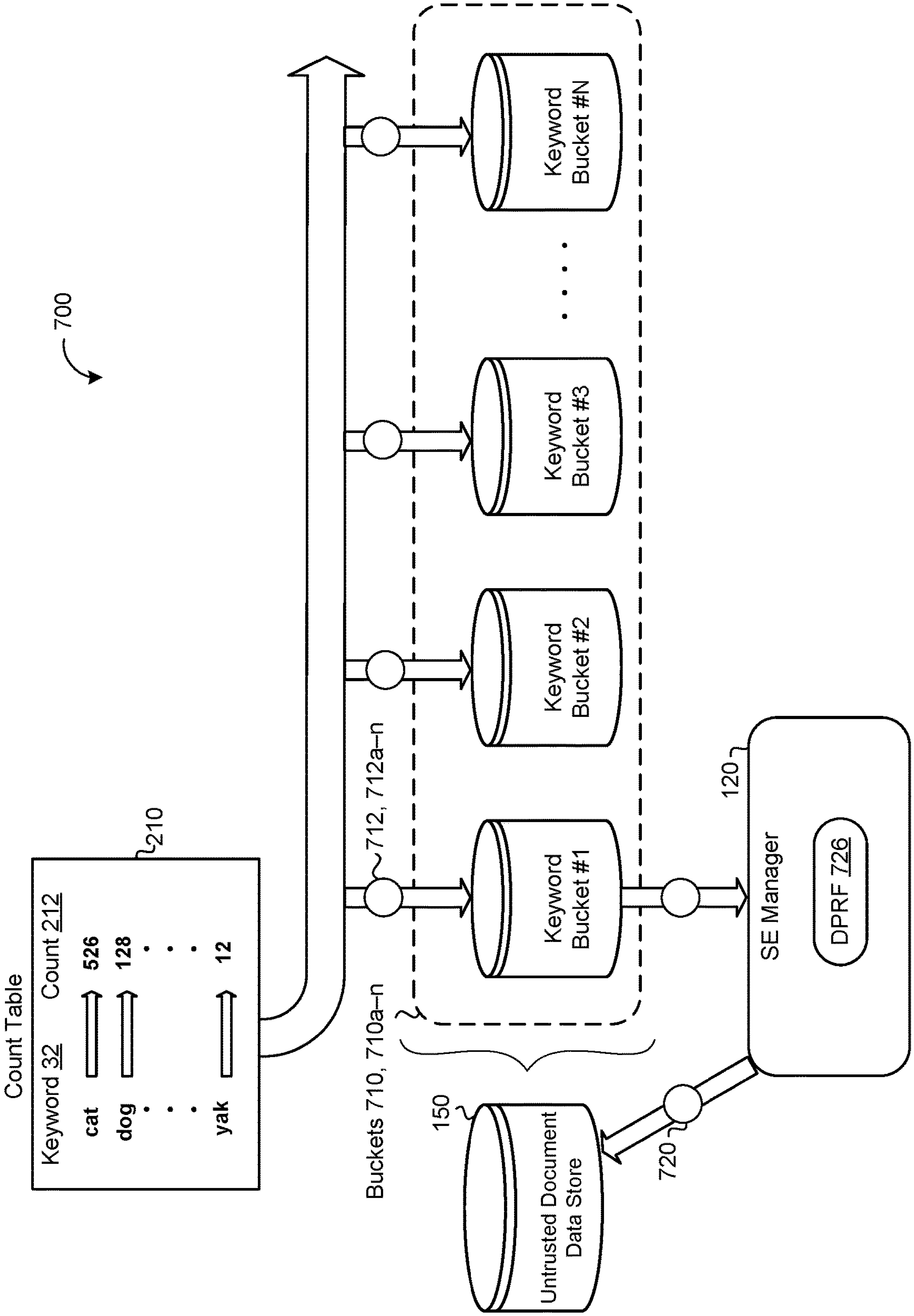

700
Count Table
Keyword 32
Count 212
cat
526
dog
128
yak
12
210
712, 712a–n
Keyword Bucket #1
Keyword Bucket #2
Keyword Bucket #3
Keyword Bucket #N
Buckets 710, 710a–n
120
SE Manager
DPRF 726
150
Untrusted Document Data Store
720

FIG. 7

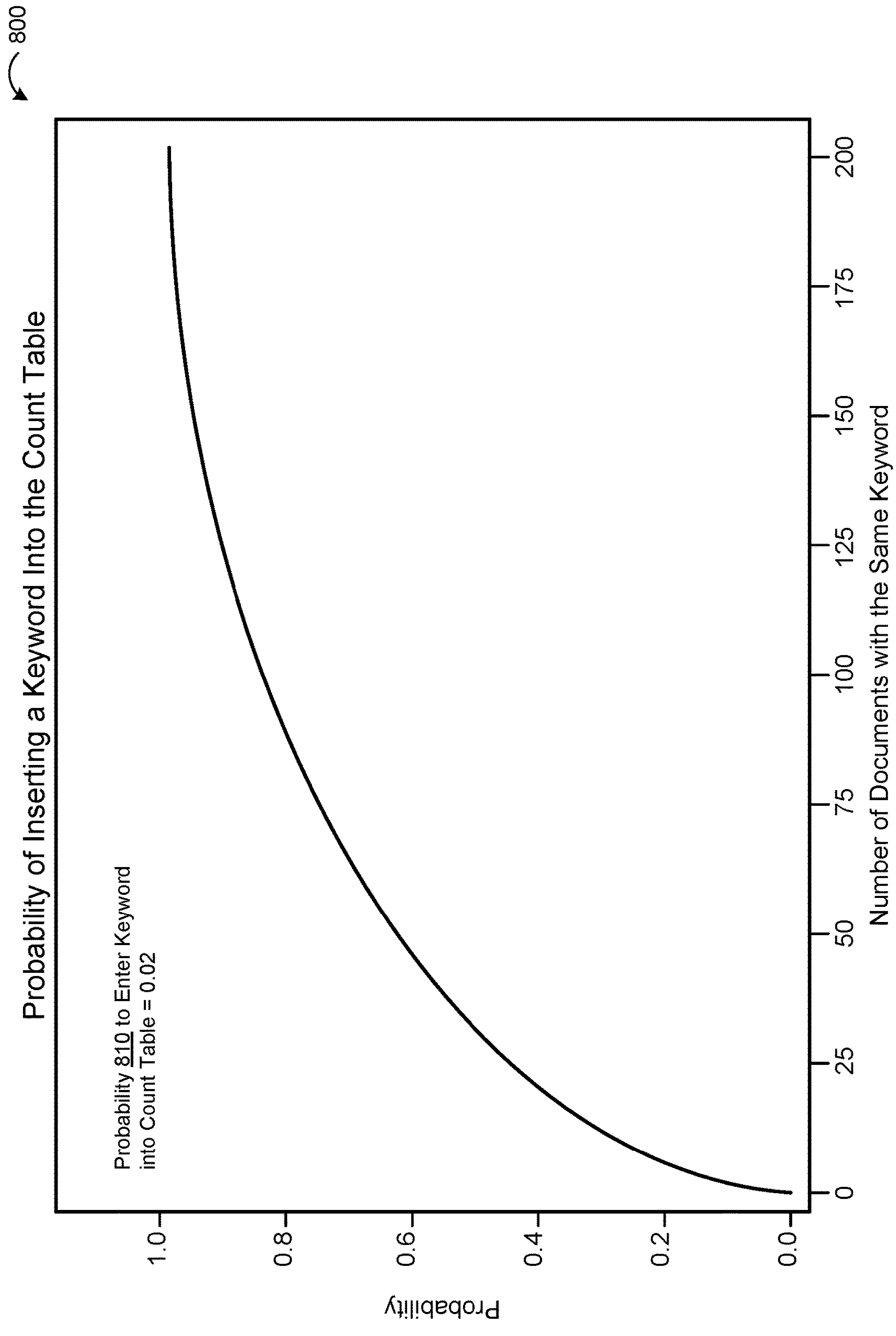

800
Probability of Inserting a Keyword Into the Count Table
Probability 810 to Enter Keyword
into Count Table = 0.02
Probability
0.0
0.2
0.4
0.6
0.8
1.0
0
25
50
75
100
125
150
175
200
Number of Documents with the Same Keyword

FIG. 8

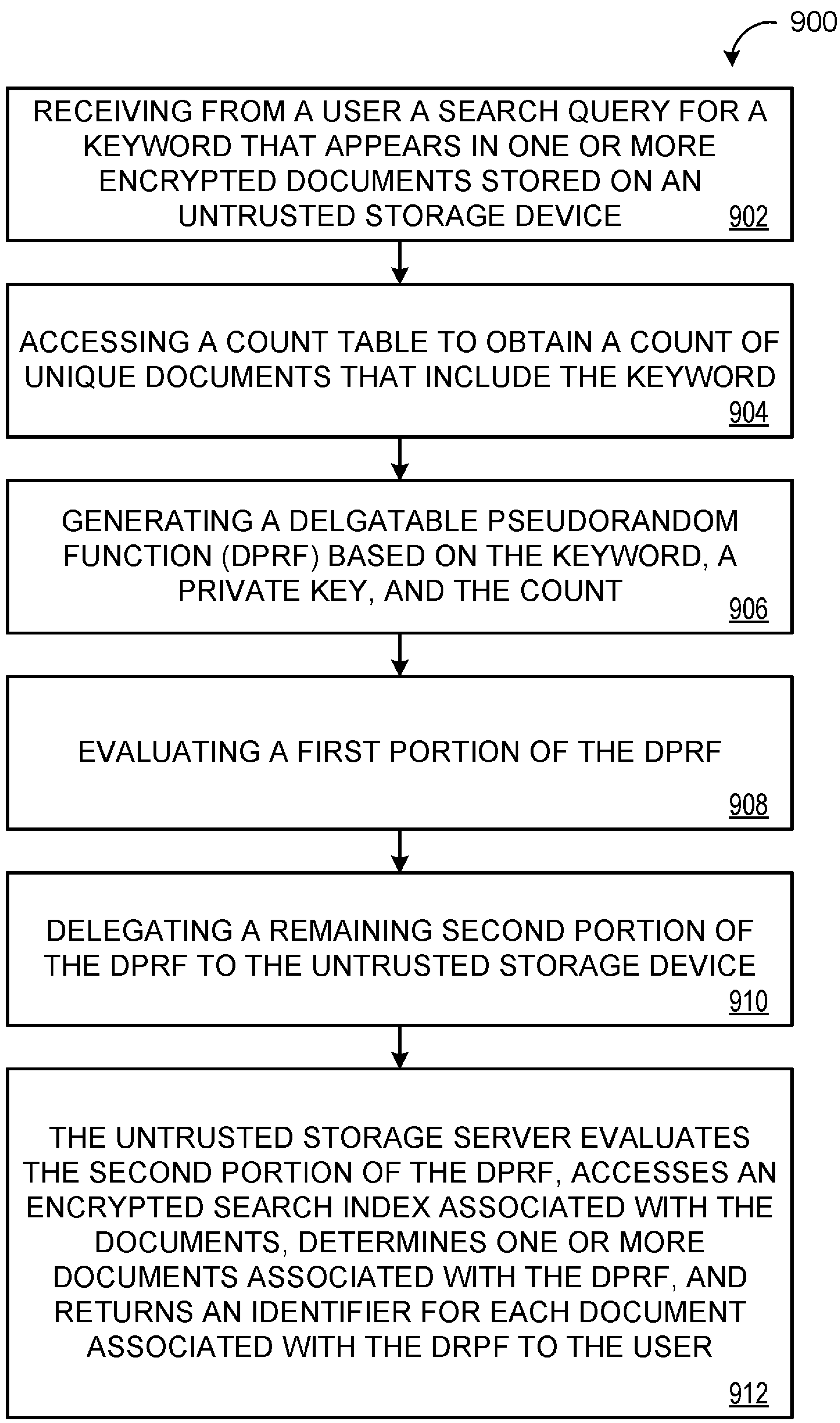

900
RECEIVING FROM A USER A SEARCH QUERY FOR A KEYWORD THAT APPEARS IN ONE OR MORE ENCRYPTED DOCUMENTS STORED ON AN UNTRUSTED STORAGE DEVICE
902
ACCESSING A COUNT TABLE TO OBTAIN A COUNT OF UNIQUE DOCUMENTS THAT INCLUDE THE KEYWORD
904
GENERATING A DELGATABLE PSEUDORANDOM FUNCTION (DPRF) BASED ON THE KEYWORD, A PRIVATE KEY, AND THE COUNT
906
EVALUATING A FIRST PORTION OF THE DPRF
908
DELEGATING A REMAINING SECOND PORTION OF THE DPRF TO THE UNTRUSTED STORAGE DEVICE
910
THE UNTRUSTED STORAGE SERVER EVALUATES THE SECOND PORTION OF THE DPRF, ACCESSES AN ENCRYPTED SEARCH INDEX ASSOCIATED WITH THE DOCUMENTS, DETERMINES ONE OR MORE DOCUMENTS ASSOCIATED WITH THE DPRF, AND RETURNS AN IDENTIFIER FOR EACH DOCUMENT ASSOCIATED WITH THE DRPF TO THE USER
912

FIG. 9

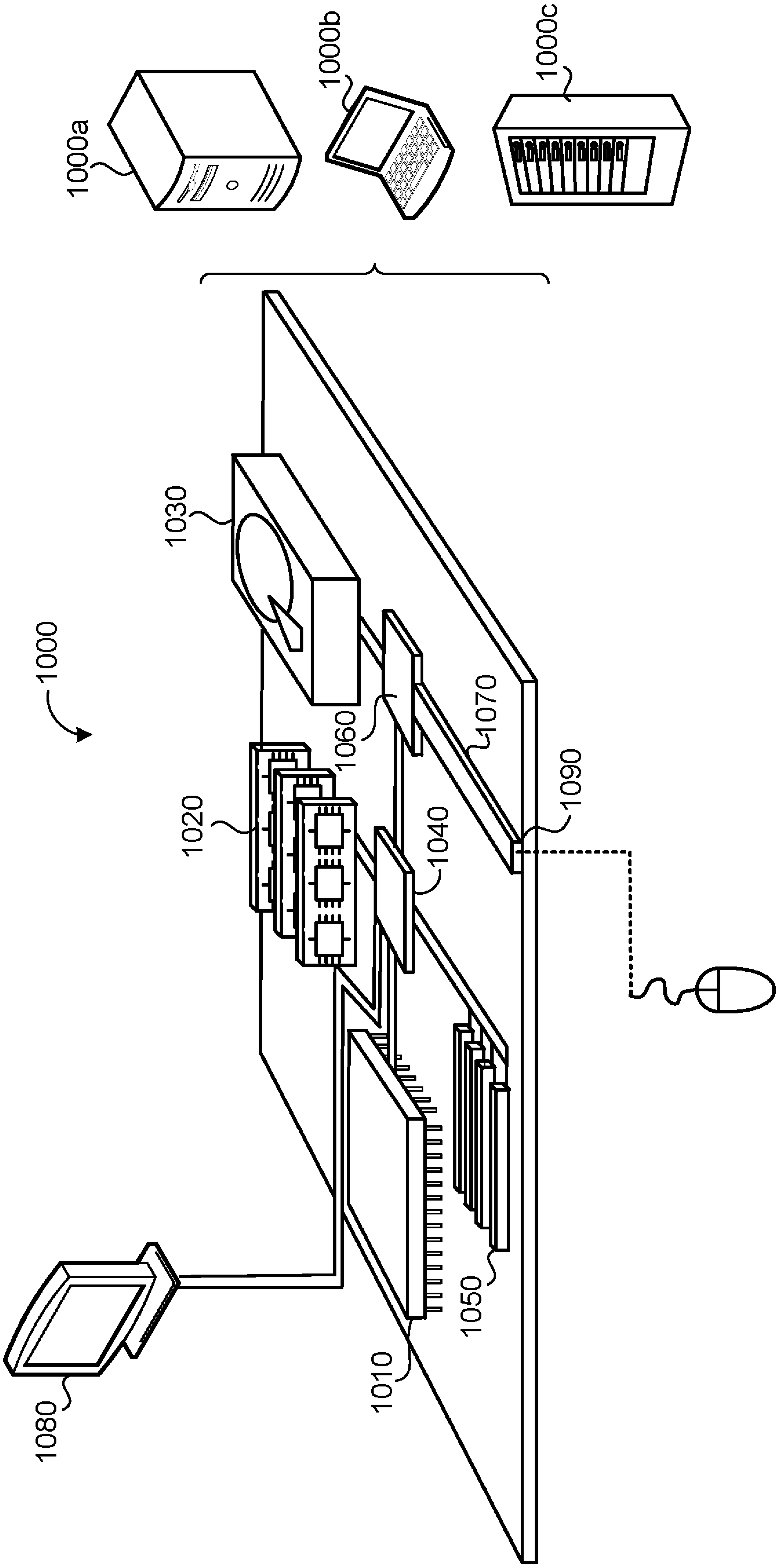

1000a
1000b
1000c
1030
1000
1060
1070
1020
1040
1090
1050
1010
1080

FIG. 10

ENCRYPTED SEARCH WITH NO ZERO-DAY LEAKAGE

CROSS REFERENCE TO RELATED APPLICATIONS

This U.S. patent application is a continuation of, and claims priority under 35 U.S.C. § 120 from, U.S. patent application Ser. No. 16/712,151, filed on Dec. 12, 2019. The disclosure of this prior application is considered part of the disclosure of this application and is hereby incorporated by reference in its entirety.

TECHNICAL FIELD

This disclosure relates to performing encrypted search with no zero-day leakage.

BACKGROUND

Searchable encryption (i.e., encrypted search) has increased in popularity as storage of large quantities of data in the cloud becomes more common. More and more, a user or client owns a large corpus of encrypted documents that are stored at a server not under the client's control (i.e., the server is untrusted). With searchable encryption, the client can store their encrypted documents on the untrusted server, but still maintain the capability of searching the documents and, for example, retrieve identifiers of all documents containing a specific keyword. However, such searchable encryption often comes with security and privacy drawbacks.

SUMMARY

One aspect of the disclosure provides a method for providing encrypted search with no zero-day leakage. The method includes receiving, at data processing hardware of a user device associated with a user, a search query for a keyword. The keyword appears in one or more encrypted documents within a corpus of encrypted documents stored on an untrusted storage device. The method also includes accessing, by the data processing hardware, a count table to obtain a count of unique documents within the corpus of encrypted documents that include the keyword and generating, by the data processing hardware, a delegatable pseudorandom function (DPRF) based on the keyword, a private cryptographic key, and the count of unique documents that include the keyword. The method also includes evaluating, by the data processing hardware, a first portion of the DPRF and delegating, by the data processing hardware, a remaining second portion of the DPRF to the untrusted storage device. The remaining second portion of the DPRF when received by the untrusted storage device causes the untrusted storage device to evaluate the remaining second portion of the DPRF and access an encrypted search index associated with the corpus of encrypted documents stored on the untrusted storage device. The untrusted storage device also determines one or more encrypted documents within the corpus of encrypted documents associated with the remaining second portion of the DPRF based on the encrypted search index and returns, to the user device, an identifier for each encrypted document of at least a portion of the one or more encrypted documents associated with the remaining second portion of the DPRF.

Implementations of the disclosure may include one or more of the following optional features. In some implementations, generating the DPRF includes generating a binary tree where the binary tree includes a set of nodes that includes a root node and a plurality of other nodes. Each other node includes a non-leaf node or a leaf node. The method may also include where a quantity of leaf nodes of the binary tree is equal to or greater than the count of unique documents that include the keyword.

In some examples, the root node of the binary tree includes a first hash of the private cryptographic key and the keyword. The root node may be associated with a first child node and a second child node, where the first child node includes a first portion of a second hash of the first hash of the private cryptographic key and the keyword, and the second child node includes a second portion of the second hash of the first hash of the private cryptographic key and the keyword. Optionally, the first portion of the second hash concatenated with the second portion of the second hash is equivalent to the second hash of the first hash of the private cryptographic key and the keyword.

Each leaf node of the set of nodes of the binary tree may be associated with a value stored in the encrypted search index. Each other node of the set of nodes of the binary tree may include a portion of a hash of a parent node associated with the corresponding other node. In some implementations, evaluating the first portion of the DPRF includes evaluating a first subset of the set of nodes of the binary tree. When the untrusted storage device evaluates the remaining second portion of the DPRF, the untrusted storage device evaluates a second subset of the set of nodes of the binary tree. The second subset includes different nodes from the set of nodes of the binary tree than the first subset.

In some examples, the method further includes, for each unique keyword of a new encrypted document uploaded by the user into the corpus of encrypted documents stored on the untrusted storage device, incrementing, by the data processing hardware, the count of unique documents within the corpus of encrypted documents that include the corresponding unique keyword in the count table and generating, by the data processing hardware, a unique keyword hash based on the private cryptographic key, the corresponding unique keyword, and the incremented count of unique documents within the corpus of encrypted documents that include the corresponding unique keyword. The method may also include generating, by the data processing hardware, a hash pair including the unique keyword hash and an encrypted document identifier associated with the new encrypted document uploaded by the user and sending, by the data processing hardware, the hash pair to the untrusted storage device.

When the untrusted storage device returns the identifier for each encrypted document of the at least the portion of the one or more encrypted documents associated with the remaining second portion of the DPRF, the untrusted storage device may return encrypted metadata associated with each returned identifier.

Another aspect of the disclosure provides a system for providing encrypted search with no zero-day leakage. The system includes data processing hardware of a user device associated with a user and memory hardware in communication with the data processing hardware. The memory hardware stores instructions that when executed on the data processing hardware cause the data processing hardware to perform operations. The operations include receiving a search query for a keyword. The keyword appears in one or more encrypted documents within a corpus of encrypted documents stored on an untrusted storage device. The operations also include accessing a count table to obtain a count of unique documents within the corpus of encrypted documents that include the keyword and generating a delegatable pseudorandom function (DPRF) based on the keyword, a private cryptographic key, and the count of unique documents that include the keyword. The operations also include evaluating a first portion of the DPRF and delegating a remaining second portion of the DPRF to the untrusted storage device. The remaining second portion of the DPRF when received by the untrusted storage device causes the untrusted storage device to evaluate the remaining second portion of the DPRF and access an encrypted search index associated with the corpus of encrypted documents stored on the untrusted storage device. The untrusted storage device also determines one or more encrypted documents within the corpus of encrypted documents associated with the remaining second portion of the DPRF based on the encrypted search index and returns, to the user device, an identifier for each encrypted document of at least a portion of the one or more encrypted documents associated with the remaining second portion of the DPRF.

This aspect may include one or more of the following optional features. In some implementations, generating the DPRF includes generating a binary tree where the binary tree includes a set of nodes that includes a root node and a plurality of other nodes. Each other node includes a non-leaf node or a leaf node. The operations may also include where a quantity of leaf nodes of the binary tree is equal to or greater than the count of unique documents that include the keyword.

In some examples, the root node of the binary tree includes a first hash of the private cryptographic key and the keyword. The root node may be associated with a first child node and a second child node, where the first child node includes a first portion of a second hash of the first hash of the private cryptographic key and the keyword, and the second child node includes a second portion of the second hash of the first hash of the private cryptographic key and the keyword. Optionally, the first portion of the second hash concatenated with the second portion of the second hash is equivalent to the second hash of the first hash of the private cryptographic key and the keyword.

Each leaf node of the set of nodes of the binary tree may be associated with a value stored in the encrypted search index. Each other node of the set of nodes of the binary tree may include a portion of a hash of a parent node associated with the corresponding other node. In some implementations, evaluating the first portion of the DPRF includes evaluating a first subset of the set of nodes of the binary tree. When the untrusted storage device evaluates the remaining second portion of the DPRF, the untrusted storage device evaluates a second subset of the set of nodes of the binary tree.

The second subset includes different nodes from the set of nodes of the binary tree than the first subset.

In some examples, the operations further include, for each unique keyword of a new encrypted document uploaded by the user into the corpus of encrypted documents stored on the untrusted storage device, incrementing the count of unique documents within the corpus of encrypted documents that include the corresponding unique keyword in the count table and generating a unique keyword hash based on the private cryptographic key, the corresponding unique keyword, and the incremented count of unique documents within the corpus of encrypted documents that include the corresponding unique keyword. The operations may also include generating a hash pair including the unique keyword hash and an encrypted document identifier associated with the new encrypted document uploaded by the user and sending the hash pair to the untrusted storage device.

When the untrusted storage device returns the identifier for each encrypted document of the at least the portion of the one or more encrypted documents associated with the remaining second portion of the DPRF, the untrusted storage device may return encrypted metadata associated with each returned identifier.

The details of one or more implementations of the disclosure are set forth in the accompanying drawings and the description below. Other aspects, features, and advantages will be apparent from the description and drawings, and from the claims.

DESCRIPTION OF DRAWINGS

FIG. **1** is a schematic view of an example system that provides encrypted search with no zero-day leakage.

FIG. **2** is a schematic view of exemplary components of a searchable encryption manager.

FIG. **3** is a schematic view of a binary tree.

FIG. **4** is a schematic view of a searchable encryption manager and advanced queries.

FIG. **5** is a schematic view of the example system adding a document to a corpus of encrypted documents.

FIG. **6** is a schematic view of the examples system deleing a document from the corpus of encrypted documents.

FIG. **7** is a schematic view of an untrusted storage device and count table bucketization.

FIG. **8** is a schematic view of a plot of a probability of inserting a keyword into the count table.

FIG. **9** is a flowchart of an example arrangement of operations for a method of providing encrypted search with no zero-day leakage.

FIG. **10** is a schematic view of an example computing device that may be used to implement the systems and methods described herein.

Like reference symbols in the various drawings indicate like elements.

DETAILED DESCRIPTION

Searchable encryption (which may also be referred to as encrypted search) has been increasing in popularity. The goal of searchable encryption is to enable a client to outsource the storage of a corpus of encrypted documents to an untrusted server. For example, the client may wish to store a large number of documents (or any other item uploaded to the server, such as pictures, emails, etc.) securely in a cloud-based storage solution. The term documents is used generally, and may represent any sort of digital files (e.g., pictures, songs, database entries, etc.). Typically, the client will want to keep the ability to efficiently search the documents (i.e., search for a specific keyword), while simultaneously maintaining the privacy and security of the documents that encryption provides. In order to maintain this privacy, information related to the contents of the documents or the queries from the client must remain hidden from the untrusted server. A common way to address this problem is the creation of a separate encrypted search index that indexes the keywords and associated document identifiers of all of the documents stored on the untrusted server.

This search index is encrypted with a key the untrusted server does not have access to, and then stored along with the documents. The client may then generate a search query that the server evaluates against the encrypted search index. The evaluation results in the encrypted document identifiers associated with the keyword of the search query, which the untrusted server returns to the client. In this way, the client receives a list of document identifiers of documents that include the keyword while minimizing information leakage (e.g., to the untrusted server).

As the untrusted server evaluates the search index in response to queries from the user, the index will gradually leak information about search patterns and, by deploying attacks such as frequency analysis, the server may eventually be able to make informed guesses on the historical searched terms with non-negligible probability. This leakage cannot be efficiently prevented as it is an inherent problem due to the searching repeatedly over the same index.

However, many searchable encryption schemes suffer from a number of additional security or privacy concerns beyond this slow leakage of using the search index. In one example, some schemes are vulnerable to zero-day attacks. A zero-day attack is an attack that reveals or leaks information to an adversary (e.g., the untrusted storage server) before any queries have been processed by the storage server. That is, search queries (i.e., searching for a keyword among the encrypted documents) typically leak at least some information to the server. However, a successful zero-day attack does not require any search queries at all to gain information about the encrypted documents.

For example, some searchable encryption schemes hash each keyword in a document into one or more small values that are attached to each encrypted document. To search for the keyword, each associated hash value may be searched. However, this scheme reveals to the server a frequency table of the number of documents (as well as the identifier of the documents) that contain a specific hash value. For example, a hash value associated with a lot of documents is likely to be a more common word than a hash value that is associated with less documents. This information is revealed to the server before any search queries have been performed. Study has shown that frequency tables can reveal a large number of keywords. While the schemes may attempt to mitigate this weakness (e.g., by adding random terms), a significant amount of noise must be added to ensure that the frequency problem is overcome, which significantly reduces the efficiency of the scheme.

Another common security issue that many searchable encryption schemes are vulnerable to are file-injection attacks. These attacks work on the premise that an adversary may send encrypted documents (e.g., emails) to a target. These emails will contain specific keywords. When the target queries for these specific keywords, the adversary may view which of the injected emails are returned and thus determine the queried keyword. In some instances, the adversary may even hide the identity of the injected emails by hiding keywords that may notify the target via, for example, invisible Hypertext Markup Language (HTML). This attack may be compounded if the adversary is able to save the queries that the target performs (or retrieve queries that were performed from a log). The adversary may then apply all these historical queries to emails that were recently injected (i.e., injected after the queries were performed) to compromise the privacy of queried keywords for historical queries. Thus, when the scheme uses the same hash for all emails in the past or future, the scheme is vulnerable to an adversary applying all previous queries into files that were only injected recently.

In order to mitigate zero-day attacks and file-injection attacks of encrypted documents while maintaining search functionality and efficiency, implementations herein are directed toward an encrypted search scheme using delegatable pseudorandom functions (DPRF) to completely hide frequency tables before any search queries have been performed.

Referring now to FIG. **1**, in some implementations, an example system **100** includes a user device **10** associated with a respective user or client **12** and in communication with an untrusted remote system **111** via a network **112**. The user device **10** may correspond to any computing device, such as a desktop workstation, a laptop workstation, or a mobile device (i.e., a smart phone). The user device **10** includes computing resources **18** (e.g., data processing hardware) and/or storage resources **16** (e.g., memory hardware).

The remote system **111** may be a single computer, multiple computers, or a distributed system (e.g., a cloud environment) having scalable/elastic computing resources **118** (e.g., data processing hardware) and/or storage resources **116** (e.g., memory hardware). An untrusted document data store **150** (i.e., a remote storage device **150**) is overlain on the storage resources **116** to allow scalable use of the storage resources **116** by one or more of the client or computing resources **118**. The document data store **150** is configured to store a corpus of documents **152**, **152***a-n*. Each document **152** includes a document identifier **154** that uniquely identifies the associated document **152** (e.g., a document name). Each document **152** also includes a set of keywords **32**. The set of keywords **32** includes all keywords that appear in the associated encrypted document **152** that the user **12** may search for. As used herein, a document **152** may refer to any item uploaded onto the remote system **111** for storage within the document data store **150**, such as, without limitation, emails, calendar events, notes, database entries, pictures, audio files, etc. In some examples, the untrusted storage device **150** stores a corpus of emails **152**, and the user **12**, via the user device **10**, accesses an inbox for receiving and composing emails. In some implementations, the user device **10** executes a Searchable Encryption (SE) manager **120** for managing access to the encrypted documents **152** within the data storage **150**.

The user **12** may interact with the SE manager **120** via a software application (e.g., a web browser) executing on the user device **10**. A software application (i.e., a software resource) may refer to computer software that causes a computing device to perform a task. In some examples, a software application may be referred to as an "application," an "app," or a "program." Example applications include, but are not limited to, system diagnostic applications, system management applications, system maintenance applications, word processing applications, spreadsheet applications, messaging applications, media streaming applications, social networking applications, and gaming applications.

The SE manager **120** receives, from the user **12**, a search query **122** for one or more keywords **32** that appear in one or more of the encrypted documents **152** stored on the untrusted storage device **150**. The SE manager **120** accesses a count table **210** to obtain a count **212** of unique documents **152** within the corpus of encrypted documents **152** that include the keyword **32**. That is, the count **212** indicates the number of unique documents **152** that the keyword **32** appears in. For example, when the queried keyword **32** is "cat", and "cat" appears in **526** different documents **152** stored on the storage device **150** and associated with the user **12**, the count **212** would be **526**.

Referring now to FIG. **2**, a schematic view **200** shows the SE manager **120** receiving the keyword count **212** of the queried keyword **32** from the count table **210**. The count table **210** includes a count **212** of how many different documents **152** that the keyword **32** appears in. In the illustrated example, the keyword "cat" appears in **526** documents **152**, the keyword "dog" appears in 128 different documents **152**, and the keyword "yak" appears in 12 different documents **152**. In some examples, the count table **210** may be encrypted and the SE manager **120** may decrypt either the count table **210** and/or count **212** using a cryptographic key. As discussed in more detail below with reference to FIG. **7**, the count table **210** may be stored locally at the user device **10** or remotely (e.g., at the untrusted storage device **150**). To maintain privacy, the count table **210** must remain secret, and therefore will generally be encrypted, especially when stored remotely from the user device **10**.

Referring back to FIG. **1**, the SE manager **120** also obtains a private cryptographic key **124**. In some examples, the SE manager **120** generates the private key **124**. In other examples, the SE manager **120** retrieves or receives the private key **124** from the user device **10** or from a third-party (e.g., a third-party key management service). The SE manager **120** generates a delegatable pseudorandom function **126** (DPRF) based on the keyword **32**, the private cryptographic key **124**, and the count **212** of unique documents **152** that include the keyword **32**. When the user **12** queries for more than one keyword **32**, the SE manager **120** may generate a separate DPRF **126** for each keyword **32**.

As used herein, a DPRF is a function that, using an in input cryptographic key K and an input x, generates an output F(K, x) that appears random to any party that does not have access to the key K. Specifically, the DPRF **126** allows for delegation of evaluation of a strict subset of the domain of the function to an untrusted proxy without the proxy being able to evaluate the function outside of the strict subset.

As an example, assume that a user desires to retrieve values stored on a server that are associated with a large number of outputs from the function F. That is, the user wants the server to retrieve or evaluate values associated with $F(K, x_1), \ldots, F(K, x_m)$ that are stored on the server. The user could simply send the function F, the key K, and the range of values for x to the server and the server could evaluate the range of values for x to obtain the outputs. However, in this scenario, the server then could evaluate the function F for any value of x, as the server has access to the key K. Another possible avenue for the user is to evaluate each value of x themselves and then send each output to the server. While this limits the information the server receives, it requires sending of m outputs, which is highly inefficient.

Ideally, the user would like to minimize the amount of information the user must send the server while also minimizing the amount of information the server learns. The DPRF **126**, as described in more detail with regards to FIG. **3** below, is a function that bounds the server from evaluating values of x outside of a specified range, thus limiting the amount of information the sever gains. For example, when the user sends the range values of x_1 to x_m for the sever to evaluate, the server will not be able to evaluate the function F for values of x less than x_1 and for values of x greater than x_m. To establish these bounds, the SE manager **120** evaluates a first portion **126**A of the DPRF **126** and delegates a remaining second portion **126**B of the DPRF to the untrusted storage device **150**.

Referring again to FIG. **2**, the SE manager **120**, in some implementations, includes a DPRF generator **218** and a DPRF evaluator **220**. The DPRF generator **218** generates the DPRF **126** for the queried keyword **32** based on the private key **124**, the keywords **32**, and the keyword count **212** received from the count table **210**. The DPRF generator **218** passes the DPRF **126** to the DPRF evaluator **220**. The DPRF evaluator **220**, as described in more detail below with reference to FIG. **3**, evaluates at least a portion of the DPRF **126** (e.g., a first portion **126***a*), and based on the portion evaluated, delegates (i.e., sends) the remaining second portion **126**B to the untrusted remote storage device **150**.

Referring back to FIG. **1**, the untrusted storage device **150** (i.e., the document data store **150** storing the encrypted documents **152** store), in response to receiving the remaining second portion **126**B of the DPRF **126** delegated by the DPRF evaluator of the SE manager **120**, evaluates the remaining second portion **126**B of the DPRF and accesses an encrypted search index **160** associated with the corpus of encrypted documents **152** stored on the untrusted storage device **150**. The storage device **150** determines one or more encrypted documents **152** within the corpus of encrypted documents that are associated with the remaining second portion **126**B of the DPRF based on the encrypted search index **160**.

The encrypted search index **160**, in some implementations, includes a list of entries **162**, **162***a-n*, where each entry **162** includes an association between a keyword **32** and at least one encrypted document identifier **154** that the keyword **32** appears in. The evaluation of the remaining second portion **126**B provides the untrusted storage device **150** with one or more of the encrypted keywords **32** associated with one or more encrypted document identifiers **154** without revealing the plaintext keyword or document identifier to the storage device **150**. The storage device **150** returns, to the user device **10**, an identifier **154** for each encrypted document **152** of at least a portion of the one or more encrypted documents **152** associated with the remaining second portion **126**B of the DPRF. That is, in some implementations, the storage device **150** does not return every identifier **154** associated with a document **152** containing the queried keyword **32**, and instead only returns a portion (e.g., fifty) of the document identifiers **154**. Subsequent queries **122** made by the user **12** may return additional results (e.g., the next fifty document identifiers **154**). In some examples, the storage device **150** returns to the user device **10** an empty set (i.e., returns no document identifiers **154**) when, for example, the queried keyword **32** does not appear in any of the documents **152**.

In some implementations, when the untrusted storage device **150** returns at least a portion of the document identifiers **154** associated with encrypted documents **152** that includes the queried keyword **32**, the untrusted storage device also returns encrypted metadata **156** associated with each returned identifier **154**. The metadata **156** may include additional relevant or contextual information for the user **12**. For example, the metadata **156** may include dates (e.g., a date the document **152** was created or uploaded), the author of the document **152**, size of the document **152**, a sentence that includes the keyword **32**, etc.

Referring now to FIG. **3**, as previously discussed, the SE manager **120** generates the DPRF **126** to solve for a range of values from $F(K, x_1), \ldots, F(K, x_m)$ by generating a binary tree **300**. In some examples, the key K is associated with a specific keyword **32** and each x value of the DPRF **126** represents one of the documents **152** that the select keyword **32** appears in. For example, if the select keyword **32** is "cat", and the count value **212** associated with "cat" is 526, then cat appears in 526 unique documents **152**. In this example, x would have a maximum size of **526** (e.g., 1 to 526) and each x would represent one of the documents **152** the keyword **32** appears in. Each value of F(K, x) is then associated with a value stored in the encrypted search index **160** that represents a document identifier **154** that the select keyword **32** appears in.

Thus, for the SE manager **120** to retrieve all of the documents **152** with the keyword "cat", the SE manager **120** and/or the untrusted storage device **150** may evaluate the DPRF **126** from F(K, 1), . . . , F(K, 526). Each of the 526 results are associated with a different value stored in the encrypted search index **160**. In another example, the SE manager **120** may retrieve only a portion of the 526 documents **152** that include the keyword "cat". In this examples, the SE manager **120** and/or the untrusted storage device **150** would evaluate only a portion of the DPRF **126**. For instance, to retrieve fifty documents **152**, the SE manager **120** and/or the untrusted storage device **150** may evaluate F(K, 1), . . . , F(K, 50). Each of the fifty results are again associated with a different value stored in the encrypted search index **160**. Similarly, to retrieve the next fifty documents, the SE manager **120** and/or the untrusted storage device **150** may evaluate F(K, 51), . . . , F(K, 100) and so on. In this way, the SE manager **120** and the untrusted storage device **150** may evaluate the DPRF **126** to obtain results associated with values within the encrypted search index **160** (i.e., entries **162**). The untrusted storage device **150** may return all or some of the values associated with the results to the SE manager **120**.

In some implementations, the SE manager **120**, in response to receiving a search query **122**, generates a DPRF **126** associated with the queried keyword **32** by generating the binary tree **300**. In other implementations, the SE manager **120** generates a binary tree **300** for each keyword **32** in the count table **210** prior to receiving a search query **122**. A binary tree is a tree data structure with a plurality of nodes where each node in the structure has at most two children. The binary tree **300** includes a set of nodes **310** that includes a root node **310**R and a plurality of other nodes **310**. The other nodes **310** are either non-leaf nodes **310**NL or leaf nodes **310**L. Each input value of x is uniquely assigned a leaf node **310**L in ascending order. A quantity of leaf nodes **310**L of the binary tree **300** may be equal to or greater than the count of unique documents **152** that include the associated keyword **32**. For example, if the keyword "cat" has a count value 212 of 526, the SE manager **120** may generate a binary tree **300** for the keyword "cat" that has at least **526** leaf nodes **310**L. Each of the **526** instances of "cat" is associated with a specific leaf node **310**L.

Each node **310** is also associated with a value **330**, **330**A-N which herein may be referred to generally as "tokens". In some implementations, the value **330** of each leaf node **310**L is associated with a value within an entry **162** of the encrypted search index **160**. That is, each value **330** of each leaf node **310**L of the binary tree **300** is associated with a value within the encrypted search index **160** that is associated with the corresponding keyword **32**. Returning to the example of the keyword **32** "cat", each of the **526** leaf nodes **310**L in the binary tree **300** generated for the keyword **32** "cat" may be associated with a value stored in the encrypted search index **160** and each of the associated values with the encrypted search index **160** corresponds to a document identifier **154** of a document **152** that includes the keyword **32** "cat".

In some implementations, the value **330** of root node **310**R of the binary tree **300** is a value of a first hash **340** of the private cryptographic key **124** and the keyword **32** associated with the binary tree **300**. Thus, each binary tree **300** will have a unique value **330**R for each root node **310**R for each binary tree **300** generated for a corresponding keyword **32**. Each root node **310**R is associated with a first child node (e.g., node 'B' in FIG. **3**) and a second child node (e.g., node 'C' in FIG. **3**). The first child node includes a first portion **330**B of a second hash **342**, **342***a* of the first hash **340** of the private cryptographic key **124** and the keyword **32**, and the second child node includes a second portion **330**C of the second hash **342** of the first hash **340** of the private cryptographic key **124** and the keyword **32**. That is, in some examples, the value **330**A of the root node **310**R is the first hash **340** of the key **124** and the keyword **32**. This value (labeled 'A' in FIG. **3**) is then hashed (e.g., using SHA256) and the resulting second hash **342***a* is split into the first portion **330**B and the second portion **330**C. As used herein, the terms "hash" and "hash function" are used to indicate any one-way function (i.e., a function where the input cannot be determined from the output) and as such, is equally applicable to encryption operations (e.g., Advanced Encryption Standard (AES)) in addition to hash operations.

In some examples, the first portion **330**B of the second hash **342** concatenated with the second portion **330**C of the second hash **342** is equivalent to the second hash **342** of the first hash **340** of the private cryptographic key **124** and the keyword **32**. As illustrated in FIG. **3**, the second hash **342** (e.g., a SHA256 hash) is a hash of **330**A (i.e., the root node **310**R value **330**A) and is equal to 'B' ||'C' (i.e., value **330**B concatenated with value **330**C). For example, the output of the SHA256 hash is a 256 bit number. The value **330**B may be equivalent to the first 128 bits of the SHA256 output while the value **330**C may be equivalent to the last 128 bits of the SHA256 output. Thus, the value **330**B concatenated with the value **330**C is equivalent to the hash **342** of the value of **330**A.

In some implementations, each other node **310** of the binary tree **300** includes a portion of a hash **342** of a parent node **310** associated with the corresponding other node **310**. That is, for each non-root node **310**R of the binary tree **300** (i.e., all non-leaf nodes **310**NL and all leaf nodes **310**L), the value **330** of the node **310** may be a portion of a hash **342** of the parent node. With continued reference to FIG. **3**, node 'B' (as with root node **310**R node 'A'') has two child nodes **310**, node 'D' and node 'E'. Node 'C' also has 2 child nodes **310**, node 'F' and node 'G'. As node 'D', node 'E', node 'F', and node 'G' have no child nodes **310**, in this example each of these four nodes is a leaf node **310**L. As previously discussed, the value **330**B of node 'B' may be the first portion of the hash **342**A of the value **330**A of node 'A'. Similarly, the value **330**B of node 'B' may be hashed (again with, for example, SHA256) and the resulting hash **342***b* may be split into a first portion **330**D and a second portion **330**E, each assigned as a value **330** of one of the two child nodes **310** (node 'D' and node 'E'). Also as previously discussed, the value **330**C of node 'C' may be the second portion of the hash **342**A of the value **330**A of the node 'A'. Likewise, the value **330**C of node 'C' may be hashed (e.g., with SHA256) and the resulting hash **342***c* may be split into a first portion **330**F and a second portion **330**G, each assigned as a value **330** of one of the two child nodes **310** (node 'F' and node 'G'). While in the illustrated example, the binary tree **300** stops at these nodes, the binary tree may continue on for any number of nodes **310** until there are a sufficient number of leaf nodes **310**L to account for the count value **212** of the associated keyword **32**.

To retrieve all of the document identifiers **154** associated with each leaf node **310**L (i.e., every document identifier **154** associated with a document **152** that includes the queried keyword **32**), the SE manager **120** may simply send the token of node 'A' (e.g., a hash of the key **124** and the keyword **32**) and the count value **212** and allow the untrusted storage device **150** to determine the value for each leaf node **310**L. In the example where the SE manager **120** needs to only retrieve a portion of the documents identifiers **154** associated with the keyword **32**, the SE manager **120** may evaluate the first portion **126**A and delegate just the second portion **126**B to the untrusted storage device **150** to limit the information leaked to the untrusted storage device **150**. For example, when the documents **152** include emails, the user **12**, when querying for a keyword **32**, may receive the **50** most recent emails that include the queried keyword **32** and only if the user indicates a desire for more results will additional emails be returned.

In some implementations, the document identifiers **154** are ordered chronologically (e.g., the document identifier **154** associated with the first leaf node **310**L is the oldest document while the document identifier **154** associated with the last leaf node **310**L is the newest document or vice versa), a range of leaf nodes **310**L starting at the bottom left or the bottom right of the binary tree may be associated with the newest or oldest documents **152** associated with the keyword **32**. This allows for returning only a portion of the document identifiers **154** associated with the queried keyword **32** (e.g., the fifty most recent documents **152**) without the need look up each keyword **32** instance in the search index **160**. This may drastically reduce the total amount of computation required. While in this example, chronological ordering is illustrated, the document identifiers **154** may of course be ordered based on any other desired criteria.

With continued reference to FIG. **3**, in the example where the SE manager **120** needs only to retrieve the document identifiers **154** associated with the tokens **330**D, **330**E of node 'D' and node 'E', it is ideal to refrain from giving the untrusted storage device the information necessary to determine the values of node 'F' and node 'G', as these nodes are unnecessary for the query **122**. In this case, the SE manager **120** may evaluate a first subset of the nodes **310** of the binary tree **300** and the untrusted storage device **150** may evaluate a second subset of the nodes **310** of the binary tree **300** that is different from the subset that that the SE manager **120** evaluated.

For example, when the SE manager **120**, instead of providing the untrusted storage device **150** with the value **330**A of the root node **310**R, provides the untrusted storage device **150** with the value **330**B of node 'B', the untrusted storage device **150** may evaluate the DPRF **126** (e.g., the binary tree **300**) using the token **330**B of node 'B' to obtain the values **330**D, **330**E of the leaf nodes **310**L node 'D' and node 'E'. Because the hash function used to obtain the token **330**B is a one-way function, the untrusted storage device **150** is not able to use that value to obtain the value **330**A of the root node **310**R and thus the tokens **330**C, **330**F, **330**G of node 'C', node 'F', and node 'G'. Thus, by determining a minimal number of nodes **310** whose union of leaf nodes **310**L covers exactly (and only) the set of values **330** that correspond to the range of document identifiers **154** to be retrieved, the amount of information provided to the untrusted storage device **150** is minimized while bandwidth requirements are kept low. To return additional document identifiers **154**, the SE manager **120** may follow up by sending additional values **330** to the untrusted storage device (e.g., the value **330**C of node 'C' to obtain the values **330**F, **330**G of node 'F' and node 'G').

In some implementations, each entry **162** of the encrypted index **160** is an association between exactly one keyword **32** and one document identifier **154**. However, in some implementations, the search index **160** may be optimized without reducing privacy. Instead of each entry **162** of the encrypted index **160** including an association between one keyword **32** and one document identifier **154**, each entry **162** may include an association between one keyword **32** and a plurality of document identifiers **154**. That is, each entry **162** associates a keyword **32** to multiple document identifiers **154** that the keyword **32** appears in. Note that if there was no limit to how many document identifiers **154** each entry **162** could associate with a single keyword **32**, the search index would risk leaking frequency table information. To mitigate this risk, each entry **162** may be limited to a maximum number of document identifiers. For example, each entry **162** may be limited to fifty or one hundred document identifiers **154**. In practice, this ensures that keywords with large frequencies (i.e., appear in many documents **152**) will be split into many different entries **162** in the search index **160**.

In some examples, the maximum number of document identifiers may be dynamically changed based on the frequency of the keywords **32**. As the frequency of the keyword **32** increases (i.e., the keyword **32** is more common in the documents **154**), the size of the maximum number of document identifiers may increase. As a result, the untrusted storage device **150** does not have to process as many hashes. The count table **210** may be used to keep track of the maximum number of document identifiers for each keyword **32** as well as the number of document identifiers **154** currently associated with each entry **162**. Optionally, instead of the count table **210** tracking the number of document identifiers **154** currently associated with each entry **162**, the SE manager **120**, each time a new keyword **32** is added, a SE manager **120** may create new entry **162** and add the keyword **32** to the new entry **162** based on a keyword probability. This leads to, on average, an expected number of document identifiers **154** to be added to the entry **1622** prior to the creation of another new entry **162**. In this way, the count table **210** does not need to track the number of document identifiers **154** assigned to each entry **162**, thus reducing the size of the count table **210**.

Referring now to the schematic view **400** of FIG. **4**, in some examples, the SE manager **120** receives a disjunctive, conjunctive, or negation search query **122**D, **122**C, **122**N. A disjunctive query **122**D includes a query of two or more keywords **32** combined with a logical OR. For example, a disjunctive query **122**D may include a query for "cat" OR "dog" and should result in returning any document identifiers **154** associated with documents **152** that include either or both the keyword "cat" and the keyword "dog". For disjunctive queries **122**D, the SE manager **120** may generate a DPRF **126** and a corresponding portion **126**B, **126**Ba-n for each keyword **32** separately. After receiving the document identifiers **154** for each keyword **32** at the user device **10**, the SE manager **120** may combine the results and, in some implementations, rank the results using any metadata **156** returned with the document identifiers **154**.

A conjunctive query **122**C includes a query of two or more keywords **32** combined with a logical AND. For example, a conjunctive query **122**C may include a query for "cat" AND "dog" and should result in returning any document identifiers **154** that are associated with documents **152** that include both "cat" and "dog". Similar to the disjunctive query **122**D, for conjunctive queries **122**C, the SE manager **120** may generate a DPRF **126** and a corresponding portion **126**B for each keyword **32** separately. After receiving the document identifiers **154** for each keyword **32** at the user device **10**, the SE manager **120** may return to the user **12** only document identifiers **154** that were returned for each keyword **32**.

A negation query **122**N includes a query for results that do not include one or more keywords **32**. For example, a negation query **122**N may include a query for all documents **152** that do not include the keyword “cat.” For negation queries **122**N, the SE manager **120** may generate a DPRF **126** and corresponding portion **126**B for the negated keyword **32**. After receiving the results for the negated keyword **32**, the SE manager **120** may retrieve all document identifiers **154** and remove from the list the identifiers **154** associated with the negated keyword **32**, and return the remaining results to the user **12**. Using the above described methods for disjunctive queries **122**D, conjunctive queries **122**C, and negation queries **122**N, complex queries **122** that combine or include multiple different types of queries may be resolved with the same techniques by splitting the complex query into multiple simpler queries.

Referring now to FIG. **5**, in some examples, the system **100** shows the user **12** adding/uploading a new document **152**N to the corpus of encrypted documents **152** stored on the untrusted storage device **150**. In this situation, the encrypted search index **160** is updated with the keywords **32** present in the newly added document **152**. The new document **152**N is associated with a new document identifier **154**N. In some implementations, for each unique keyword **32** of the new encrypted document **152**N uploaded by the user **12** into the corpus of encrypted documents **152** stored on the untrusted storage device **150**, the SE manager **120** increments the count **212** of unique documents **152** within the corpus of encrypted documents **152** that include the corresponding unique keyword **32** in the count table **210**. For example, when the new document **152**N includes the keyword “cat”, and the current count **212** associated with the keyword “cat” is 526, the count **212** is incremented to 527.

The SE manager **120**, in some examples, generates a unique keyword hash **520** based on the private cryptographic key **124**, the corresponding unique keyword **32**, and the incremented count **212** of unique documents **152** within the corpus of encrypted documents that include the corresponding unique keyword **32**. For example, the SE manager **120** may use a hash function **510** to compute $H_{kw}=F(K\|kw, cnt_{kw})$, where H_{kW} represents the hash value **520**, K represents the private key **126**, kw represents the keyword **32**, and cnt_{kw} represents the incremented count **212**. Any suitable one-way function or algorithm may be used to hash or encrypt the keyword **32** (e.g., SHA256).

The SE manager **120** may also generate a hash pair **522** that includes the unique keyword hash **520** and an encrypted document identifier **154**N (i.e., the SE manager **120** hashes or encrypts the new document identifier **154**N) associated with the new encrypted document **152** uploaded by the user **12**. The SE manager **120** sends the hash pair **522** to the untrusted storage device **150**. The SE manager **120** may generate a separate and unique hash pair **522** for each unique keyword **32** within the newly uploaded document **152**N.

Draft documents **152** (e.g., emails that are saved without sending or are actively being composed) are typically saved frequently (e.g., every few seconds) by the user device **10**. The SE manager **120** may update the search index **160** at the same frequency as the draft is saved or at a different frequency. For example, when the draft is saved every 5 seconds, the SE manager **120** may update the encrypted search index **160** every 5 minutes. In some implementations, the SE manager **120** may update the encrypted search index **160** at the same rate as the draft is saved, but update the count table **210** at a slower frequency. In this case, tokens **330** may temporarily be reused for updating the search index **160** until the count table **210** is updated at a future time.

When the documents **152** stored on the untrusted storage device **150** are emails, the SE manager **120** may automatically add received emails at the user device **10** to the corpus of encrypted emails on the untrusted storage device. In some examples, emails that have been received, but not yet opened, are not added to the search index **160**. That is, in some examples, the SE manager **120** automatically adds opened emails to the search index **160**. In this way, an email may be revoked by the sender without the SE manager **120** and/or the untrusted storage device **150** inferring content of the revoked email from the keywords **32**.

Referring now to FIG. **6**, similar to adding a document **152**, the system **100** shows the SE manager **120**, in some implementations, receiving a deletion request **630** to delete a document **152** from the untrusted storage device **150**. In this case, the SE manager **120** retrieves each keyword **32** present in the document **152** to be deleted (e.g., from the untrusted storage device **150**) and, for each keyword **32**, decrements the corresponding count **212** in the count table **210**. The SE manager then instructs the untrusted storage device to delete the values within the encrypted search index associated with the deleted document **152**D. For example, the SE manager **120** may generate a hash **620** of the private key **124**, the keyword **32**, and the appropriate count **212** (or other identifier) using a hash function **610** to generate a hash pair **622** with the document identifier **154**. The SE manager **120** may send the hash pairs **622** to the untrusted storage device **150** to indicate to the untrusted storage device which entries within the encrypted search index **160** to delete. The untrusted storage device **150** may run a periodic task to update the search index **160** at regular intervals. In some implementations, the untrusted storage device **150** keeps a list of all document identifiers **154** of deleted documents **152**, and prior to returning results from a search query **122**, removes any document identifiers **154** that are associated with deleted documents **152**.

Optionally, the untrusted storage device **150** may periodically compress (e.g., perform garbage collection) the search index **160** after one or more documents **152** have been deleted. After a document is deleted, the deleted document may create a “hole” at the count **212** associated with the deleted document **152**. The untrusted storage device **150** may move or shift entries in the search index **160** with higher counts **212** to ones of lower counts as the lower counts become available from document deletions. The resulting empty higher count entries may then be deleted from the search index **160**.

In some scenarios, the user **12** may desire to delete portions of a document **152** without deleting the entire document **152**. In this situation, some keywords **32** are removed from the document **152** and the encrypted search index **160** no longer accurately reflects the keywords **32** present in the modified documents **152**. In some implementations, a deletion index **660** includes reference to keywords **32** deleted from documents **152** stored within the corpus of encrypted documents on the untrusted data storage **150**. The deletion index **660** may be generated and maintained similarly to when new document keywords **32** are added to the search index **160**. Prior to the untrusted storage device **150** returning the document identifiers **154** associated with the queried keyword, the untrusted storage device may reference the deletion index **660** to determine if the deletion index **660** indicates that any of the document identifiers **154** include keywords **32** that have been deleted. The untrusted storage device **150** may remove document identifiers **154** that the deletion index indicates the queried keyword **32** was deleted from.

In order to prevent zero-day leakage (e.g., frequency table attacks), it is important that the plaintext of the count table

210 is not available to anyone other than the user **12**. However, it is also desirable that the user **12** have easy access to the count table **210** from a variety of user devices **10** simultaneously. There are a variety of methods for storing the count table **210** that address these concerns to varying degrees. For example, the count table may be stored only locally on the user device **10**. However, this implementation has significant drawbacks in that the user is limited to only the user device **10** that the count table **210** is stored on, and it would be difficult if not impossible to recover the count table **210** if the user device **10** loses it (e.g., the user device **10** crashes).

Another implementation is storing the count table **210** in an encrypted format on the untrusted storage device **150**. The count table **210** may be encrypted with a second private cryptographic key that is different from the private cryptographic key **124**, or alternatively the count table **210** may be encrypted with the same private key **124**. The user device **10** may then, when performing a query, first download the encrypted count table **210** from the untrusted storage server **150**, decrypt it, and perform the query. The user device **10** may send to the untrusted storage device **150** an updated count table **210** each time a document **152** is added or removed from the corpus of encrypted documents. This allows for synchronization between multiple user devices **10** and ensures backups in case a user device crashes, however, the bandwidth requirements may be significant, especially for some user devices (e.g., mobile phones). At the cost of greatly increased complexity, the untrusted storage device **150** may instead store incremental backups of the count table **210**. For example, the backup may be uploaded at regular intervals (e.g., once a day or every few hours). User devices may upload changes to the count table **210** (e.g., adding or deleting a document **152**) and the untrusted storage device **150** may track these changes to the count table **210** until the next backup upload.

Yet another implementation for storing the count table **210** involves storing an encrypted count table **210** on the untrusted storage device **150** and accessing encrypted entries of the count table **210**. For example, for each keyword **32**, the untrusted storage device **150** may store an identifier encrypted with a unique key that points to an encryption of the count **212** for that keyword. When the user **12** adds a document **152**, the user **12** requests the untrusted storage device to return the encrypted counts **212** associated with the identifier. The user device **10** may then perform a search as described above using the recovered counts **212**, and then send encrypted incremented counts back to the untrusted storage device **150** for the untrusted storage device **150** to update. This implementation provides protection from crashed user devices and minimizes the bandwidth required. However, logs of accessing the encrypted counts, if not properly deleted, may leak frequency information. This frequency information may allow for the generation of a frequency table which may be used in an attack.

In yet another implementation, the count table **210** is instead replaced with a single max count integer. The max count integer may be set to the largest count **212**. That is, the max count integer may be set the count **212** of the keyword **32** with the highest count **212** (i.e., appears in the most documents **152**). When searching for a keyword **32**, the SE manager **120** may delegate to the untrusted storage device **150** a DPRF **126** over the entire range up to the max count integer. The untrusted storage device may perform a search (e.g., a binary search) over the encrypted search index **160** to obtain the actual count **212** of the queried keyword **32**. For example, the untrusted storage device **150** may determine that the largest count value that matches a result in the encrypted search index **160** is the actual count **212** of the keyword. This implementation removes the need for the count table **210**, but increases the number of lookups the untrusted storage device **150** must perform on the encrypted search index **160** while also potentially degrading privacy, as logs of the search may leak a frequency of counts of keywords **32**.

In yet another implementation, the count table **210** is partitioned into a plurality of different access buckets. Here, the partitioning may use k-anonymity, whereby k-anonymity refers to a property of anonymized data where a specific member of a population cannot be readily identified or distinguished from the data.

Referring now to the schematic view **700** of FIG. **7**, in some implementations, the SE manager **120** divides the count table **210** into a plurality of buckets **710**, **710***a-n* and stores the buckets **710** on the untrusted storage device **150**. Here, each bucket **710** stores one or more counts **212** of unique documents **152** within the corpus of encrypted documents **152** that include a respective keyword **32**. That is, each keyword **32** and associated count **212** pair **712**, **712***a-n* (e.g., "cat" and **526**) are encrypted and assigned to a bucket **710** and each bucket is stored on the untrusted storage device **150**. The untrusted storage device **150** may host any number of buckets **710** and each bucket **710** may store any number of keyword-count pairs **712**, however each keyword-count pair **712** is only assigned to a single bucket **710**. The SE manager **120** may request a specific pair **712** (e.g., a count **212** for a specific keyword **32**) by generating and sending a bucket request **720** to the untrusted storage device **150** that indicates a specific bucket **710** of the plurality of buckets **710**. In response, the untrusted storage device **150** returns each pair **712** stored in the specific bucket **710**. In this way, the untrusted storage device **150** cannot discern the specific pair **712** from the bucket of pairs that the untrusted storage device **150** returned to the SE manager **120**. The SE manager **120** may determine which bucket **710** a pair **712** is assigned to by generating second DPRF **726** whose output domain is simply the number of buckets **710**.

The bandwidth required for bucketization is balanced against the strength of the anonymity the bucketization provides. That is, the greater the number of keyword and count pairs **712** per bucket **710** (i.e., when the total number of buckets **710** is small), the greater number of pairs **712** returned for each query **122**, the greater the anonymity, and the greater the bandwidth consumption. Conversely, the fewer the number of keyword and count pairs **712** per bucket **710** (i.e., when the total number of buckets **710** is large), the fewer number of pairs **712** returned for each query **122**, the less the anonymity, and the less the bandwidth consumption. This implementation ensures that, even if logs generated by the untrusted storage device are not deleted, the leakage is mitigated by the k-anonymity techniques. In particular, the leakage of frequencies occur at the granularity of buckets (which typically will contain k encrypted pairs **712**) and therefore the frequency leakage only leaks frequencies for groups of approximately k keywords **32**.

In some examples, the total number of buckets **710** is fixed. That is, the number of buckets **710** in use does not change and new keyword count pairs **712** are continually added to the same buckets **710**. Over time, as the number of keyword count pairs **712** per bucket increases, the overall bandwidth consumption of the bucketization technique similarly increases. In other examples, the number of buckets **710** is not fixed (i.e., dynamic bucketization). In this case, the output domain of the second DPRF **726** is a maximum number of buckets that may be deployed (e.g., **1024**). As with the fixed bucket implementation, the second DPRF **726** is used to assign the keyword count pair **712** to the buckets **710**. To reduce the number of bucket **710** from the maximum amount assigned by the second DPRF **726** to a desired amount, different possible outputs of the second DPRF **726** may be combined into a single bucket **710**. That is, two or more buckets **710** may be dynamically associated together.

For example, if 1,024 is the maximum number of buckets, but the target number of buckets is 64, every 16 buckets **710** may be combined, such that when a keyword-count pair **712** from one of the 16 buckets is requested, the untrusted storage device **150** will return all of the pairs **712** from each of the 16 buckets. Note that each group of buckets **710** does not have to constitute the same number of buckets **710**. For example, one group may be 16 buckets, while another group is 32 buckets. To increase or decrease the number of buckets **710**, the SE manager **120** may simply change the number of buckets **710** that are combined. This allows the SE manager **120** to dynamically change the number of buckets **710** in use without physically changing the underlying count table **210**. When the count table **210** is stored in a sorted order, dynamic bucketization also ensures that counts **212** that are placed into the same bucket **710** are logically nearby for efficiency purposes.

FIG. **8** shows a plot **800** depicting a likelihood of inserting a new keyword **32** into the count table **210** when a probability **810** to enter keyword is 0.02. The plot **800** has an x-axis denoting a number of documents **152** with the same new keyword **32** and a y-axis denoting a probability or likelihood that the new keyword **32** is added to the count table **210**. As is apparent from the plot **800**, as the number of documents **152** with the new keyword **32** approaches 200, the probability that the keyword **32** is entered approaches 100 percent. In some implementations, a size of the count table **210** is reduced by adding new keywords **32** to the count table **210** based on a probability. That is, when a new document **152**N (FIG. **5**) is added to the corpus of encrypted documents stored on the untrusted storage device **150**, when the new document **152**N contains a keyword **32** that is not already in the count table **210**, the SE manager **120** may determine whether to add the keyword **32** to the count table **210** based on a probability **810**. For example, the probability **810** that a new keyword **32** is added to the count table **210** may be 1 in 50 (i.e., 2 percent). When the SE manager **120** determines, based on the probability **810**, that the keyword **32** is to be added to the count table **210** (e.g., 2% of the time), the keyword **32** is added as described with regards to FIG. **5**. When the SE manager **120** determines, based on the probability **810**, that the keyword **32** is not to be added to the count table **210**, the SE manager **120** may instead randomly assign the keyword **32** to a token **330** within a threshold range. In some examples, the threshold range may be the default number of documents identifiers **154** that are retrieved in response to a search query (e.g., fifty).

For example, when the SE manager **120** determines to not add a new keyword **32** to the count table **210**, the SE manager **120** may instead generate a hash pair **522** as described with regard to FIG. **5** using a random count value **212** between one and fifty. The new keyword **32**, as it is used in additional documents, will eventually be added to the count table **210** (i.e., eventually, based on the probability **810**, the keyword **32** will be added to the count table **210**).

While there is a chance that some tokens **330** will be used for multiple documents **152**, i.e., when randomly selecting the count value **212** between 1 and 50, the same number is randomly selected more than once, due to the nature of the infrequent keyword **32** and the strong likelihood that the keyword **32** will eventually be added to the count table **210**, the amount of information leaked from sharing the token **330** is minimal. At most, the untrusted storage device **150** may learn that each document **152** that shares the same token **300** has a keyword **32** in common. The untrusted storage device **150** does not learn what the keyword **32** is or the total number of documents **152** that include the keyword **32**. This technique may drastically reduce the size of the count table **210**, as rarely used keywords (e.g., symbols, acronyms, names, etc.) will not be included. This decreases both the storage cost of storing the count table **210** and the communication costs during count table operations (e.g., with regards to FIG. **7**).

FIG. **9** is a flowchart of an exemplary arrangement of operations for a method **900** of providing encrypted search with no zero-day leakage. The method **900** includes, at step **902**, receiving, at data processing hardware **18** of a user device **10** associated with a user **12**, a search query **122** for a keyword **32**. The keyword **32** appears in one or more encrypted documents **152** within a corpus of encrypted documents **152** stored on an untrusted storage device **150**. The method **900** includes, at step **904**, accessing, by the data processing hardware **18**, a count table **210** to obtain a count **212** of unique documents **152** within the corpus of encrypted documents **152** that include the keyword **32** and, at step **906**, generating, by the data processing hardware **18**, a delegatable pseudorandom function (DPRF) **126** based on the keyword **32**, a private cryptographic key **124**, and the count **212** of unique documents **152** that include the keyword **32**.

At step **908**, the method **900** includes evaluating, by the data processing hardware **18**, a first portion of the DPRF **126**A, and at step **910**, delegating, by the data processing hardware **18**, a remaining second portion of the DPRF **126**B to the untrusted storage device **150**. The remaining second portion of the DPRF, when received by the untrusted storage device **150**, causes the untrusted storage device **150** to, at step **912**, evaluate the remaining second portion of the DPRF **126**B, access an encrypted search index **160** associated with the corpus of encrypted documents **152** stored on the untrusted storage device **150**, and determine one or more encrypted documents **152** within the corpus of encrypted documents **152** associated with the remaining second portion of the DPRF **126**B based on the encrypted search index **160**. The untrusted storage device **150** also returns, to the user device **10**, an identifier **154** for each encrypted document **152** of at least a portion of the one or more encrypted documents **152** associated with the remaining second portion of the DPRF **126**B.

FIG. **10** is schematic view of an example computing device **1000** that may be used to implement the systems and methods described in this document. The computing device **1000** is intended to represent various forms of digital computers, such as laptops, desktops, workstations, personal digital assistants, servers, blade servers, mainframes, and other appropriate computers. The components shown here, their connections and relationships, and their functions, are meant to be exemplary only, and are not meant to limit implementations of the inventions described and/or claimed in this document.

The computing device **1000** includes a processor **1010**, memory **1020**, a storage device **1030**, a high-speed interface/controller **1040** connecting to the memory **1020** and high-speed expansion ports **1050**, and a low speed interface/controller **1060** connecting to a low speed bus **1070** and a storage device **1030**. Each of the components **1010**, **1020**, **1030**, **1040**, **1050**, and **1060**, are interconnected using various busses, and may be mounted on a common motherboard or in other manners as appropriate. The processor **1010** can process instructions for execution within the computing device **1000**, including instructions stored in the memory **1020** or on the storage device **1030** to display graphical information for a graphical user interface (GUI) on an external input/output device, such as display **1080** coupled to high speed interface **1040**. In other implementations, multiple processors and/or multiple buses may be used, as appropriate, along with multiple memories and types of memory. Also, multiple computing devices **1000** may be connected, with each device providing portions of the necessary operations (e.g., as a server bank, a group of blade servers, or a multi-processor system).

The memory **1020** stores information non-transitorily within the computing device **1000**. The memory **1020** may be a computer-readable medium, a volatile memory unit(s), or non-volatile memory unit(s). The non-transitory memory **1020** may be physical devices used to store programs (e.g., sequences of instructions) or data (e.g., program state information) on a temporary or permanent basis for use by the computing device **1000**. Examples of non-volatile memory include, but are not limited to, flash memory and read-only memory (ROM)/programmable read-only memory (PROM)/erasable programmable read-only memory (EPROM)/electronically erasable programmable read-only memory (EEPROM) (e.g., typically used for firmware, such as boot programs). Examples of volatile memory include, but are not limited to, random access memory (RAM), dynamic random access memory (DRAM), static random access memory (SRAM), phase change memory (PCM) as well as disks or tapes.

The storage device **1030** is capable of providing mass storage for the computing device **1000**. In some implementations, the storage device **1030** is a computer-readable medium. In various different implementations, the storage device **1030** may be a floppy disk device, a hard disk device, an optical disk device, or a tape device, a flash memory or other similar solid state memory device, or an array of devices, including devices in a storage area network or other configurations. In additional implementations, a computer program product is tangibly embodied in an information carrier. The computer program product contains instructions that, when executed, perform one or more methods, such as those described above. The information carrier is a computer- or machine-readable medium, such as the memory **1020**, the storage device **1030**, or memory on processor **1010**.

The high speed controller **1040** manages bandwidth-intensive operations for the computing device **1000**, while the low speed controller **1060** manages lower bandwidth-intensive operations. Such allocation of duties is exemplary only. In some implementations, the high-speed controller **1040** is coupled to the memory **1020**, the display **1080** (e.g., through a graphics processor or accelerator), and to the high-speed expansion ports **1050**, which may accept various expansion cards (not shown). In some implementations, the low-speed controller **1060** is coupled to the storage device **1030** and a low-speed expansion port **1090**. The low-speed expansion port **1090**, which may include various communication ports (e.g., USB, Bluetooth, Ethernet, wireless Ethernet), may be coupled to one or more input/output devices, such as a keyboard, a pointing device, a scanner, or a networking device such as a switch or router, e.g., through a network adapter.

The computing device **1000** may be implemented in a number of different forms, as shown in the figure. For example, it may be implemented as a standard server **1000*a*** or multiple times in a group of such servers **1000*a***, as a laptop computer **1000*b***, or as part of a rack server system **1000*c***.

Various implementations of the systems and techniques described herein can be realized in digital electronic and/or optical circuitry, integrated circuitry, specially designed ASICs (application specific integrated circuits), computer hardware, firmware, software, and/or combinations thereof. These various implementations can include implementation in one or more computer programs that are executable and/or interpretable on a programmable system including at least one programmable processor, which may be special or general purpose, coupled to receive data and instructions from, and to transmit data and instructions to, a storage system, at least one input device, and at least one output device.

These computer programs (also known as programs, software, software applications or code) include machine instructions for a programmable processor, and can be implemented in a high-level procedural and/or object-oriented programming language, and/or in assembly/machine language. As used herein, the terms "machine-readable medium" and "computer-readable medium" refer to any computer program product, non-transitory computer readable medium, apparatus and/or device (e.g., magnetic discs, optical disks, memory, Programmable Logic Devices (PLDs)) used to provide machine instructions and/or data to a programmable processor, including a machine-readable medium that receives machine instructions as a machine-readable signal. The term "machine-readable signal" refers to any signal used to provide machine instructions and/or data to a programmable processor.

The processes and logic flows described in this specification can be performed by one or more programmable processors, also referred to as data processing hardware, executing one or more computer programs to perform functions by operating on input data and generating output. The processes and logic flows can also be performed by special purpose logic circuitry, e.g., an FPGA (field programmable gate array) or an ASIC (application specific integrated circuit). Processors suitable for the execution of a computer program include, by way of example, both general and special purpose microprocessors, and any one or more processors of any kind of digital computer. Generally, a processor will receive instructions and data from a read only memory or a random access memory or both. The essential elements of a computer are a processor for performing instructions and one or more memory devices for storing instructions and data. Generally, a computer will also include, or be operatively coupled to receive data from or transfer data to, or both, one or more mass storage devices for storing data, e.g., magnetic, magneto optical disks, or optical disks. However, a computer need not have such devices. Computer readable media suitable for storing computer program instructions and data include all forms of non-volatile memory, media and memory devices, including by way of example semiconductor memory devices, e.g., EPROM, EEPROM, and flash memory devices; magnetic disks, e.g., internal hard disks or removable disks; magneto optical disks; and CD ROM and DVD-ROM disks. The processor and the memory can be supplemented by, or incorporated in, special purpose logic circuitry.

To provide for interaction with a user, one or more aspects of the disclosure can be implemented on a computer having a display device, e.g., a CRT (cathode ray tube), LCD (liquid crystal display) monitor, or touch screen for displaying information to the user and optionally a keyboard and a pointing device, e.g., a mouse or a trackball, by which the user can provide input to the computer. Other kinds of devices can be used to provide interaction with a user as well; for example, feedback provided to the user can be any form of sensory feedback, e.g., visual feedback, auditory feedback, or tactile feedback; and input from the user can be received in any form, including acoustic, speech, or tactile input. In addition, a computer can interact with a user by sending documents to and receiving documents from a device that is used by the user; for example, by sending web pages to a web browser on a user's client device in response to requests received from the web browser.

A number of implementations have been described. Nevertheless, it will be understood that various modifications may be made without departing from the spirit and scope of the disclosure. Accordingly, other implementations are within the scope of the following claims.

What is claimed is:

1. A computer-implemented method when executed by data processing hardware causes the data processing hardware to perform operations comprising:
- receiving a search query for a keyword appearing in one or more encrypted documents within a corpus of encrypted documents stored on an untrusted storage device;
- generating a delegatable pseudorandom function (DPRF) based on the keyword, a private cryptographic key, and a count of unique documents within the corpus of encrypted documents that include the keyword;
- evaluating a first portion of the DPRF;
- delegating a remaining second portion of the DPRF to the untrusted storage device; and
- receiving, from the untrusted storage device, an identifier for each encrypted document of at least a portion of the one or more encrypted documents associated with the remaining second portion of the DPRF.

2. The computer-implemented method of claim 1, wherein the operations further comprise accessing a count table to obtain the count of unique documents within the corpus of encrypted documents that include the keyword.

3. The computer-implemented method of claim 1, wherein delegating the remaining second portion of the DPRF to the untrusted storage device causes the untrusted storage device to:
- evaluate the remaining second portion of the DPRF;
- access an encrypted search index associated with the corpus of encrypted documents stored on the untrusted storage device; and
- determine one or more encrypted documents within the corpus of encrypted documents associated with the remaining second portion of the DPRF based on the encrypted search index.

4. The computer-implemented method of claim 1, wherein generating the DPRF comprises:
- generating a binary tree comprising a set of nodes that includes a root node and a plurality of other nodes, each other node comprising a non-leaf node or a leaf node,
- wherein a quantity of leaf nodes of the binary tree is equal to or greater than the count of unique documents that include the keyword.

5. The computer-implemented method of claim 4, wherein:
- the root node of the binary tree comprises a first hash of the private cryptographic key and the keyword; and
- the root node is associated with a first child node and a second child node, the first child node comprising a first portion of a second hash of the first hash of the private cryptographic key and the keyword, and the second child node comprising a second portion of the second hash of the first hash of the private cryptographic key and the keyword.

6. The computer-implemented method of claim 5, wherein the first portion of the second hash concatenated with the second portion of the second hash is equivalent to the second hash of the first hash of the private cryptographic key and the keyword.

7. The computer-implemented method of claim 4, wherein each leaf node of the set of nodes of the binary tree is associated with a value stored in an encrypted search index.

8. The computer-implemented method of claim 4, wherein each other node of the set of nodes of the binary tree comprises a portion of a hash of a parent node associated with the corresponding other node.

9. The computer-implemented method of claim 4, wherein evaluating the first portion of the DPRF comprises evaluating a first subset of the set of nodes of the binary tree.

10. The computer-implemented method of claim 9, wherein the untrusted storage device evaluates a second subset of the set of nodes of the binary tree, the second subset comprising different nodes from the set of nodes of the binary tree than the first subset.

11. A system comprising:
- data processing hardware of a user device associated with a user; and
- memory hardware in communication with the data processing hardware, the memory hardware storing instructions that when executed on the data processing hardware cause the data processing hardware to perform operations comprising:
  - receiving a search query for a keyword appearing in one or more encrypted documents within a corpus of encrypted documents stored on an untrusted storage device;
  - generating a delegatable pseudorandom function (DPRF) based on the keyword, a private cryptographic key, and a count of unique documents within the corpus of encrypted documents that include the keyword;
  - evaluating a first portion of the DPRF;
  - delegating a remaining second portion of the DPRF to the untrusted storage device; and
  - receiving, from the untrusted storage device, an identifier for each encrypted document of at least a portion of the one or more encrypted documents associated with the remaining second portion of the DPRF.

12. The system of claim 11, wherein the operations further comprise accessing a count table to obtain the count of unique documents within the corpus of encrypted documents that include the keyword.

13. The system of claim 11, wherein delegating the remaining second portion of the DPRF to the untrusted storage device causes the untrusted storage device to:
- evaluate the remaining second portion of the DPRF;
- access an encrypted search index associated with the corpus of encrypted documents stored on the untrusted storage device; and
- determine one or more encrypted documents within the corpus of encrypted documents associated with the remaining second portion of the DPRF based on the encrypted search index.

14. The system of claim 11, wherein generating the DPRF comprises:

generating a binary tree comprising a set of nodes that includes a root node and a plurality of other nodes, each other node comprising a non-leaf node or a leaf node,

wherein a quantity of leaf nodes of the binary tree is equal to or greater than the count of unique documents that include the keyword.

15. The system of claim 14, wherein:

the root node of the binary tree comprises a first hash of the private cryptographic key and the keyword; and

the root node is associated with a first child node and a second child node, the first child node comprising a first portion of a second hash of the first hash of the private cryptographic key and the keyword, and the second child node comprising a second portion of the second hash of the first hash of the private cryptographic key and the keyword.

16. The system of claim 15, wherein the first portion of the second hash concatenated with the second portion of the second hash is equivalent to the second hash of the first hash of the private cryptographic key and the keyword.

17. The system of claim 14, wherein each leaf node of the set of nodes of the binary tree is associated with a value stored in an encrypted search index.

18. The system of claim 14, wherein each other node of the set of nodes of the binary tree comprises a portion of a hash of a parent node associated with the corresponding other node.

19. The system of claim 14, wherein evaluating the first portion of the DPRF comprises evaluating a first subset of the set of nodes of the binary tree.

20. The system of claim 19, wherein the untrusted storage device evaluates a second subset of the set of nodes of the binary tree, the second subset comprising different nodes from the set of nodes of the binary tree than the first subset.

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