US010385656B2

(12) United States Patent
Samuel et al.

(10) **Patent No.: US 10,385,656 B2**
(45) **Date of Patent: Aug. 20, 2019**

(54) **METHODS AND SYSTEMS FOR DETERMINING MANUFACTURING AND OPERATING PARAMETERS FOR A DEVIATED DOWNHOLE WELL COMPONENT**

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(*) Notice: Subject to any disclaimer, the term of this patent is extended or adjusted under 35 U.S.C. 154(b) by 613 days.

(21) Appl. No.: **14/895,165**

(22) PCT Filed: **Jan. 17, 2014**

(86) PCT No.: **PCT/US2014/011998**
§ 371 (c)(1),
(2) Date: **Dec. 1, 2015**

(87) PCT Pub. No.: **WO2014/204521**
PCT Pub. Date: **Dec. 24, 2014**

(65) **Prior Publication Data**

US 2016/0108704 A1 Apr. 21, 2016

Related U.S. Application Data

(60) Provisional application No. 61/837,986, filed on Jun. 21, 2013.

(51) **Int. Cl.**

E21B 41/00	(2006.01)
E21B 7/04	(2006.01)

(Continued)

(52) **U.S. Cl.**
CPC ***E21B 41/00*** (2013.01); ***E21B 7/04*** (2013.01); ***E21B 17/00*** (2013.01); ***G06F 17/16*** (2013.01)

(58) **Field of Classification Search**
CPC . E21B 41/00; E21B 7/04; E21B 17/00; E21B 17/003; E21B 17/16; E21B 49/00; G06F 17/16
See application file for complete search history.

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(57) **ABSTRACT**

Systems and methods for determining manufacturing or operating parameters for a deviated downhole well component, including a method that includes representing a tubular string as nodes separated by segments, determining transfer matrices for determining an i^{th} node's state vector from an $i^{th}-1$ node's state vector, and defining initial state vector values for the reference node. The nodes are numerable from 1 to N with an initial, mechanically constrained reference node representable with i=0, and each is associated with a

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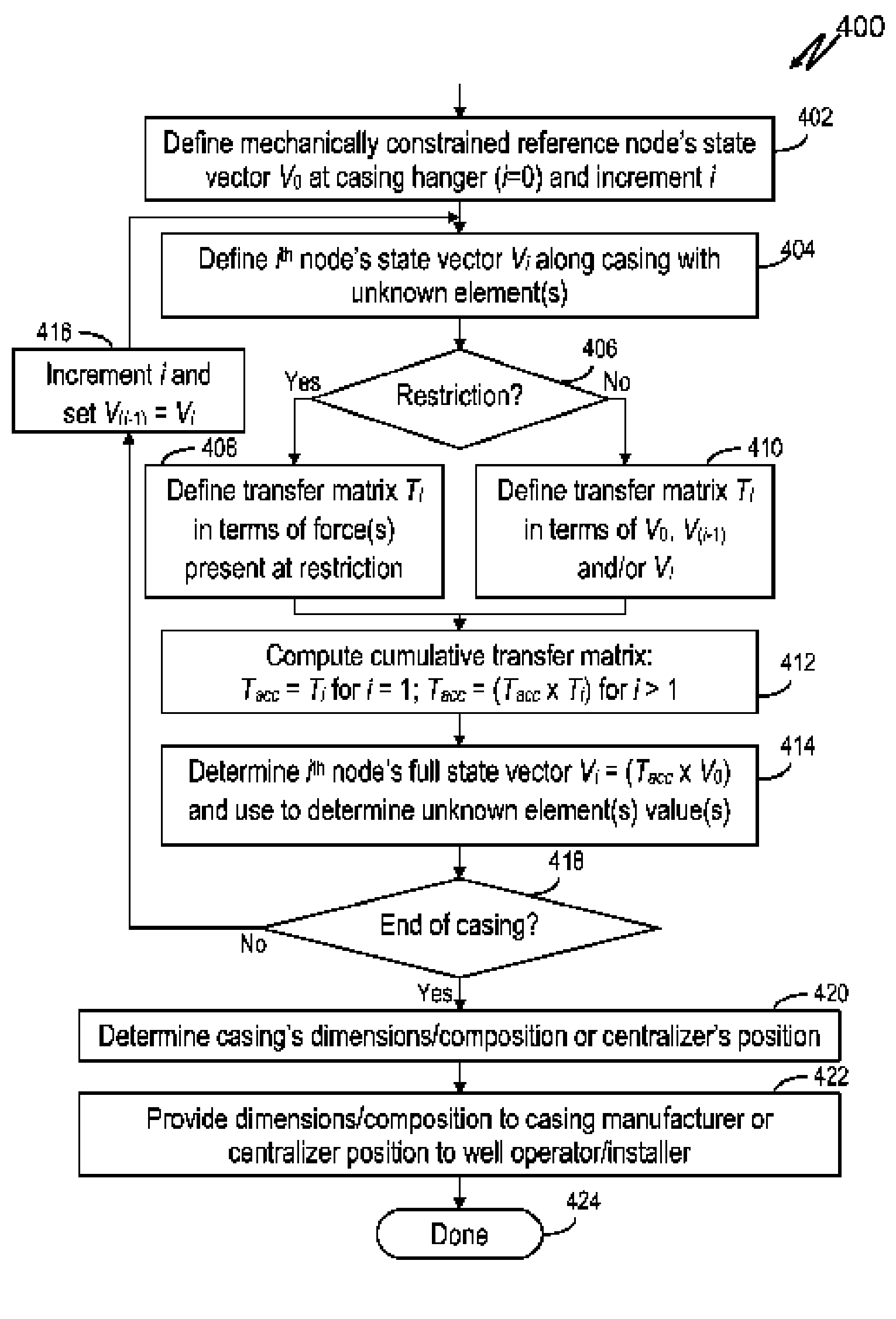

state vector describing a corresponding node position and one or more forces present at said node. The method further includes applying the transfer matrices to obtain each of the state vectors' values, deriving from at least one of the state vectors a parameter value for said component, and specifying a component having said parameter value. The parameter value can include a centralizer or stabilizer composition, manufacturing dimensions, or position.

24 Claims, 4 Drawing Sheets

(51) **Int. Cl.**

G06F 17/16	(2006.01)
E21B 17/00	(2006.01)

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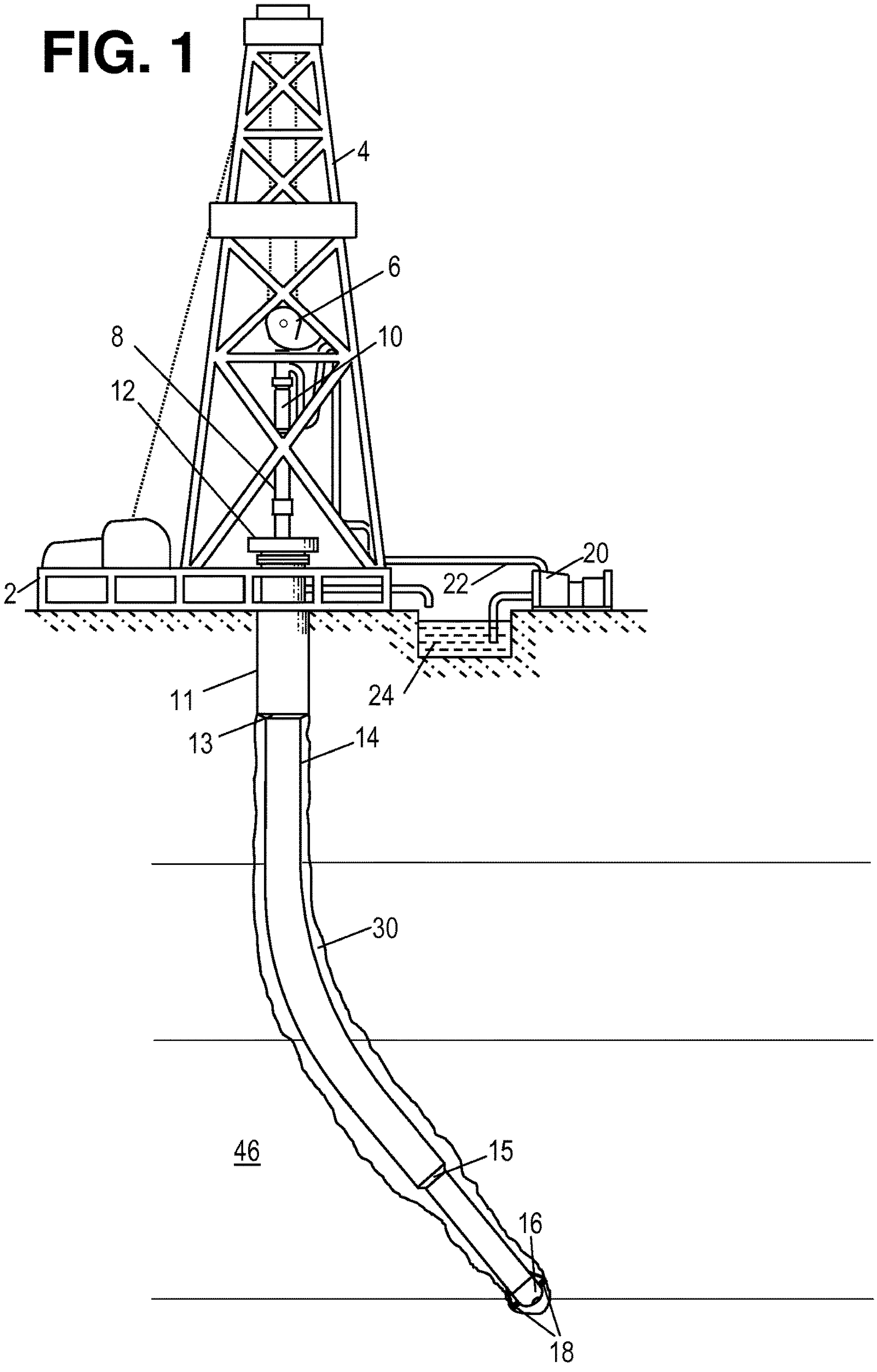
FIG. 1



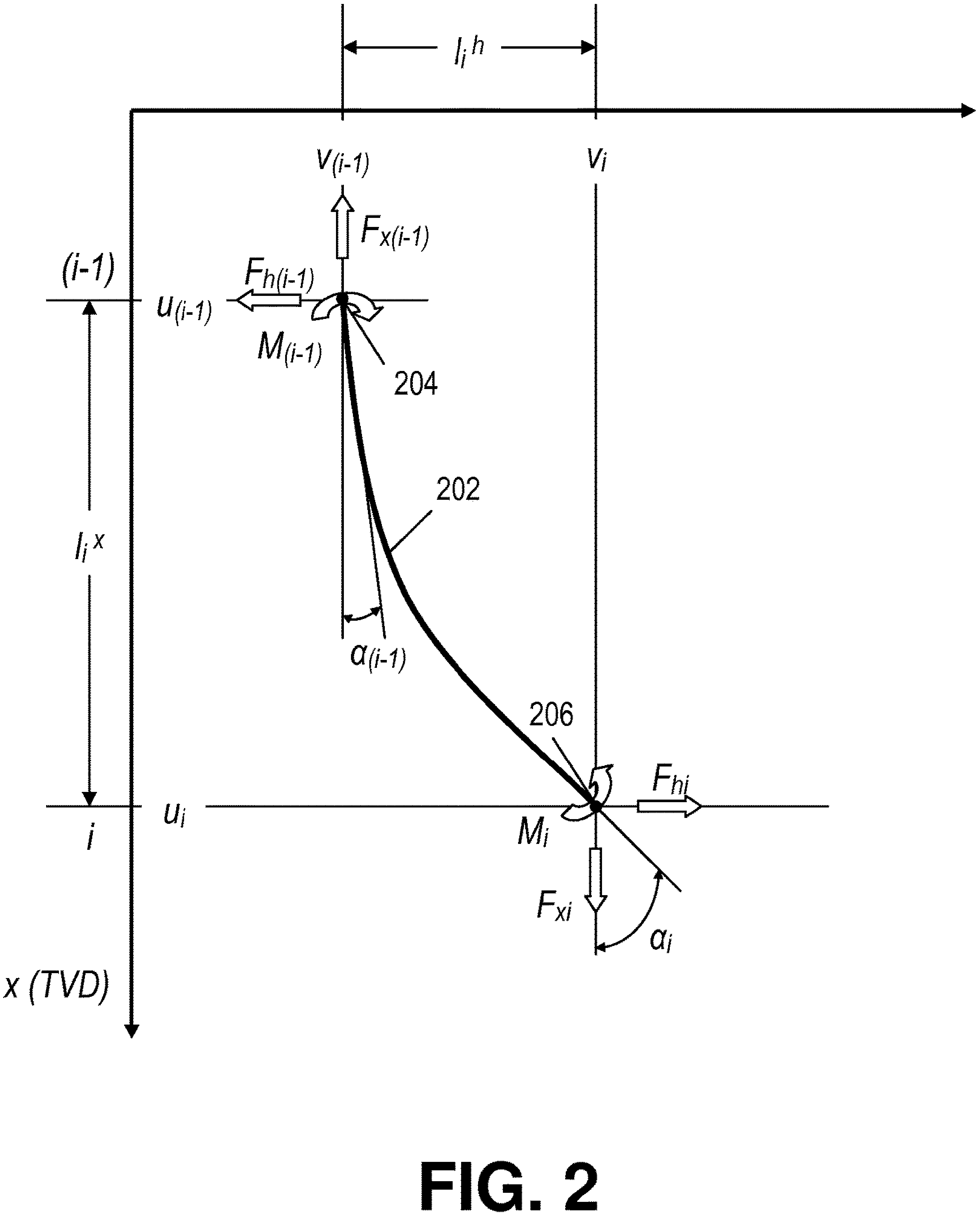
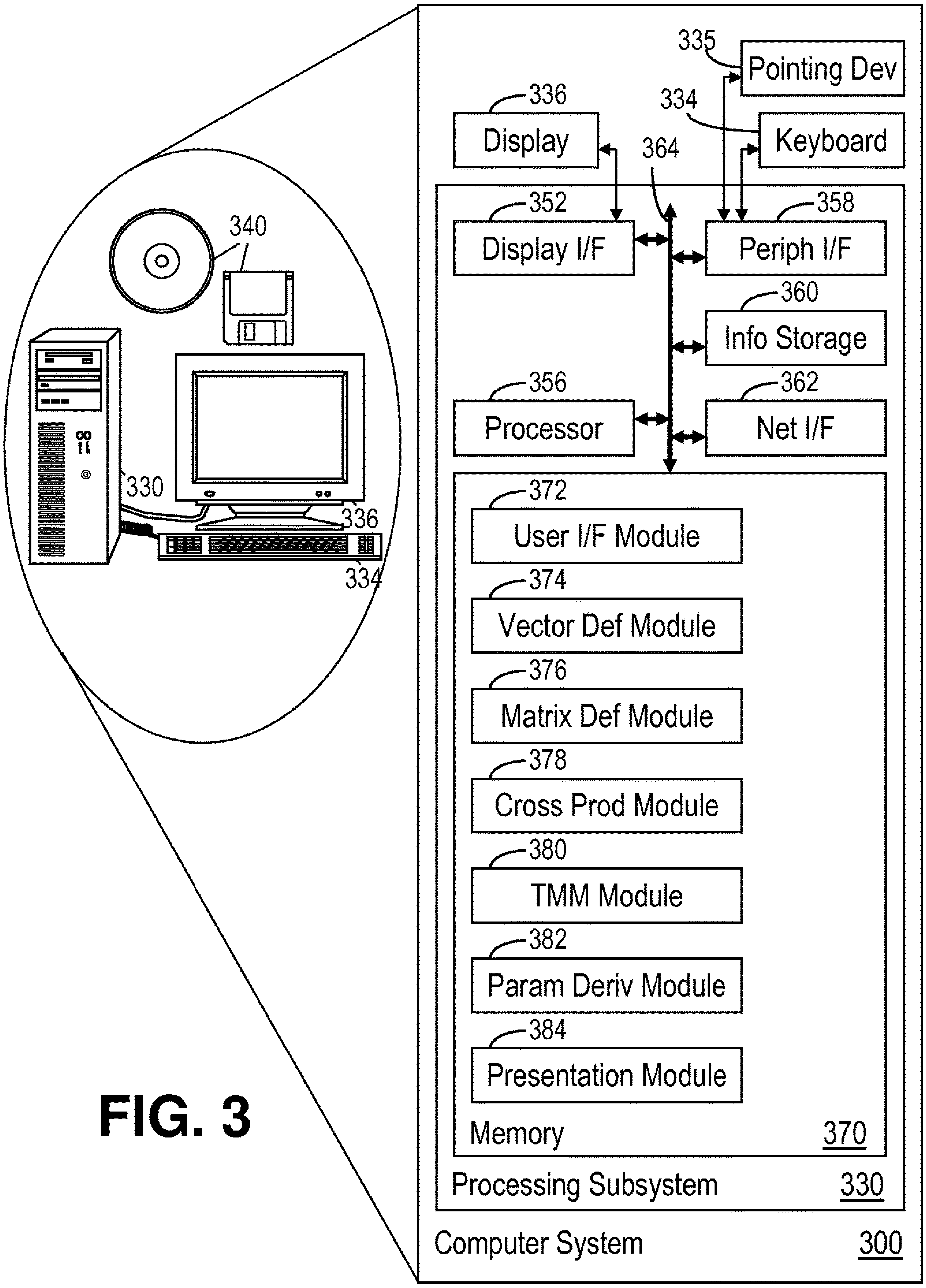

l_i^h
$v_{(i-1)}$
v_i
$F_{x(i-1)}$
(i-1)
$F_{h(i-1)}$
$u_{(i-1)}$
$M_{(i-1)}$
204
202
l_i^x
$\alpha_{(i-1)}$
206
F_{hi}
u_i
i
M_i
F_{xi}
α_i
x (TVD)

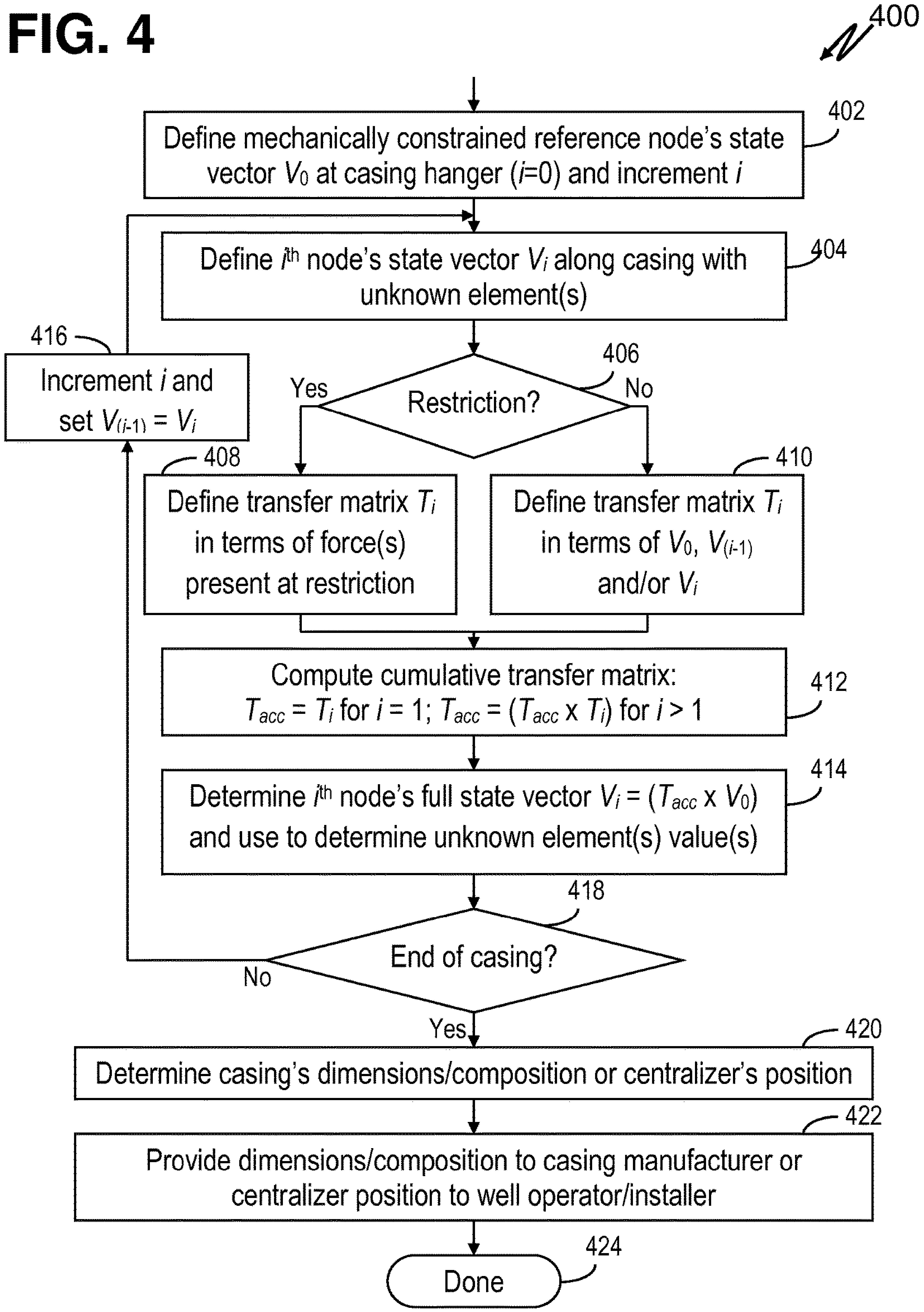
FIG. 2



335
Pointing Dev
336
334
Display
Keyboard
364
352
Display I/F
358
Periph I/F
340
360
Info Storage
356
Processor
362
Net I/F
330
372
User I/F Module
336
334
374
Vector Def Module
376
Matrix Def Module
378
Cross Prod Module
380
TMM Module
382
Param Deriv Module
384
Presentation Module
Memory 370
Processing Subsystem 330
Computer System 300

FIG. 3

FIG. 4


400
402
Define mechanically constrained reference node's state vector V_0 at casing hanger (i=0) and increment i
404
Define i^{th} node's state vector V_i along casing with unknown element(s)
406
Restriction?
Yes
No
408
Define transfer matrix T_i in terms of force(s) present at restriction
410
Define transfer matrix T_i in terms of V_0, $V_{(i-1)}$ and/or V_i
412
Compute cumulative transfer matrix: $T_{acc} = T_i$ for i = 1; $T_{acc} = (T_{acc} \times T_i)$ for i > 1
414
Determine i^{th} node's full state vector $V_i = (T_{acc} \times V_0)$ and use to determine unknown element(s) value(s)
416
Increment i and set $V_{(i-1)} = V_i$
418
End of casing?
No
Yes
420
Determine casing's dimensions/composition or centralizer's position
422
Provide dimensions/composition to casing manufacturer or centralizer position to well operator/installer
424
Done

METHODS AND SYSTEMS FOR DETERMINING MANUFACTURING AND OPERATING PARAMETERS FOR A DEVIATED DOWNHOLE WELL COMPONENT

CROSS-REFERENCE TO RELATED APPLICATION

This application claims priority to Provisional U.S. Application Ser. No. 61/837,986, titled "Methods and Systems for Modeling a Deviated Downhole Well Component" and filed Jun. 21, 2013 by Robello Samuel and Zhenying Wang, which is incorporated herein by reference.

BACKGROUND

As world demand for petrochemical products has continued to increase, oil and gas companies have had to expand their exploration and production efforts into developing increasingly deep wells. As a result, the structures constructed to form a well must be capable of operating under larger loads and stresses than ever before. Because failures can have costly consequences, it is important to design all well structures with appropriate safety margins.

One example of such a structure is the well casing. A well casing is a tubular structure generally made of a steel pipe surrounded by a concrete layer that secures the steel pipe to the surrounding formation, thus defining the outside wall of the well. The concrete provides support to the steel pipe, as well as additional isolation layer between the formation and fluids flowing within the casing. In order to determine the correct materials and dimensions for the various casing components, engineers frequently perform computer simulations to model various casing configurations under simulated downhole conditions. The simulations provide the engineer with information regarding the various loads and stresses to which the casing might be subjected, and enable potential designs to be evaluated.

But casing designs are only as good as the underlying simulation model. While simulations of single section casings in vertical wells are generally well understood and produce accurate results, tapered casings, deviated casings and casings with fluid flow restrictions represent indeterminate complex mechanical systems that can be very difficult or impractical to model using existing techniques. While methods do exist wherein these more complex systems are modeled as simpler single-section vertical wells with the results being adjusted to include additional safety margins, such methods can incur a significant risk, given the lack of quantifiable data to support the selected margins.

BRIEF DESCRIPTION OF THE DRAWINGS

A better understanding of the various disclosed embodiments can be obtained when the following detailed description is considered in conjunction with the attached drawings, in which:

FIG. **1** shows an illustrative downhole well with a well casing modeled using the disclosed systems and methods.

FIG. **2** shows the various parameters describing the forces operating on an illustrative well casing string segment.

FIG. **3** shows an illustrative computer system suitable for performing the disclosed methods.

FIG. **4** shows an illustrative example of the disclosed methods.

It should be understood that the drawings and corresponding detailed description do not limit the disclosure, but on the contrary, they provide the foundation for understanding all modifications, equivalents, and alternatives falling within the scope of the appended claims.

DETAILED DESCRIPTION

The paragraphs that follow describe illustrative systems and methods for determining manufacturing or operating parameters for a deviated downhole well component. Examples are provided within the context of a tapered and deviated well casing that is mechanically constrained at opposite ends. The mechanics of such a casing are explained and illustrated, and matrices are presented that mathematically describe the known and unknown forces acting at various points along the casing. Finally, methods and systems are described that combine the various matrices to compute the forces present along the casing.

The disclosed systems and methods are best understood when described in an illustrative usage context. Accordingly, FIG. **1** shows an illustrative drilling environment. A drilling platform **2** supports a derrick **4** having a traveling block **6** for raising and lowering a drillstring **8** into borehole **30**. A top drive **10** supports and rotates the drillstring **8** as it is lowered through the wellhead **12**. A pump **20** circulates drilling fluid through a feed pipe **22** to top drive **10**, downhole through the interior of drill string **8**, through orifices in a downhole tool (not shown), back to the surface via the annulus around drillstring **8**, and into a retention pit **24**. The drilling fluid aids in maintaining the borehole integrity.

Because boreholes are routinely drilled to ten thousand feet or more in depth and can be steered horizontally for perhaps twice that distance, a well casing string is inserted into the borehole and is cemented to the borehole wall to provide support to the borehole and isolation between the formation and the fluids flowing within the well casing string. In the example of FIG. **1**, the upper end of well casing string **14** is attached to and mechanically constrained by a casing hanger, located at the end of casing header **11**. As well casing string **14** extends downhole it may be tapered to provide additional support between the casing string and the borehole wall and to reduce the overall weight of the string. Well casing string **14** of FIG. **1**, for example, includes tapered reductions **13** and **15**. Well casing string **14** also curves to conform to the shape of the deviated well shown. A shoe **16** is located at the end of well casing string **14**, wherein shoe **16** and stabilizers **18** fix and mechanically constrain the lower end of well casing string **14** within borehole **30**.

As can be seen in the illustrative example of FIG. **1**, prior to being cemented to the borehole wall, well casing string **14** is supported and mechanically constrained by just two points at either end of the well casing string. The deviation of the well casing string, as well as the reductions in the cross-sectional are of the well casing string at the reductions, produce complex three-dimensional forces that act upon the well. The resulting service load distribution on the well casing string causes the string to become a statically complicated indeterminate mechanical system. Piston forces acting on plugs within the casing (e.g., cementing plugs) also produce forces similar to those at the reduction, further complicating the system.

In at least some illustrative embodiments, the forces acting on the above-described well casing string are modeled by dividing the system into a series of N subsystems that each only interact with adjacent subsystems, and then determining the forces acting on each subsystem in sequence. FIG. **2** shows an illustrative well casing string subsystem identified as segment **202** that is defined by two nodes **204** and **206**. Each node is described by a state vector that includes information regarding the location of the node relative to a reference node, and also includes information describing the forces present at the node. Each illustrative state vector is defined as,

$$V_i=[u_i v_i \alpha_i F_{xi} F_{hi} M_i -1]_i^T \qquad (1)$$

where,

u_i is the true vertical depth (TVD) of node i relative to the reference node;

v_i is the horizontal distance of node i from the reference node;

α_i is the inclination angle of the casing segment at node i;

F_{xi} is the vertical force present at node i;

F_{hi} is the horizontal force present at node i; and

M_i is the bending moment present at node i.

The reference node of the illustrative example is located at the casing hanger and is designated as node 0, and the node at the opposite end of the well casing string and furthest away from node 0 is designated as node N.

After dividing the illustrative well casing string into segments, the unknown forces acting on each segment's downhole node are determined by starting at the reference node at one end of the first segment (where all elements of the state vector are known) and determining the state vector for the next node at the opposite end of the segment. The state vector for the downhole node of the first segment is determined by computing a cross product of a transfer matrix and the reference node's state vector. This transfer matrix is defined based upon the known elements of the downhole node's state vector, the previous node's state vector and/or the reference node's state vector. For the first segment the previous node is also the reference node (i.e., i−1=0). In at least some illustrative embodiments, the transfer matrix T_i for a well casing string segment defined between nodes i and i−1 is described using known state vector elements as,

$$\begin{bmatrix} 1 & 0 & -l_i^h & \left(\frac{(l_i^h)^3}{6\times(EI)_i}+\frac{l_i^x}{(EA)_i}\right) & 0 & -\frac{(l_i^h)^2}{2\times(EI)_i} & l_i^x+\Delta\alpha_{i-1}^0\times l_i^h \\ 0 & 1 & l_i^x & 0 & \left(-\frac{(l_i^x)^3}{6\times(EI)_i}+\frac{l_i^h}{(EA)_i}\right) & \frac{(l_i^x)^2}{2\times(EI)_i} & l_i^h-\Delta\alpha_{i-1}^0\times l_i^x \\ 0 & 0 & 1 & -\frac{(l_i^h)^2}{2\times(EI)_i} & \frac{(l_i^x)^2}{2\times(EI)_i} & \frac{\sqrt{(l_i^x)^2+(l_i^h)^2}}{(EI)_i} & \Delta\alpha_i^0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & l_i^h & -l_i^x & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_i \qquad (2)$$

where,

l_i^h is the horizontal distance defined as (v_i-v_{i-1});

l_i^x is the vertical distance defined as (u_i-u_{i-1});

$\Delta\alpha_{i-1}^0$ is the change in inclination angle at the $i^{th}-1$ node defined as $(\alpha_{i-1}-\alpha_0)$;

$\Delta\alpha_i^0$ is the change in inclination angle at the i^{th} node defined as $(\alpha_i-\alpha_0)$;

$(EI)_i$ is the product of Young's modulus and a moment of inertia of the component at the i^{th} node; and

$(EA)_i$ is the product of Young's modulus and a cross-sectional area of the component at the i^{th} node.

For the first node, i=1 and the state vector V_1 is expressed as,

$$V_1=T_1\times V_0 \qquad (3).$$

Equation (3) expresses the state vector V_1 as a set of constrained linear equations for the well casing segment that can be solved to determine the unknown forces present at node 1. For each subsequent node i, the cross product of the node's transfer matrix and the reference node's state vector is combined with prior cross products for nodes 1 through i−1 to determine the state vector for node i, thus determining the unknown forces at each node i (i.e., F_{xi}, F_{hi} and/or M_i). Once these forces are known, the axial force at a node can be computed as,

$$F_{\alpha i}=F_{xi}\times\cos(\alpha_i)+F_{hi}\times\sin(\alpha_i) \qquad (4).$$

In at least some illustrative embodiments, a transfer matrix method (TMM) is used to perform the combination of cross products. Using TMM, the combination of cross products producing the i^{th} node's state vector is expressed as,

$$V_i=(\Pi_1{}^i T_i)\times V_0 \qquad (5).$$

Because each of the prior products of products for each node 1 through i−1 have already been calculated, the product of products does not need to be recalculated for each node. Instead, each node's transfer matrix can be combined with a cumulative transfer matrix, thus avoiding duplicative computations. For example, for a three node string this would be expressed as,

$$T_{acc}=T_1;\ V_1=T_{acc}\times V_0 \qquad (6),$$

$$T_{acc}=T_{acc}\times T_2;\ V_2=T_{acc}\times V_0=(T_1\times T_2)\times V_0 \qquad (7), \text{ and}$$

$$T_{acc}=T_{acc}\times T_3;\ V_3=T_{acc}\times V_0=(T_1\times T_2\times T_3)\times V_0 \qquad (8).$$

The cross product $T_1\times T_2$ has already been calculated in equation (7) and saved as cumulative transfer matrix T_{acc}, which is reused without recalculation in equation (8).

It should be noted that the transfer matrix is not limited to the specific embodiment of equation (2). For example, plugs such as cementing plugs present within the casing string and reductions in the cross-sectional area of the casing such as reduction **15** of FIG. **1** may be represented by much simpler transfer matrices. In at least some illustrative embodiments, such plugs and reductions located at a node i are represented as,

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & F_p^x \\ 0 & 0 & 0 & 0 & 0 & 1 & F_p^h \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_i \qquad (8)$$

where,

F_p^x is a vertical force present on a plug or reduction located at the i^{th} node; and

F_p^h is a horizontal force present on the plug or reduction.

Other transfer matrices may include, for example, parameters that describe the load imposed on a casing string by a salt formation (i.e., "salt loading"). A wide variety of transfer matrices suitable for use with the methods described herein will become apparent to those of ordinary skill in the art, and all such variations of transfer matrices are within the scope of the present disclosure.

The algorithmic approach of equations (6) through (8) for computing equation (5) is suitable for implementation by software executing on a computer system such as the illustrative system shown in FIG. **3**. Both hardware and software components of computer system **300** are shown, which in at least some illustrative embodiments implement at least part of the matrix-based well casing string modeling shown as method **400** in FIG. **4** (described in more detail below). Computer system **300** operates in accordance with software (which may be stored on non-transitory information storage media **340**) and enables a user to interact with the system via keyboard **334**, pointing device **335** (e.g., a mouse) and display **336** to configure, control and monitor the execution of the matrix-based well casing string modeling.

Located within processing subsystem **330** of computer system **300** is a display interface **352**, a processor **356**, a peripheral interface **358**, an information storage device **360**, a network interface **362** and a memory **370**. Bus **364** couples each of these elements to each other and transports their communications. Network interface **362** enables communications with other systems (e.g., via the Internet with a central database server housing additional modeling parameters and suitable for saving the results of the modeling). In accordance with user input received via peripheral interface **358** and program instructions from memory **370** and/or information storage device **360**, processor **356** processes input from the user and applies it to the well casing string data to perform the disclosed methods and present the results to the user. Storage device **360** may be implemented using any number of known non-transitory information storage media, including but not limited to magnetic disks, solid-state storage devices and optical storage disks.

Various software modules are shown loaded into memory **370** of FIG. **3**, where they are each accessed by processor **356** for execution. These modules include: user interface module, which processes user inputs provided with keyboard **334** and pointing device **335** via peripheral interface **358**; vector definition module **374**, which defines the state vector for each node; matrix definition module **376**, which defines the transfer matrix for a segment; cross product module **378**, which computes the cross product that updates the cumulative transfer matrix; transfer matrix method module **380**, which uses the cumulative transfer matrix from cross product module **378** to determine unknown state vector elements; parameter derivation module **382**, which derives well component manufacturing parameters or well component centralizer positions; and presentation module **384** which provides the derived manufacturing parameters or centralizer positions to manufacturing or operations personnel (e.g., by graphically presenting the dimensions of casing segments or of centralizer positions along the length of the casing prior to cementing into place).

FIG. **4** shows an illustrative method that implements the above-described matrix-based modeling, at least part of which may be implemented by software executing on computer system **300**. It should be noted that although the embodiment of FIG. **3** shows various software modules executing on computer system **300**, in other illustrative embodiments some or all of the modules may execute on two or more computers within a networked and/or distributed system. Referring to both FIGS. **3** and **4**, the state vector for the reference node (node 0) at the start of a casing string is defined (block **402**; vector definition module **374**), either via user input (user interface module **372**) or using previously stored data (e.g., data stored on information storage device **360**). Node index i is incremented from 0 to 1 and the total product is initialized to zero (block **402**; TMM module **382**). A state vector for the current i^{th} node (here node 1) is defined in a manner similar to that used for the reference node (block **404**; vector definition module **374**), but with at least one unknown state vector element.

If the segment associated with the current node is one that restricts fluid flow, such as a plug or a casing cross-sectional area reduction (block **406**; matrix definition module **376**) the transfer matrix is defined in terms of one or more forces present at the restriction (block **408**; matrix definition module **376**). If the segment associated with the current node is a well casing string, the transfer matrix is defined in terms of the reference node's state vector, the previous node's state vector and/or the known elements of the current node's state vector (block **410**; matrix definition module **376**). Once the transfer matrix is defined for node i, the cumulative transfer matrix T_{acc} is either initialized as T_1 for i=1, or updated as the cross product of the current node's transfer matrix T_i and T_{acc} for i>1 (block **412**; cross product module **380**). The cross product is then utilized to determine the node's full state vector, which is used to determine the unknown element values of the current node's state vector (block **414**; TMM module **380**). In at least some illustrative embodiments, block **414** is skipped for the first node (i=1) if all the state vector values are already known (e.g., if the values are readily measurable at a starting point at the surface).

If additional well casing string segments remain (block **418**; TMM module **380**) node index i is incremented and the current node's state vector becomes the prior node's state vector (block **416**; TMM module **380**). The above-described process is then repeated for the next segment and node along the well casing string (blocks **404** through **418**). If there are no additional well casing string segments (block **418**; TMM module **380**), the previously unknown and now calculated elements of each state vector are subsequently used as a basis for determining casing manufacturing parameters such as dimensions and composition, or as a basis for locating centralizers to position the casing within a borehole (block **420**, parameter derivation module **382**). The resulting manufacturing parameters or centralizer position are respectively provided as composition and/or dimensional specifications to manufacturing personnel, or as a position for a centralizer or stabilizer to well operators/component installers (block **422**, presentation module **384**), ending the method (block **424**).

In at least some illustrative embodiments, the calculated forces at each node along a casing string are indicated on a graphical representation of the well casing string. Once the forces have been determined, the axial load (e.g., tension) present at a given node can be computed, for example by using equation (4). Casing string parameters such as segment lengths, wall thickness and material compositions may then be determined from the computed axial load. These parameters provide the required casing safety margins at a reduced cost when compared to existing methods that overestimate the required casing string parameters. In other illustrative embodiments, the positions of one or more centralizers (determined based upon the computed and displayed forces) are on the casing string's graphical representation. The calculated forces at each node are used to determine the side force to which a casing segment i (located between two nodes i and i+1) is subjected, for example by using the equation,

$$\text{SideForce}_i=\sqrt{(F_{x(i+1)}-F_{xi})^2+(F_{h(i+1)}-F_{hi})^2} \quad (9).$$

The contact points between the casing and borehole wall can be determined from the state vector V_i and the well trajectory. The computed side force and the contact points are then used to determine a suitable centralizer(s) and the best centralizer position(s), for example, at the contact point(s) between the casing segment and the borehole wall.

Numerous other modifications, equivalents, and alternatives, will become apparent to those skilled in the art once the above disclosure is fully appreciated. For example, although the embodiments described use the TMM to determine the transfer matrices, other analytical methods are also suitable to determine transfer matrices used to determine the unknown elements of a node of interest. Further, although the examples provided are applied within the context of static modeling, static snapshots repeatedly performed over time may be combined to provide dynamic as well as near-real-time or real-time modeling of the well casing string (e.g., to predict the loading on a well casing string as a cementing plug progresses down the string). Additionally, although the disclosed embodiments describe modeling well casing strings, any of a wide variety of well components may be modeled to determine the manufacturing and/or operating parameters of said components. These well components include but are not limited to drillstrings, workstrings, production strings and coiled tubing stings. Other well components that restrict fluid flow within a running tubular string (e.g., packers) are also within the scope of the disclosure. It is intended that the following claims be interpreted to embrace all such modifications, equivalents, and alternatives where applicable.

What is claimed is:

1. A method for determining manufacturing or operating parameters for a deviated downhole well component, the method comprising:

representing a tubular string prior to cementing in a borehole as a sequence of nodes separated by tapered segments, said nodes being numerable from i=1 to N with an initial, mechanically constrained reference node located at a casing hanger representable with i=0, a final, mechanically constrained N^{th} node located at a shoe representable with i=N, and each node being associated with a state vector describing a position of the corresponding node in the borehole and one or more forces present on the tubular string at said corresponding node;

determining a sequence of transfer matrices enabling the determination of an i^{th} node's state vector from an $i^{th}-1$ node's state vector;

defining values of an initial state vector for the reference node;

applying the transfer matrices to obtain values for each of the state vectors;

deriving from at least one of the state vectors a parameter value for said component, the parameter value being in a set consisting of a composition, manufacturing dimensions, and a position for a centralizer or stabilizer of the tubular string; and

specifying a component having said parameter value, wherein said parameter value facilitates manufacture of the component or positioning of the component for cementing in the borehole.

2. The method of claim **1**, wherein said specifying includes providing the composition or a dimensional specification to a manufacturer of said component.

3. The method of claim **1**, wherein said specifying includes providing the position for the centralizer or stabilizer to an installer of said component.

4. The method of claim **1**, wherein each state vector comprises a vertical position u_i, a horizontal position v_i, and an inclination angle α_i, associated with node i; and further comprises a vertical force F_{xi}, a horizontal force F_{hi} and a bending moment M_i present at node i; and wherein the state vector is representable as $[u_i, v_i, \alpha_i, F_{xi}, F_{hi}, M_i, 1]^T$.

5. The method of claim **4**, wherein the transfer matrix for the i^{th} node, said i^{th} node associated with a tubular segment, is representable as:

$$\begin{bmatrix} 1 & 0 & -l_i^h & \left(\frac{(l_i^h)^3}{6\times(EI)_i}+\frac{l_i^x}{(EA)_i}\right) & 0 & -\frac{(l_i^h)^2}{2\times(EI)_i} & l_i^x+\Delta\alpha_{i-1}^0\times l_i^h \\ 0 & 1 & l_i^x & 0 & \left(-\frac{(l_i^x)^3}{6\times(EI)_i}+\frac{l_i^h}{(EA)_i}\right) & \frac{(l_i^x)^2}{2\times(EI)_i} & l_i^h-\Delta\alpha_{i-1}^0\times l_i^x \\ 0 & 0 & 1 & -\frac{(l_i^h)^2}{2\times(EI)_i} & \frac{(l_i^x)^2}{2\times(EI)_i} & \frac{\sqrt{(l_i^x)^2+(l_i^h)^2}}{(EI)_i} & \Delta\alpha_i^0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & l_i^h & -l_i^x & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_i$$

where,

l_i^h is a horizontal distance defined as (v_i-v_{i-1});

l_i^x is a vertical distance defined as (u_i-u_{i-1});

$\Delta\alpha_{i-1}^0$ is a change in inclination angle at the $i^{th}-1$ node defined as $(\alpha_{i-1}-\alpha_0)$;

$\Delta\alpha_i^0$ is a change in inclination angle at the i^{th} node defined as $(\alpha_i-\alpha_0)$;

$(EI)_i$ is a product of Young's modulus and a moment of inertia of the component at the i^{th} node; and

$(EA)_i$ is a product of Young's modulus and a cross-sectional area of the component at the i^{th} node.

6. The method of claim **4**, wherein said deriving includes deriving an axial force $F_{\alpha i}$ present at the i^{th} node.

7. The method of claim **4**, wherein the transfer matrix for the ith node, said ith node associated with a flow restriction, is representable as:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & F_p^x \\ 0 & 0 & 0 & 0 & 0 & 1 & F_p^h \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_i$$

where,

- F_p^x is a vertical force present on a plug located at the ith node; and
- F_p^h is a horizontal force present on the plug.

8. The method of claim **1**, wherein the Nth node is also mechanically constrained.

9. The method of claim **1**, wherein the component comprises a running string selected from the group consisting of a well casing string, a drillstring, a production string and a coil tubing.

10. The method of claim **9**, wherein the running string comprises a tapered segment, a cross-section size change, a packer or a plug.

11. The method of claim **1**, further comprising cementing the component having said parameter value in the borehole.

12. The method of claim **1**, further comprising manufacturing the component having said parameter value or positioning the component having said parameter value in the borehole.

13. A system that determines manufacturing and operating parameters for a deviated downhole well component, the system comprising:

- a memory having deviated downhole well component modeling software; and
- one or more processors coupled to the memory, the software causing the one or more processors to:
  - represent a tubular string prior to cementing in a borehole as a sequence of nodes separated by tapered segments, said nodes being numerable from i=1 to N with an initial, mechanically constrained reference node located at a casing hanger representable with i=0, a final, mechanically constrained Nth node located at a shoe representable with i=N, and each node being associated with a state vector describing a position of the corresponding node in the borehole and one or more forces present on the tubular string at said corresponding node;
  - determine a sequence of transfer matrices enabling the determination of an ith node's state vector from an ith−1 node's state vector;
  - define values of an initial state vector for the reference node;
  - apply the transfer matrices to obtain values for each of the state vectors;
  - derive from at least one of the state vectors a parameter value for said component, the parameter value being in a set consisting of a composition, manufacturing dimensions, and a position for a centralizer or stabilizer of the tubular string; and
  - specify a component having said parameter value, wherein said parameter value facilitates manufacture of the component or positioning of the component for cementing in the borehole.

14. The system of claim **13**, wherein the one or more processors specify said component at least in part by providing the composition or a dimensional specification to a manufacturer of said component.

15. The system of claim **14**, wherein the transfer matrix for the ith node, said ith node associated with a tubular segment, is representable as:

$$\begin{bmatrix} 1 & 0 & -l_i^h & \left(\frac{(l_i^h)^3}{6\times(EI)_i}+\frac{l_i^x}{(EA)_i}\right) & 0 & -\frac{(l_i^h)^2}{2\times(EI)_i} & l_i^x+\Delta\alpha_{i-1}^0\times l_i^h \\ 0 & 1 & l_i^x & 0 & \left(-\frac{(l_i^x)^3}{6\times(EI)_i}+\frac{l_i^h}{(EA)_i}\right) & \frac{(l_i^x)^2}{2\times(EI)_i} & l_i^h-\Delta\alpha_{i-1}^0\times l_i^x \\ 0 & 0 & 1 & -\frac{(l_i^h)^2}{2\times(EI)_i} & \frac{(l_i^x)^2}{2\times(EI)_i} & \frac{\sqrt{(l_i^x)^2+(l_i^h)^2}}{(EI)_i} & \Delta\alpha_i^0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & l_i^h & -l_i^x & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_i$$

where,

- l_i^h is a horizontal distance defined as $(v_i - v_{i-1})$;
- l_i^x is a vertical distance defined as $(u_i - u_{i-1})$;
- $\Delta\alpha_{i-1}^0$ is a change in inclination angle at the ith−1 node defined as $(\alpha_{i-1}-\alpha_0)$;
- $\Delta\alpha_i^0$ is a change in inclination angle at the ith node defined as $(\alpha_i-\alpha_0)$;
- $(EI)_i$ is a product of Young's modulus and a moment of inertia of the component at the ith node; and
- $(EA)_i$ is a product of Young's modulus and a cross-sectional area of the component at the ith node.

16. The system of claim **13**, wherein the one or more processors specify said component at least in part by providing the position for the centralizer or stabilizer to an installer of said component.

17. The system of claim **13**, wherein each state vector comprises a vertical position u_i, a horizontal position v_i, and an inclination angle α_i, associated with node i; and further comprises a vertical force F_{xi}, a horizontal force F_{hi} and a bending moment M_i present at node i; and wherein the state vector is representable as $[u_i, v_i, \alpha_i, F_{xi}, F_{hi}, M_i, 1]^T$.

18. The system of claim **17**, wherein the one or more processors derive said parameter value at least in part by deriving an axial force $F_{\alpha i}$ present at the ith node.

19. The system of claim **17**, wherein the transfer matrix for the ith node, said ith node associated with a flow restriction, is representable as:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & F_p^x \\ 0 & 0 & 0 & 0 & 0 & 1 & F_p^h \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_i$$

where,

F_p^x is a vertical force present on a plug located at the ith node; and

F_p^h is a horizontal force present on the plug.

20. The system of claim **13**, wherein the Nth node is also mechanically constrain ed.

21. The system of claim **13**, wherein the component comprises a running string selected from the group consisting of a well casing string, a drillstring, a production string and a coil tubing.

22. The system of claim **21**, wherein the running string comprises a tapered segment, a cross-section size change, a packer or a plug.

23. The system of claim **13**, further comprising cementing the component having said parameter value in the borehole.

24. The system of claim **13**, further comprising manufacturing the component having said parameter value or positioning the component having said parameter value in the borehole.

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