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(54) XX COUPLER FOR FLUX QUBITS

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H03K 19/195	(2006.01)
B82Y 10/00	(2011.01)

(52) **U.S. Cl.**
CPC ***G06N 99/002*** (2013.01); ***H03K 19/195*** (2013.01); *B82Y 10/00* (2013.01)

(58) **Field of Classification Search**
CPC G06N 99/002; H03K 19/195; B82Y 10/00
See application file for complete search history.

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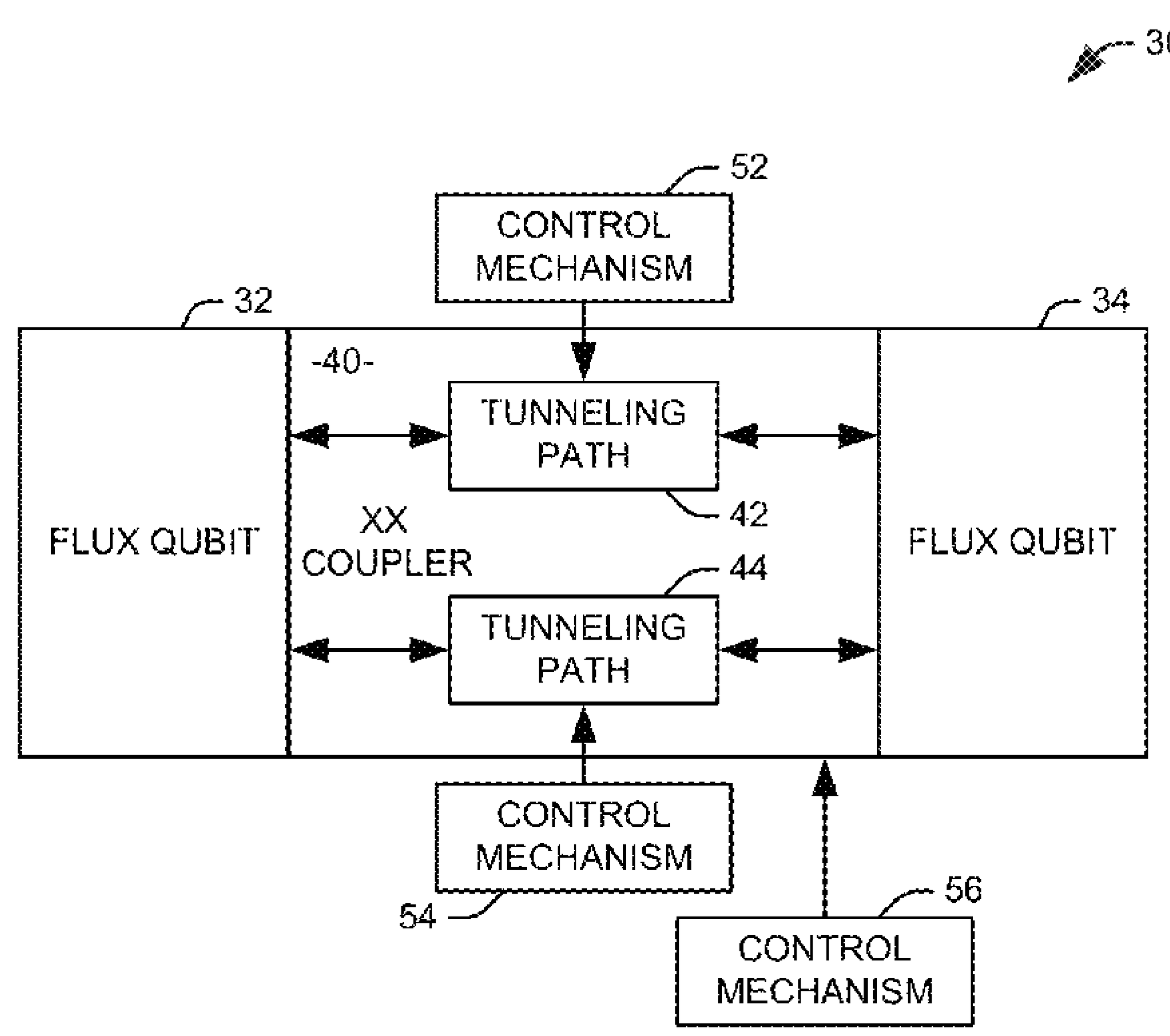
(57) **ABSTRACT**

Systems and methods are provided for coupling two flux qubits. A quantum circuit assembly includes a first flux qubit, having at least two potential energy minima, and a second flux qubit, having at least two potential energy minima. A system formed by the first and second qubits has at least four potential energy minima prior to coupling, each of the four potential energy minima containing at least one eigenstate of a system comprising the first flux qubit and the second flux qubit. A coupler creates a first tunneling path between a first potential energy minimum of the system and a second potential energy minimum of the system, and a second tunneling path between a third potential energy minimum of the system and a fourth potential energy minimum of the system. The coupler creates the first and second tunneling paths between potential energy minima representing states of equal bit parity.

10 Claims, 4 Drawing Sheets



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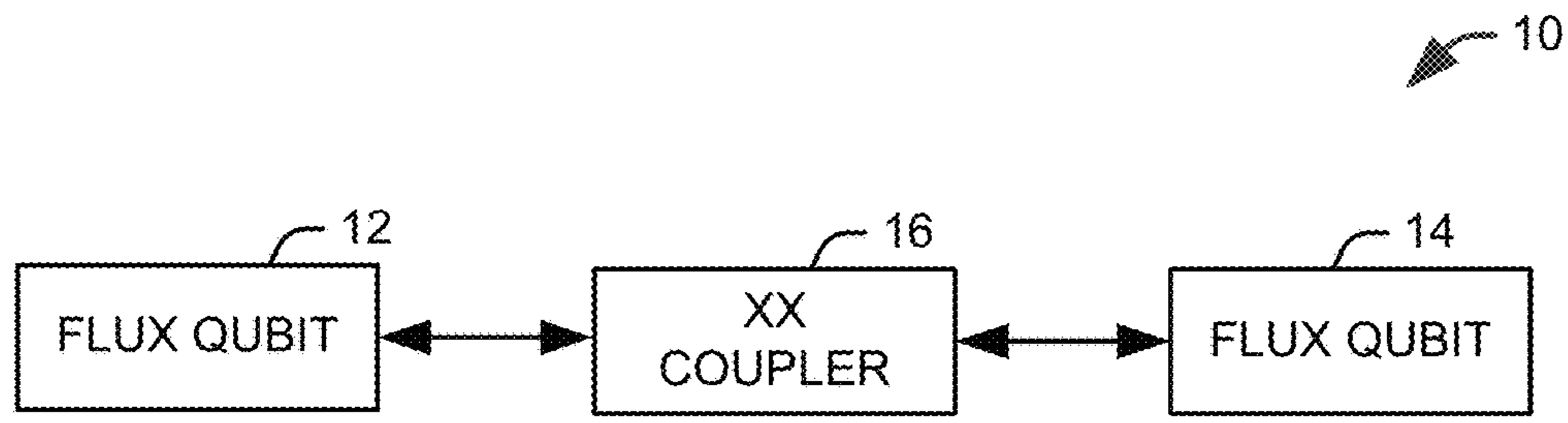

10
12
FLUX QUBIT
16
XX
COUPLER
14
FLUX QUBIT

FIG. 1

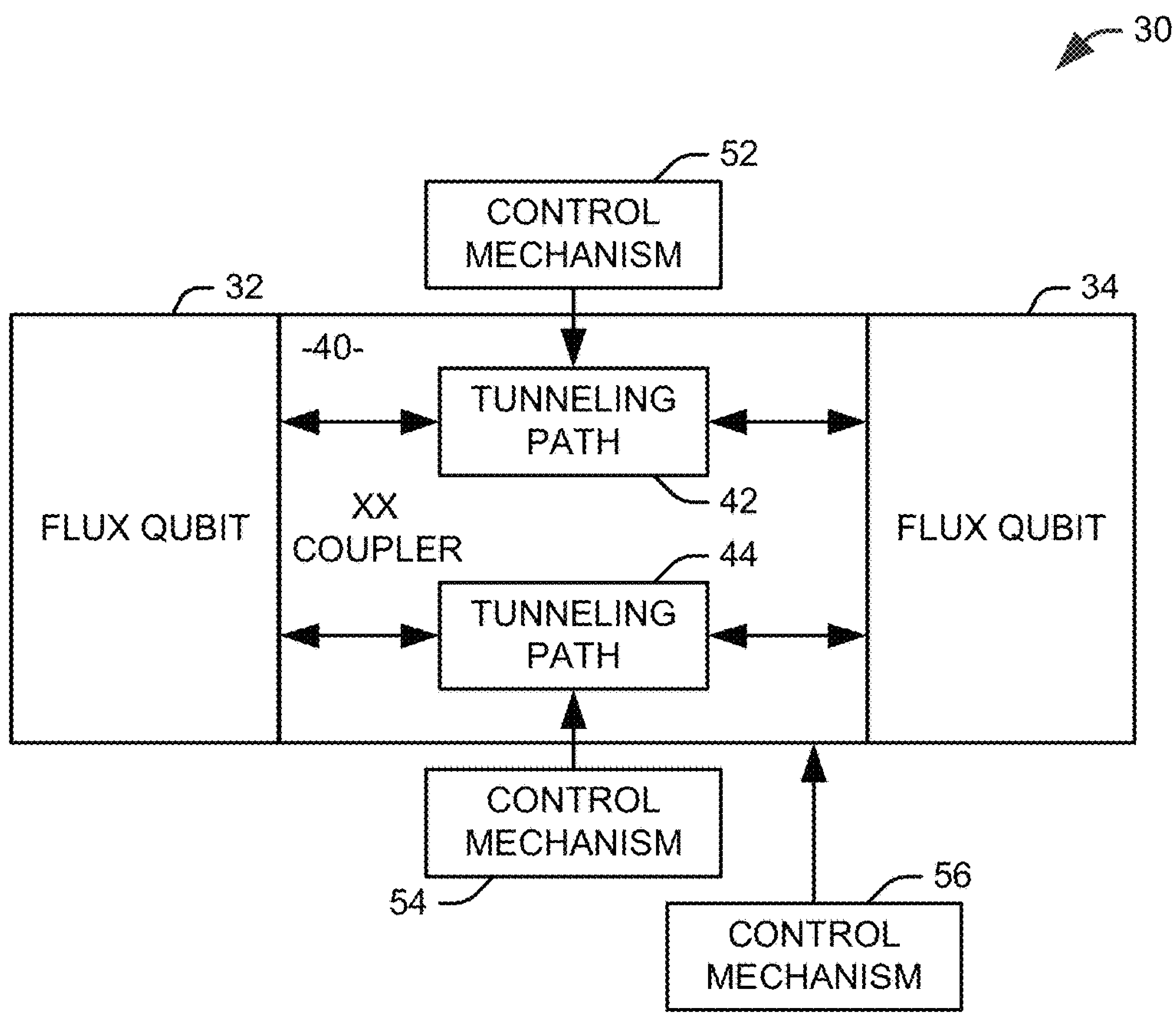

30
52
CONTROL
MECHANISM
32
34
-40-
TUNNELING
PATH
42
FLUX QUBIT
XX
COUPLER
44
TUNNELING
PATH
FLUX QUBIT
CONTROL
MECHANISM
54
56
CONTROL
MECHANISM

FIG. 2

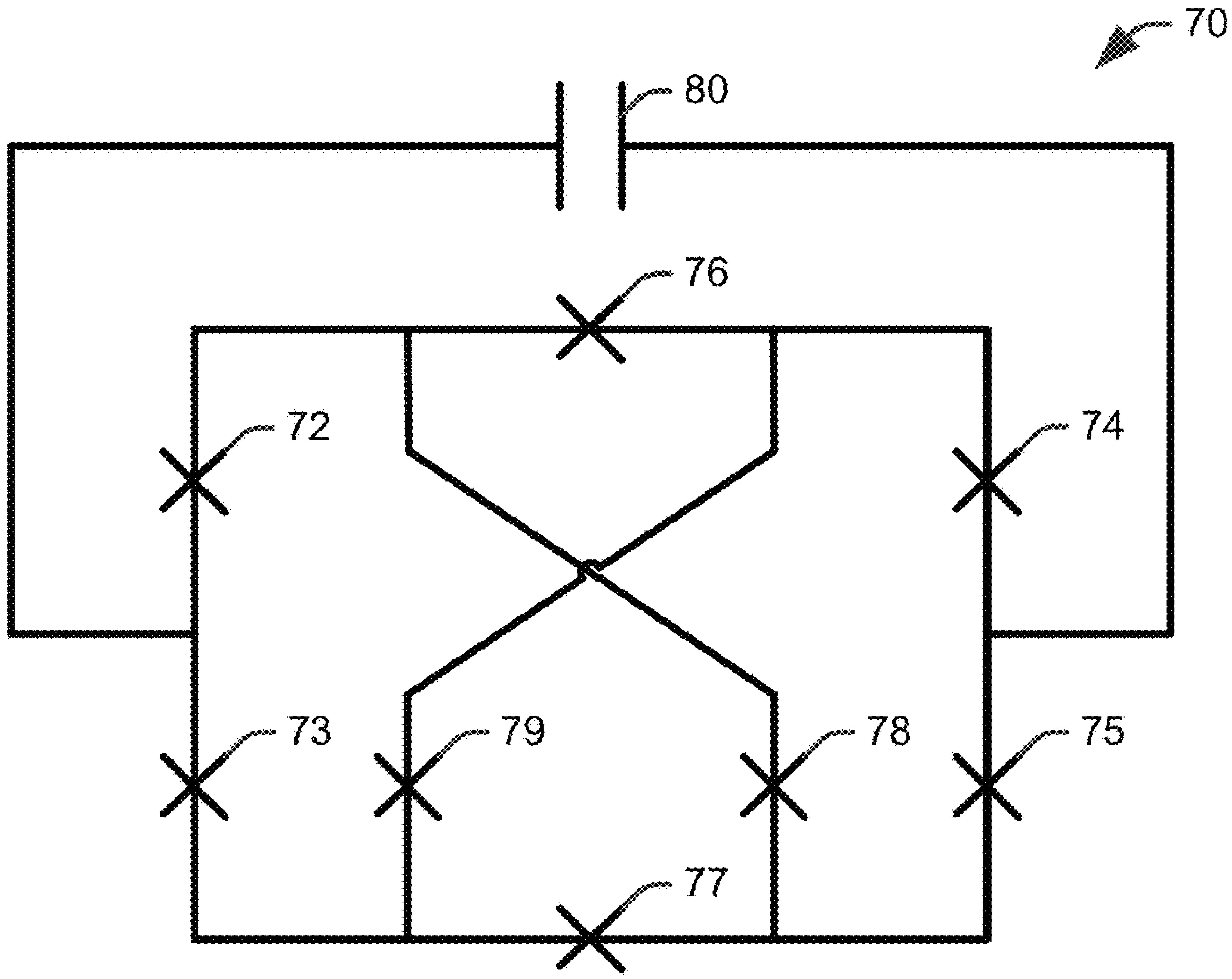


FIG. 3

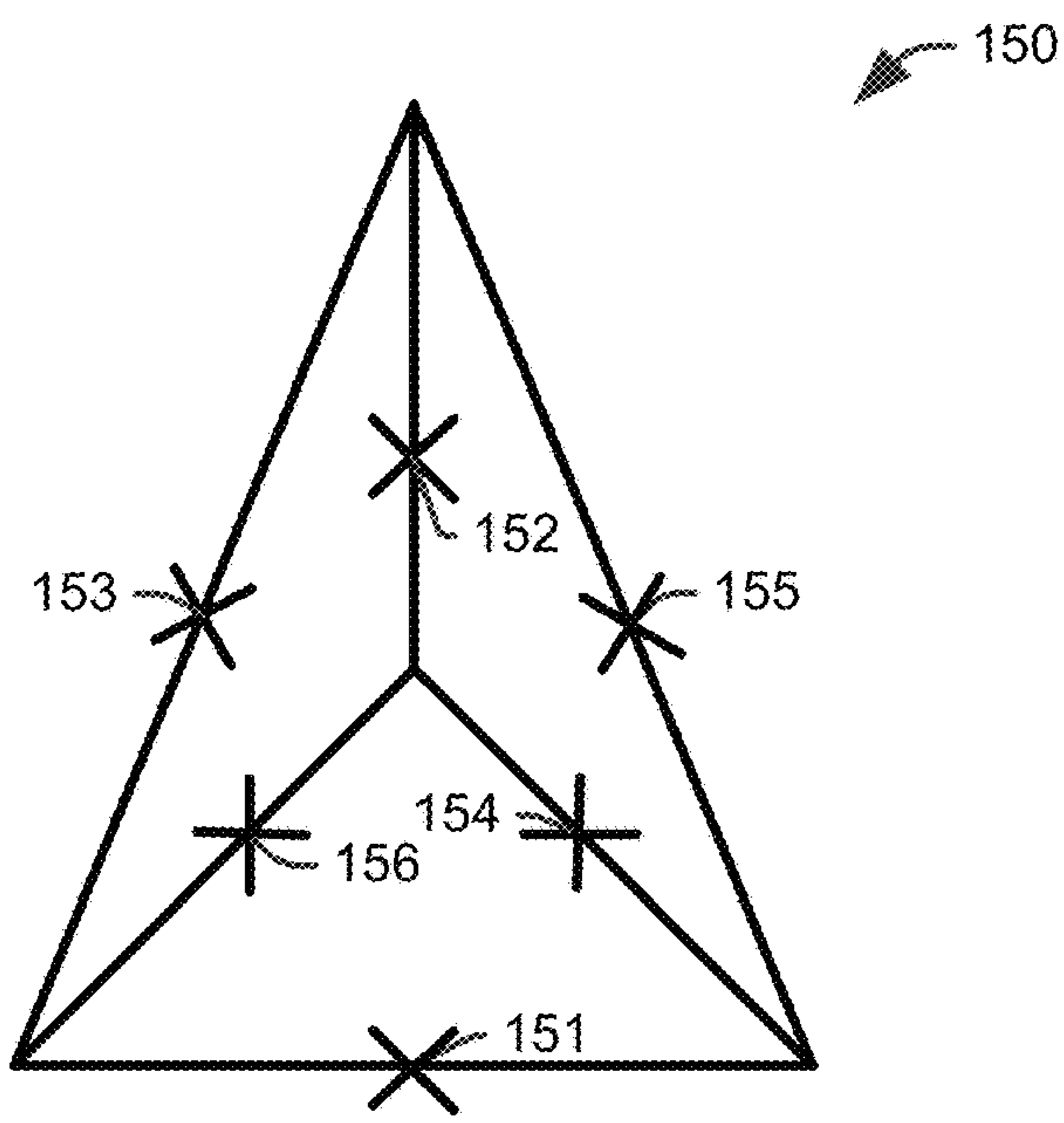
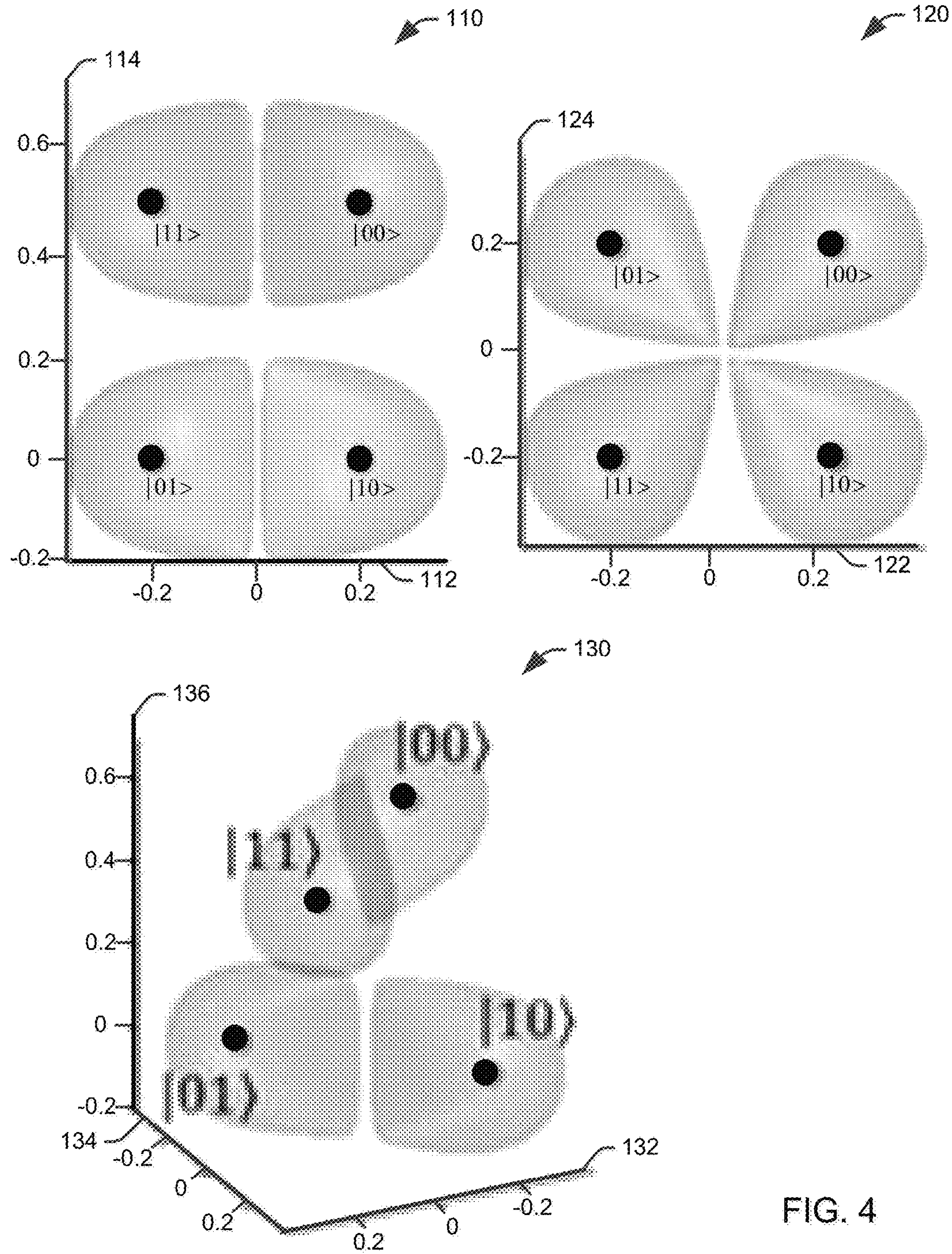
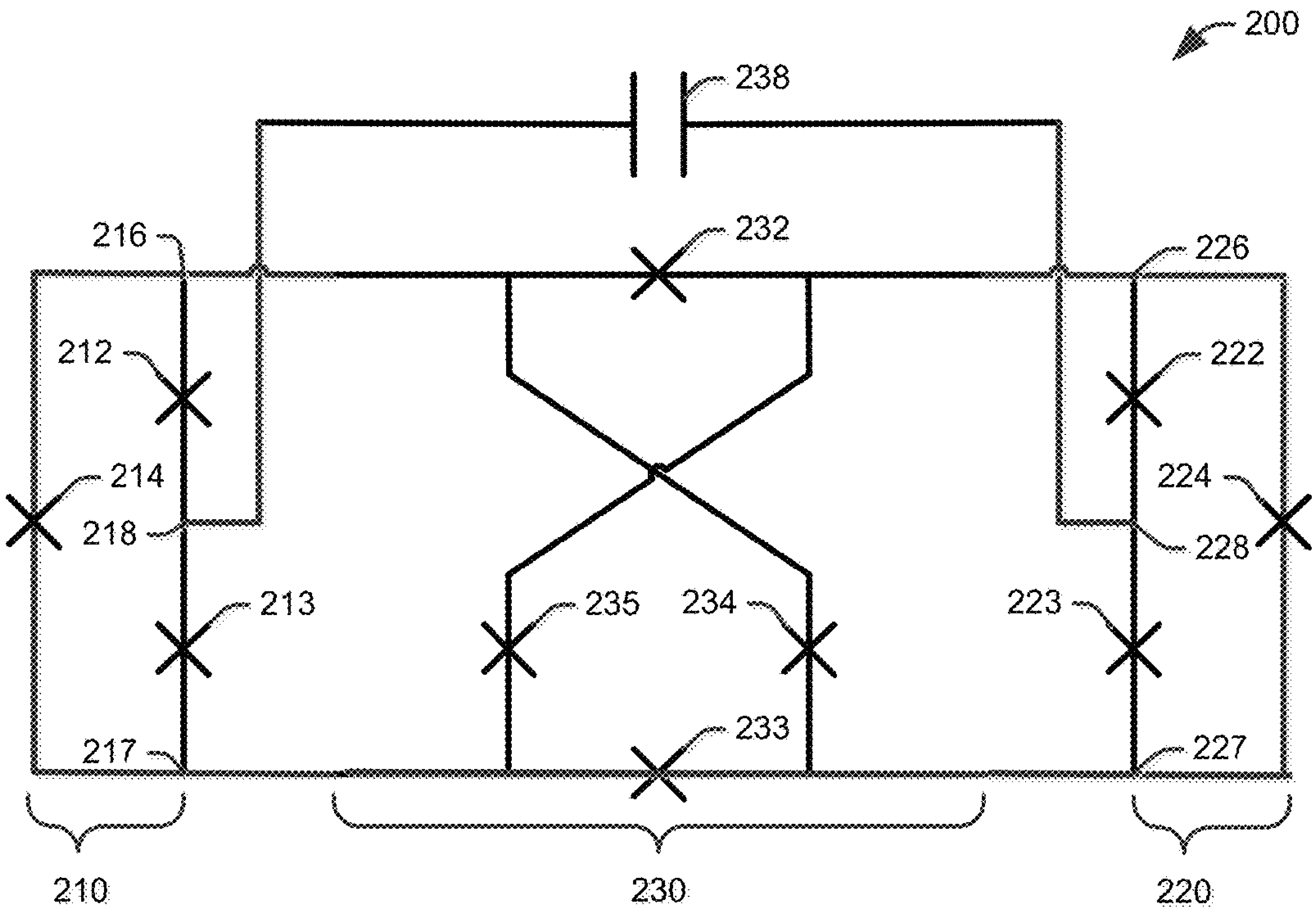


FIG. 5



110
114
0.6
0.4
0.2
0
-0.2
|11>
|00>
|01>
|10>
-0.2
0
0.2
112
120
124
0.2
0
-0.2
|01>
|00>
|11>
|10>
-0.2
0
0.2
122
130
136
0.6
0.4
0.2
0
-0.2
|00⟩
|11⟩
|10⟩
|01⟩
134
-0.2
0
0.2
132
0.2
0
-0.2

FIG. 4


200
238
216
232
226
212
222
214
224
218
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213
235
234
223
217
233
227
210
230
220

FIG. 6

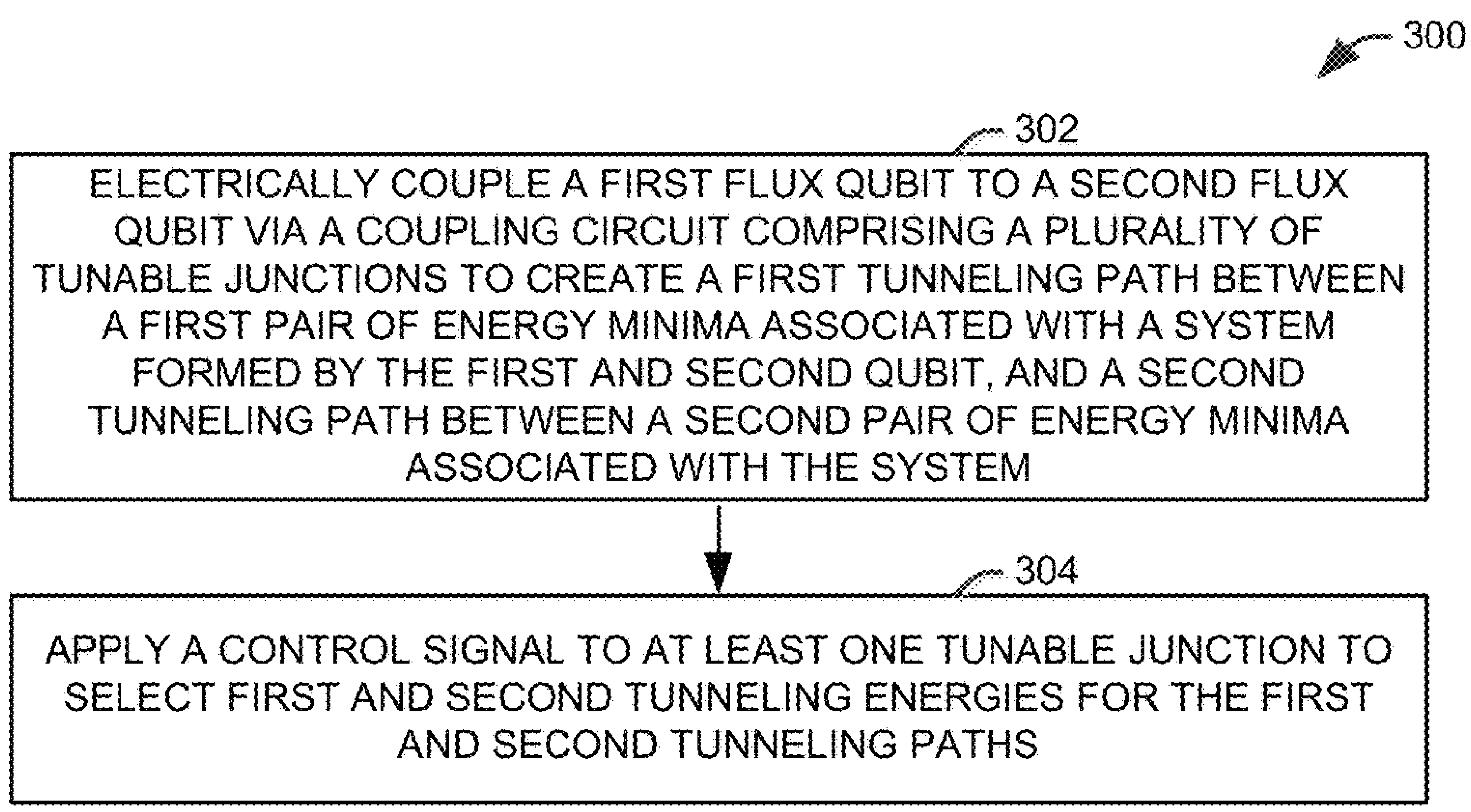
300
302
ELECTRICALLY COUPLE A FIRST FLUX QUBIT TO A SECOND FLUX QUBIT VIA A COUPLING CIRCUIT COMPRISING A PLURALITY OF TUNABLE JUNCTIONS TO CREATE A FIRST TUNNELING PATH BETWEEN A FIRST PAIR OF ENERGY MINIMA ASSOCIATED WITH A SYSTEM FORMED BY THE FIRST AND SECOND QUBIT, AND A SECOND TUNNELING PATH BETWEEN A SECOND PAIR OF ENERGY MINIMA ASSOCIATED WITH THE SYSTEM
304
APPLY A CONTROL SIGNAL TO AT LEAST ONE TUNABLE JUNCTION TO SELECT FIRST AND SECOND TUNNELING ENERGIES FOR THE FIRST AND SECOND TUNNELING PATHS

FIG. 7

XX COUPLER FOR FLUX QUBITS

TECHNICAL FIELD

This invention was made with government support under Federal Government Contract Number 30069353. The government may have certain rights in the invention

TECHNICAL FIELD

This invention relates to quantum computing, and more particularly, to a coupler for coupling the X basis states of flux qubits.

BACKGROUND

A classical computer operates by processing binary bits of information that change state according to the laws of classical physics. These information bits can be modified by using simple logic gates such as AND and OR gates. The binary bits are physically created by a high or a low signal level occurring at the output of the logic gate to represent either a logical one (e.g., high voltage) or a logical zero (e.g., low voltage). A classical algorithm, such as one that multiplies two integers, can be decomposed into a long string of these simple logic gates. Like a classical computer, a quantum computer also has bits and gates. Instead of using logical ones and zeroes, a quantum bit ("qubit") uses quantum mechanics to occupy both possibilities simultaneously. This ability and other uniquely quantum mechanical features enable a quantum computer can solve certain problems exponentially faster than that of a classical computer.

SUMMARY OF THE INVENTION

In accordance with an aspect of the present invention, a quantum circuit assembly includes a first flux qubit, having at least two potential energy minima, and a second flux qubit, having at least two potential energy minima. A system formed by the first qubit and the second qubit has at least four potential energy minima prior to coupling, each of the four potential energy minima containing at least one eigenstate of a system comprising the first flux qubit and the second flux qubit. A coupler creates a first tunneling path between a first potential energy minimum of the system and a second potential energy minimum of the system, and a second tunneling path between a third potential energy minimum of the system and a fourth potential energy minimum of the system. The coupler creates the first and second tunneling paths between potential energy minima representing states of equal bit parity, such that the first potential energy minimum represents the state $|01\rangle$, the second potential energy minimum represents the state, $|10\rangle$, the third potential energy minimum represents the state $|00\rangle$, and the fourth potential energy minimum represents the state, $|11\rangle$.

In accordance with another aspect of the present invention, a method is provided for coupling quantum states of two flux qubits. A first flux qubit is electrically coupled to a second flux qubit via a coupler comprising at least one tunable Josephson junctions to create a first tunneling path, between a first pair of potential energy minima associated with a system formed by the first and second qubit, and a second tunneling path, between a second pair of potential energy minima associated with the system. A control signal is applied to the at least one tunable junction to tune one of

a first tunneling energy associated with the first tunneling path and a second tunneling energy associated with the second tunneling path.

In accordance with yet another aspect of the present invention, a quantum circuit assembly includes a first flux qubit, having at least two potential energy minima and a second flux qubit, having at least two potential energy minima. A system formed by the first qubit and the second qubit has at least four potential energy minima prior to coupling, each of the four potential energy minima containing a quantum state of a system comprising the first flux qubit and the second flux qubit. A coupler, comprising a plurality of tunable Josephson junctions, creates a first tunneling path between a first potential energy minimum of the system and a second potential energy minimum of the system, and a second tunneling path between a third potential energy minimum of the system and a fourth potential energy minimum of the system. The coupler is tunable via a control signal applied to at least one of the plurality of tunable junctions to tune a first tunneling energy associated with the first tunneling path and a second tunneling energy associated with the second tunneling path.

BRIEF DESCRIPTION OF THE DRAWINGS

FIG. **1** illustrates one example of system comprising two coupled flux qubits;

FIG. **2** illustrates a high level schematic of a quantum circuit for generating an XX interaction;

FIG. **3** is one example of a quantum circuit for generating an XX interaction between two flux qubits;

FIG. **4** is an energy diagram representing the circuit of FIG. **3** in a phase basis;

FIG. **5** is another example of a quantum circuit for generating an XX interaction between two flux qubits;

FIG. **6** is still another example of a quantum circuit for generating an XX interaction between two flux qubits; and

FIG. **7** illustrates one example of a method for coupling quantum states of two flux qubits.

DETAILED DESCRIPTION

Systems and methods are providing XX coupling between two flux qubits. An XX coupling between two qubits makes it energetically favorable for the states of the first and second qubits to align in the same direction along the X-axis, both pointing either in the +X direction or both in the −X direction. Each axis corresponds to a specific quantum state defined on the Bloch sphere of the qubit. XX interactions have multiple uses including generating non-stoquastic Hamiltonians, generally in conjunction with ZZ and Z Hamiltonian terms that are used on quantum annealing machines, various quantum logic gates, such as those described in co-pending application Ser. No. 15/225,102 filed Aug. 1, 2016 and titled "Quantum Gates Via Multi-Step Adiabatic Drag", which is hereby incorporated by reference, and various passive noise suppression schemes, such as those described in co-pending application Ser. No. 15/225,210 filed Aug. 1, 2016 and titled "Quantum Operations with Passive Noise Suppression", which is hereby incorporated by reference.

FIG. **1** illustrates one example of system **10** comprising two coupled flux qubits. The system includes a first flux qubit **12** and a second flux qubit **14** operatively coupled to the first flux qubit via an XX coupler **16**. A flux qubit, in general terms, is a superconducting loop interrupted by some number of Josephson junctions. While a biasing element is not illustrated in the simplified example of FIG. **1**, in general operation, a flux qubit is biased by a flux in units of the superconducting flux quantum Φ_0. When the applied bias flux is near one-half of a flux quantum and for suitable device parameters, the potential energy of the system exhibits two minima, one corresponding to clockwise and the other to counterclockwise current flow in the superconducting loop. The two possible directions of current flow represent the lowest energy quantum states of the system. While it is also possible to have a single potential well even at half a flux quantum of bias flux, the double-well regime described here highlights the unique capability of the inventive coupler to function even with energetically degenerate states.

A quantum system comprising the two flux qubits **12** and **14** has four energy minima, assuming both qubits are biased appropriately. Using $|0\rangle$ to refer to a first direction (e.g., clockwise) of current flow and $|1\rangle$ to apply to a second direction (e.g., counter-clockwise) in the standard basis, the four states representing the energy minima are $|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$. The energy minima are separated by potential barriers, such that a transition from one minima to another generally requires, in the absence of quantum tunneling, application of energy to the system to bring one or both qubits into an excited state and then allow the excited qubit or qubits to fall back into one of the energy minima.

A quantum circuit can be designed such that there is a non-zero probability that the state of a given qubit can change without the application of energy. In general, the Josephson junctions in a flux qubit loop create a potential with two or more minima and a barrier through which the multi-dimensional phase wave-function can tunnel. In accordance with an aspect of the present invention, the XX coupler **16** creates a plurality of tunneling paths between the potential minima associated with the multiple states of the first and second flux qubits **12** and **14**, such that a tunneling path between pairs of ground states having equal bit parity are created. In other words, the XX coupler **16** allows the system formed by the two qubits to tunnel between the states $|00\rangle$ and $|11\rangle$ as well as between the states $|01\rangle$ and $|10\rangle$. Effectively, a first tunneling path creates a first interaction $g_1(|01\rangle\langle 10|+|10\rangle\langle 01|)$, where g_1 is the strength, or tunneling energy of the first interaction, and a second tunneling path creates a second interaction $g_2(|00\rangle\langle 11|+|11\rangle\langle 00|)$, where g_2 is the strength of the second interaction. The interaction strength, g_i, for a given tunneling path depends on the height of the tunneling barrier between the two states is and is equal to half the energy splitting between the ground states and excited states of the coupling term. The sum of the two interactions is the XX interaction as written in the standard, or Z, basis.

An advantage of the proposed XX coupler **16** is that it can provide an XX interaction without coupling the qubits along other axes of the Bloch sphere or introducing single qubit effects, such as single qubit tunneling. When the coupler Josephson junctions have slightly different critical currents due to fabrication variation, the coupler can produce an interaction $g_{XX}XX+g_{YY}YY+g_{ZZ}ZZ$, where the signs of g_{YY} and g_{ZZ} can be positive or negative depending on the relative values of the coupler junctions' critical currents. The magnitudes of g_{YY} and g_{ZZ} can be tuned to zero by replacing one or more junctions with tunable junctions, such as compound junctions. For example, where compound junctions are used, and the coupling strengths can be tuned by adjusting the flux in the compound junction loops. If the junction variation is small, only a single tunable junction may be needed to tune g_{YY} and g_{ZZ} to zero. For large junction variations, multiple junctions may be replaced with tunable junctions to tune g_{YY} and g_{ZZ} to zero. This also allows the XX coupling strength to be adjusted and even set to zero if desired. Where a pure ZZ coupling is desired, the tunneling barriers can be raised using a first set of control fluxes, thereby shutting off all tunneling between potential minima, and both 00 and 11 minima can be raised or lowered in energy relative to the 01 and 10 minima using a second set of control fluxes. Further, the proposed coupler can be used for qubits having degenerate energy states, that is, energy states having the same energy. Flux qubits are a common example of a qubit that can be operated with degenerate ground states. The inventors have found that, given current fabrication techniques, coupling strengths as high as two gigahertz between two flux qubits can be achieved via the proposed coupler.

FIG. **2** illustrates a high level schematic of a quantum circuit **30** for generating an XX interaction. It will be appreciated that, in contrast to the more specific examples of FIGS. **3**, **5**, and **6**, the illustrated circuit is provided at a conceptual level to better explain the concepts involved. The circuit **30** includes a first flux qubit **32** and a second flux qubit **34** joined by an XX coupler **40**. The coupler **40** is configured to create a first tunneling path **42** and a second tunneling path **44** between potential energy minima, representing quantum states of the system including by the two flux qubits **32** and **34**. A first control mechanism **52** creates one or more control fluxes that change the tunneling strength along the first tunneling path **42** and the absolute energies of states coupled via the first tunneling path. A second control mechanism **54** creates one or more control fluxes that change the tunneling strength along the second tunneling path **44** and the absolute energies of states coupled via the second tunneling path. A third control mechanism **56** provides voltages signals that set offset charge values on specific nodes of the quantum circuit, comprising both flux qubits and coupler. The control of offset charge enables the sign of the XX coupling to be adjusted to either positive or negative.

FIG. **3** is one example of a quantum circuit **70** for generating an XX interaction between two flux qubits. In the illustrated implementation, the two flux qubits are not tunable, and are integrated at least partially into the coupler assembly itself, and the circuit **70** can be conceptualized as a single assembly with, for suitable circuit parameters, doubly degenerate ground states

$$\frac{|00\rangle+|11\rangle}{\sqrt{2}} \text{ and } \frac{|01\rangle+|10\rangle}{\sqrt{2}}.$$

Here, the state (0 or 1) of a first flux qubit represents the direction of the current passing through first and second Josephson junctions **72** and **73**, and a state (0 or 1) of a second flux qubit represents the direction of the current passing through third and fourth Josephson junctions **74** and **75**. While, as described above, the flux qubits are integral with the coupler, the coupler can be considered to include fifth, sixth, seventh, and eighth Josephson junctions **76-79** as well as a capacitor **80**. It should be noted that any number of junctions could be replaced with a tunable junction, such as a flux-tunable compound junction. Incorporating two tunable junctions is sufficient for a high purity XX interaction in the presence of moderate junction asymmetry. The circuit of FIG. **3** can also be viewed as instance of the circuit in FIG. **6** where the two junctions, **214** and **224**, have been replaced by compound junctions and tuned to nearly zero

Josephson energy. In this case the two junctions, **214** and **224**, can be omitted from the circuit, producing the simplified circuit if FIG. **3**.

Each Josephson junction **72-79** as well as the capacitor **80** has a superconducting phase, δ_i, across the component. For the purpose of example, each of the first and second Josephson junctions **72** and **73** will be assumed to have a same superconducting phase of δ_1, each of the third and fourth Josephson junctions **74** and **75** will be assumed to have a same superconducting phase of δ_2. Given this assumption, a potential, U_C, due to the coupler can be written as:

$$U_C=-E_5\cos(\delta_1+\delta_2-\delta_3)-E_6\cos(\delta_1+\delta_2+\delta_3-2\pi f_1)-E_7\cos(-\delta_1+\delta_2+\delta_3+2\pi f_2)-E_8\cos(\delta_1-\delta_2+\delta_3+2\pi f_3) \quad \text{Eq. 1}$$

where E_5 is a Josephson energy of the fifth Josephson junction **76**, E_6 is a Josephson energy of the sixth Josephson junction **77**, E_7 is a Josephson energy of the seventh Josephson junction **78**, E_8 is a Josephson energy of the eighth Josephson junction **79**, δ_3 is a superconductive phase across the capacitor **80**, f_1 is the flux, in flux quanta, through the loop of the assembly containing junctions **72** through **77**, f_2 is the flux, in flux quanta, through the loop of the assembly containing the junctions **72**, **73**, **77**, and **78**, and f_3 is the flux, in flux quanta, through the loop of the assembly containing junctions **74**, **75**, **77** and **79**.

For $E_5=E_6=E_7=E_8=E$, $f_1=0$, and $f_2=f_3=0.5$, the potential can be rewritten as:

$$U_C=4E\sin(\delta_1)\sin(\delta_2)\cos(\delta_3) \quad \text{Eq. 2}$$

This potential has the desirable property that for $\delta_3=0$, energy is positive along the line $\delta_1=\delta_2$ and negative along the line $\delta_1=-\delta_2$. At $\delta_3=\pi$, the opposite is true—energy is negative along the line $\delta_1=\delta_2$ and positive along the line $\delta_1=-\delta_2$. Now, two uncoupled flux qubits, having Josephson junctions with phase variables δ_1 and δ_2, respectively, will have a potential with four minima at $[\pm\delta_0, \pm\delta_0]$, where $|\delta_0|$ is the magnitude of the phase at the minima of the double well potential for each flux qubit. Introducing the coupler raises the energy of the two states $\pm[\delta_0, \delta_0]$ relative to the two states $[\pm\delta_0, \mp\delta_0]$ at $\delta_3=0$ and vice versa at $\delta_3=\pi$. Accordingly, pairs of minima are located in separate planes of constant δ_3.

FIG. **4** is an energy diagram representing the circuit of FIG. **3** in terms of the superconducting phases δ_1, δ_2, and δ_3. In the energy diagram, the grey shapes represent surfaces having a same energy, and the smaller solid black shapes represent the four minima of the potential. The four minima are labeled based on the sign of the phases of δ_1 and δ_2 at the minima, which determine the direction of current flow in the qubits. A first view **110** of the energy diagram depicts a projection of the energy diagram into the δ_2-δ_3 plane. A horizontal axis **112** represents a value for the superconducting phase, δ_2, of the Josephson junctions **74** and **75** associated with the second qubit, in radians divided by 2π and a vertical axis **114** represents a value for the superconducting phase, δ_3, of the capacitor **80** associated, in radians divided by 2π. It will be appreciated that two of the minima, $|01\rangle$ and $|10\rangle$, are located on the plane $\delta_3=0$, and the other two minima, $|00\rangle$ and $|11\rangle$, are located on the plane corresponding to $\delta_3/2\pi=0.5$, or more simply, $\delta_3=\pi$.

A second view **120** of the energy diagram depicts a projection of the energy diagram into the δ_1-δ_2 plane. A horizontal axis **122** represents a value for the superconducting phase, δ_1, of the Josephson junctions **72** and **73** associated with the first qubit, in radians divided by 2π and a vertical axis **124** represents a value for the superconducting phase, δ_2, of the Josephson junctions **74** and **75** associated with the second qubit, in radians divided by 2π. It will be appreciated that two of the minima, $|01\rangle$ and $|10\rangle$, are located on the plane $\delta_1=-\delta_2$, and the other two minima, $|00\rangle$ and $|11\rangle$ are located on the plane, $\delta_1=\delta_2$. A third view **130** of the energy diagram depicts a perspective view. As with the other views **110** and **120**, a first axis **132** represents a value for the superconducting phase, δ_1, of the Josephson junctions **72** and **73** associated with the first qubit, in radians divided by 2π, a second axis **134** represents a value for the superconducting phase, δ_2, of the Josephson junctions **74** and **75** associated with the second qubit, in radians divided by 2π, and a third axis **136** represents a value for the superconducting phase, δ_3, of the capacitor **80** associated, in radians divided by 2π.

As can be seen from the diagram, if the wave function spread is large in the δ_1-δ_2 plane, to enable tunnel-coupling, but small in the δ_3 direction, the desired ground states, $(|00\rangle+|11\rangle)/\sqrt{2}$ and $(|01\rangle+|10\rangle)/\sqrt{2}$, will form. The strength of the −XX interaction is given by the strength of the tunnel coupling between potential minima. When there are multiple tunneling paths from one minima to another, it is possible for offset charges to affect the tunneling energy due to interference from the Aharonov-Casher effect. Further, it will be appreciated that a capacitance of the capacitor **80** can be selected to decrease the wave-function spread in one direction. This decrease in the spread of the wave function can decouple the two sets of minima, allowing for the coupler to achieve the two aforementioned ground states. It will be appreciated that functional coupler can also be constructed where one or more of junctions **72-75** in FIG. **3** are replaced with inductors of suitable value. It will also be appreciated that, by adding suitable inductors, this circuit can couple two flux qubits via mutual inductance such that the qubits and couplers are electrically isolated from each other.

FIG. **5** is another example of a quantum circuit **150**, comprising a plurality of Joesphson junctions **151-156**, for generating an XX interaction between two flux qubits. As in FIG. **3**, the two flux qubits are not tunable, and are integrated into the coupler assembly itself. In general terms, however, a state of a first flux qubit is represented by the direction of the current passing through junction **153**, and a state of a second flux qubit is represented by the direction of the current passing through junction **155**. While the flux qubits are integral with the coupler, the coupler can be considered to include first and second Josephson junctions **151** and **152**. It will be appreciated, however, that any number of junctions could be replaced with a compound Josephson junction or other element having a tunable Josephson energy. Incorporating tunable junctions is sufficient for a high purity XX interaction in the presence of junction asymmetry, for example, due to minor variances in the fabrication process.

FIG. **6** is yet another example of a quantum circuit **200** for generating an XX interaction between two flux qubits **210** and **220**. In the illustrated implementation, each flux qubit **210** and **220** is tunable via an applied flux, such that either or both of a relative energy of the energy levels of the qubit and a barrier height between the energy states can be tuned. A first flux qubit **210** comprises three Josephson junctions **212-214** arranged in a loop enclosing nominally one half of a flux quantum. A second flux qubit **220** comprises three Josephson junctions **222-224** arranged in a loop enclosing nominally one half of a flux quantum.

A coupler **230** comprises a first Josephson junction **232** connected to each of the first reference node **216** and the fourth reference node **226**, and a second Josephson junction **233** connected to each of the second reference node **217** and the fifth reference node **227**. A third Josephson junction **234** is connected to each of the first reference node **216** and the fifth reference node **227**, and a fourth Josephson junction **235** is connected to each of the second reference node **217** and the fourth reference node **226**, such that the coupler forms a "twisted loop" comprising the four Josephson junctions. A capacitor **238** is connected to each of the third reference node **218** and the sixth reference node **228**.

It will be appreciated that the Josephson energy of a Josephson junction is generally static. In one implementation, one or more of the Josephson junctions **232-235** comprising the coupler **230** can be replaced with a tunable element having a Josephson energy that is tunable via an applied flux or other control signal. One example of such an element is a compound Josephson junction. In practice, at least one tunable junction is advisable to correct for variance in the fabrication process even under the best of circumstance, and in practice, two tunable junctions can be used for this purpose. In one implementation, all of the Josephson junction **232-235** can be made tunable such that the tunneling energies of the tunneling paths created by the coupler can be tuned to alter or eliminate the coupling provided by the device. For example, the tunneling energies can be reduced to near zero to eliminate the XX coupling or made unequal to add an element of ZZ coupling. In another implementation, the capacitor **238** can be omitted and offset charges, controlled by gate voltages, can be used to suppress undesired tunneling and control the sign of the coupling. This is possible via Aharonov-Casher interference, whereby offset charge on a superconducting island in the circuit induces a phase difference between two tunneling paths from on minimum to another. When the offset charge is 0.5 Cooper pairs, the interference is destructive and tunneling does not occur. When the offset charge is between 0.5 and 1 Cooper pair, the tunneling energy can be negative leading to an anti-symmetric ground state as is the case for a positive XX coupling.

In view of the foregoing structural and functional features described above in FIGS. **1-6**, example methods will be better appreciated with reference to FIG. **7**. While, for purposes of simplicity of explanation, the method of FIG. **7** is shown and described as executing serially, it is to be understood and appreciated that the present invention is not limited by the illustrated order, as some actions could in other examples occur in different orders and/or concurrently from that shown and described herein.

FIG. **7** illustrates one example of a method **300** for coupling quantum states of two flux qubits. At **302**, a first flux qubit is electrically coupled to a second flux qubit via a coupler comprising a plurality of Josephson junctions, with at least one being tunable, in addition to capacitive and/or mutual inductive and/or galvanic interactions. The coupler creates a first tunneling path, between a first pair of energy minima associated with a system formed by the first and second qubit, and a second tunneling path, between a second pair of energy minima associated with the system. In one implementation, tunneling paths are formed between minima representing states of the system having equal bit parity, that is, between the states $|00\rangle$ and $|11\rangle$ and between the states $|01\rangle$ and $|10\rangle$.

At **304**, a control signal, such as current, producing flux or voltage, is applied to at least one of the one or more tunable junctions to select a first tunneling energy associated with the first tunneling path and a second tunneling energy associated with the second tunneling path. In accordance with an aspect of the present invention, the selection of the tunneling energies via the applied signal can control the coupling behavior of the coupler. For example, if the control signal is applied such that the first and second coupling energies are substantially equal, an XX coupling between the first flux qubit and the second flux qubit is produced. To maintain a pure XX coupling, one or more other tunable Josephson junctions may be adjusted with a control signal to ensure that single qubit tunneling effects and YY and ZZ couplings are avoided. In another example, the control signal can be applied such that the first and second coupling energies are not equal to provide an XX coupling, a YY coupling and/or a ZZ coupling between the first flux qubit and the second flux qubit. Finally, the control signal can be applied such that the first and second coupling energies are substantially equal to zero as to selectively decouple the first flux qubit and the second flux qubit. Accordingly, the coupling provided by the coupler can be controlled for quantum logic gate operations and other applications.

What have been described above are examples of the present invention. It is, of course, not possible to describe every conceivable combination of components or methodologies for purposes of describing the present invention, but one of ordinary skill in the art will recognize that many further combinations and permutations of the present invention are possible. Accordingly, the present invention is intended to embrace all such alterations, modifications, and variations that fall within the scope of the appended claims.

What is claimed is:

1. A quantum circuit assembly comprising:

a first flux qubit, having at least two potential energy minima;

a second flux qubit, having at least two potential energy minima, such that a system formed by the first qubit and the second qubit has at least four potential energy minima prior to coupling, each of the four potential energy minima containing at least one eigenstate of a system comprising the first flux qubit and the second flux qubit; and

a coupler that creates a first tunneling path between a first potential energy minimum of the system and a second potential energy minimum of the system, and a second tunneling path between a third potential energy minimum of the system and a fourth potential energy minimum of the system, the coupler creating the first and second tunneling paths between potential energy minima representing states of equal bit parity, such that the first potential energy minimum represents the state $|01\rangle$, the second potential energy minimum represents the state, $|10\rangle$, the third potential energy minimum represents the state $|00\rangle$, and the fourth potential energy minimum represents the state, $|11\rangle$.

2. The quantum circuit assembly of claim **1**, wherein the coupler creates the first and second tunneling paths such that a first tunneling energy between the first and second potential energy minima is equal to a second tunneling energy between the third and fourth potential energy minima.

3. A quantum circuit assembly comprising:

a first flux qubit, having at least two potential energy minima;

a second flux qubit, having at least two potential energy minima, such that a system formed by the first qubit and the second qubit has at least four potential energy minima prior to coupling, each of the four potential energy minima containing at least one eigenstate of a system comprising the first flux qubit and the second flux qubit; and

a coupler that creates a first tunneling path between a first potential energy minimum of the system and a second potential energy minimum of the system, and a second tunneling path between a third potential energy minimum of the system and a fourth potential energy minimum of the system, the coupler creating the first and second tunneling paths between potential energy minima representing states of equal bit parity, such that the first potential energy minimum represents the state $|01\rangle$, the second potential energy minimum represents the state, $|10\rangle$, the third potential energy minimum represents the state $|00\rangle$, and the fourth potential energy minimum represents the state, $|11\rangle$, the coupler comprising:

a plurality of Josephson junctions; and

a capacitor, operatively connected to each of the first flux qubit and the second flux qubit, that decreases a spread of a wave function of the system to decouple the first potential energy minimum of the system and the second potential energy minimum of the system from the third potential energy minimum of the system and the fourth potential energy minimum of the system.

4. The quantum circuit assembly of claim **3**, wherein, in a phase basis defined by a first superconducting phase, δ_1, associated with the first flux qubit, a second superconducting phase, δ_2, associated with the second flux qubit, and a third superconducting phase, δ_3, associated with capacitor, the coupler introduces a potential energy to the system that is positive along a line $\delta_1=\delta_2$ and negative along a line $\delta_1=-\delta_2$ in a plane $\delta_3=0$, and negative along the line $\delta_1=\delta_2$ and positive along the line $\delta_1=-\delta_2$ in a plane $\delta_3=\pi$.

5. The quantum circuit assembly of claim **3**, wherein the coupler comprises a first Josephson junction connected to each of a first reference node associated with the first flux qubit and a second reference node associated with the second flux qubit, a second Josephson junction connected to each of a third reference node associated with the first flux qubit and a fourth reference node associated with the second flux qubit, a third Josephson junction connected to each of the first reference node and the fourth reference node, and a fourth Josephson junction connected to each of the third reference node and the second reference node.

6. The quantum circuit assembly of claim **3**, wherein at least one of the plurality of Josephson junctions comprising the coupler is implemented as part of a tunable element, such that a Josephson energy of the Josephson junction can be tuned via an applied flux.

7. The quantum circuit assembly of claim **6**, wherein the tunable element is a compound Josephson junction.

8. The quantum circuit assembly of claim **6**, wherein each of the plurality of Josephson junctions comprising the coupler is implemented as part of a tunable element, such that the coupler can be selectively activated via the applied flux.

9. The quantum circuit assembly of claim **6**, wherein the Josephson energies of the Josephson junctions are tuned such that a tunneling barrier within each qubit is sufficiently large to prevent single qubit tunneling effects.

10. A quantum circuit assembly comprising:

a first flux qubit, having at least two potential energy minima;

a second flux qubit, having at least two potential energy minima, such that a system formed by the first qubit and the second qubit has at least four potential energy minima prior to coupling, each of the four potential energy minima containing at least one eigenstate of a system comprising the first flux qubit and the second flux qubit; and

a coupler that creates a first tunneling path between a first potential energy minimum of the system and a second potential energy minimum of the system, and a second tunneling path between a third potential energy minimum of the system and a fourth potential energy minimum of the system, the coupler creating the first and second tunneling paths between potential energy minima representing states of equal bit parity, such that the first potential energy minimum represents the state $|01\rangle$, the second potential energy minimum represents the state, $|10\rangle$, the third potential energy minimum represents the state $|00\rangle$, and the fourth potential energy minimum represents the state, $|11\rangle$, the coupler being configured such that the first tunneling path is the only tunneling path associated with either of the first and second potential energy minima and the second tunneling path is the only tunneling path associated with either of the third and fourth potential energy minima.

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